Graph-Regularized Generalized Low Rank Models

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Properties of Images

- High Dimensionality
Properties of Images

- High Dimensionality

- Noise and Occlusions
Noise and Occlusions
Properties of Images

- High Dimensionality

- Noise and Occlusions

- Graph Structure
Graph Structure
Previous Work

- Generalized Low Rank Models (GLRM)

- Spectral Embedding
Low Rank Models

- Approximate a data matrix as the product of two low-rank factors

\[ Y \approx X' W \]

("Narrow" factor)  
("Wide" factor)
Low Rank Models

- Objective Function:

\[
\min_{X,W} \sum_{(i,j) \in \Omega} l_j(Y_{ij}, x_i^T w_j) + r(X) + \tilde{r}(W)
\]
Low Rank Models

- Using squared error recovers truncated SVD (PCA if centered and scaled)

\[
\underset{X, Y}{\text{argmin}} \| A - X^T Y \|_F^2
\]
Spectral Embedding

- Maximizes similarity along a graph using Laplacian matrix $L$
Spectral Embedding - Laplacian Matrix

(Path Graph)

\[
\begin{vmatrix}
1 & -1 \\
-1 & 2 & -1 \\
-1 & 2 & -1 \\
-1 & 1 \\
\end{vmatrix}
\]
Spectral Embedding - Laplacian Matrix

(Complete Graph)
Spectral Embedding

- Maximizes similarity along a graph using Laplacian matrix $L$

$$\min_X \text{tr}(X L X^T) \quad s.t. \quad X X^T = I.$$
Graph GLRM

- Objective Function:

\[
\min_{X, W} \sum_{(i,j) \in \Omega} l_j(Y_{ij}, x_i^T w_j) + \alpha \text{tr}(X L X^T) + \tilde{r}(W)
\]
Fitting

GraphGLRM uses modified Proximal Alternating Linearized Minimization (PALM) to fit factors; it takes two alternating proximal gradient steps per iteration, one per factor.
Software Implementation

- Implementation in Julia language available at

https://github.com/mihirparadkar/GraphGLRM.jl
Software Implementation

- User specifies the data table, loss function, regularizers on factors, and rank, along with optional parameters like a list of indices of known values

\[
gm = \text{GGLRM}(A_{\text{missing}}, \text{loss}, r_x, r_y, k, \text{obs}=\text{obs})
\]
Experiments

- Imputation of block occlusions

- Classification of faces into male/female with occluded images
# Results - Classification Experiment

<table>
<thead>
<tr>
<th>Embedding Method</th>
<th>Precision</th>
<th>Recall</th>
<th>F1-Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>None</td>
<td>0.727</td>
<td>0.4</td>
<td>0.516</td>
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<tr>
<td>PCA</td>
<td>0.381</td>
<td>0.4</td>
<td>0.390</td>
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<tr>
<td>Spectral Embedding</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Vanilla GLRM</td>
<td>0.714</td>
<td>0.25</td>
<td>0.370</td>
</tr>
<tr>
<td>Graph GLRM</td>
<td>1</td>
<td>0.5</td>
<td>0.667</td>
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</tbody>
</table>
# Results - Imputation Experiment

<table>
<thead>
<tr>
<th>Method</th>
<th>MSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>PCA</td>
<td>15032</td>
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<tr>
<td>Spectral Embedding</td>
<td>3415.4</td>
</tr>
<tr>
<td>Vanilla GLRM</td>
<td>634.63</td>
</tr>
<tr>
<td><strong>Graph GLRM</strong></td>
<td><strong>554.48</strong></td>
</tr>
</tbody>
</table>
Results - Imputation Experiment
Results - Imputation Experiment

![Error on Imputation Experiment Chart]

- PCA
- Spectral Embedding
- Vanilla GLRM
- Graph GLRM

Mean-Squared Error
Results - Imputation Experiment
Conclusions

- Combines linear embedding of GLRM with non-linear dimensionality reduction through graph Laplacian

- Freely-available and performant software implementation for fitting

- Can improve performance in reconstructing missing values and in classification with noisy data
Thank You!