A Cognitive Assessment of Topological Spatial Relations: Results from an Empirical Investigation

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Abstract

Whether or not a formal approach to spatial relations is a cognitively adequate (the term will be explicated in this paper) model of human spatial knowledge is more often based on the intuition of the researchers than on empirical data. In contrast, the research reported here is concerned with an empirical assessment of one of the three general classes of spatial relations, namely topological knowledge. In the reported empirical investigation, subjects had to group numerous spatial configurations consisting of two circles with respect to their similarity. As is well known, such tasks are solved on the basis of underlying spatial concepts. The results were compared with the *RCC-theory* and *Egenhofer's approach* to topological relations and support the assumption that both theories are cognitively adequate in a number of important aspects.

Introduction

A large number of spatial relations we consider in everyday contexts are topological and new empirical results from a series of experiments have shown that topological relations are an important part of human spatial knowledge (Knauff, 1997; Knauff, Rauh, Schlieder & Strube, in press). However, there is certainly room for disagreement when asking which kind of topological information is used if people conceptualize spatial arrangements. Whereas psychological research does not pay much attention to this question, in several fields of computer science, in particular in the so called qualitative spatial reasoning (QSR) community and in geographical information systems (GIS), a lot of systems of topological relations have been developed. This is due to the fact that this kind of information is easily accessible and remains invariant under different transformations, such as translation, rotation, or scaling. Two of the most important approaches in these research areas are given by the *RCC-theory* of Randell et al. (Randell, Cohn & Cui, 1992; Randell, Cui & Cohn, 1992) and the very similar work of Egenhofer et al. (Egenhofer, 1991; Egenhofer & Franzosa, 1991; Egenhofer & Herring 1994). Both sets of basic topological relations are equivalent in fact, but have been defined in two quite different ways: the RCC-theory is an axiomatization using the C(x,y) relation ("connects"), whereas Egenhofer's work is based on a set theoretical characterization and topological constraints (see below).

From a psychological point of view such approaches are an empirical challenge. The question is whether or not they are useful models of human spatial cognition as well. Some of the questions are: Does it make sense to postulate similar representations of topological spatial concepts in our mind? Which kind of topological information is used if people conceptualize spatial arrangements? How are these concepts used in spatial reasoning tasks, and are the cognitive processes comparable to the formal reasoning mechanism?

In recent years such questions in the QSR-community have been discussed under the keyword "cognitive adequacy", although there is no agreement on what this term exactly means. Secondly, the question of whether or not a formal approach to spatial relations is a cognitively adequate model of human spatial knowledge is more often based only on the intuition of the researchers than on empirical data.

Firstly, we will give a little more insight into the important characteristics of the RCC-theory and Egenhofer's approach. Then, we will try to give a specialization of what the cognitive adequacy of spatial relational systems should be. Finally, we report our empirical research on the cognitive adequacy of topological spatial relations and discuss the results with respect to its psychological and computational consequences.

What does "cognitive adequacy" mean?

In a strong sense cognitive adequacy must at least mean that something is a model of human cognition. Strube (1992) proposed *to range* it from an absolutely strong (idealized) meaning down to a very weak notion of conforming to well known ergonomic standards. The first case means that a formal approach (or implementation) is claimed to be an adequate model of human knowledge and reasoning mechanisms in all or some relevant aspects. In the second case the claim is reduced to the assumption that an implementation of a formal approach can be characterized as ergonomic and user-friendly (Strube, 1992).

Orthogonal to this view, we argued in a previous paper that another distinction is very important as well. To our mind, it is often neglected that cognitive adequacy can be claimed for the basic (topological, ordinal or metrical) relations or for the reasoning mechanism. Therefore, it is essential to differentiate between two kinds of cognitive adequacy, namely *conceptual* and *inferential cognitive adequacy* (Knauff, Rauh & Schlieder, 1995).

Inferential cognitive adequacy in our meaning can be claimed if and only if the reasoning mechanism of the calculus is structurally similar to the way people reason about space. Independently from this, ***conceptual cognitive adequacy*** can be claimed if and only if empirical evidence supports the assumption that a system of relations is a model of people's conceptual knowledge of spatial relationships.

For the moment it is sufficient to understand, first, that the notion of cognitive adequacy is of course an idealization and the question of whether or not an approach can be characterized as strongly adequate is important in particular from a psychological and/or cognitive science point of view. On the other hand weak adequacy can be seen as sufficient for numerous subfields of computer science and empirical results can be fed back into the development of new formal models. Second, the distinction between inferential and conceptual adequacy is associated with different psychological perspectives. Empirical investigations on inferential adequacy are very close to the part of general psychology that is traditionally called the psychology of thinking, whereas conceptual adequacy overlaps a number of research topics in the psychology of knowledge.

One of the main advantages of using formal models such as the RCC-theory or Egenhofer's approach in psychology is that they can be used to define stimulus material that distinguishes all possible situations with respect to the topological aspects. In this way, the results may support the assumption that people use similar information when they conceptualize spatial arrangements or the outcomes reject a formal approach as a cognitive inadequate model of human spatial cognition. Last but not least, the conceptual/inferential distinction is also important for weak (ergonomic) adequacy and for the development of user-friendly software systems, because the reasons for ergonomic disadvantages can be traced back to users' problems with both parts of the systems, e.g. the knowledge representation or the reasoning algorithms. However, it is important to keep in mind that we only focus on the aspect of conceptual adequacy in this paper.

Psychological background and previous work

As already mentioned in the introduction, psychological research has not paid much attention to the mental representation of topological knowledge. One reason for this may be that the formal distinction between topological, ordering, and metric spatial relations in psychology is not a matter of course. In general, psychological research is concerned with spatial relations which have a direct equivalent in natural language and most research does not distinguish between linguistic representations and knowledge structures. For this reason it is not surprising that psychological research focuses in particular on the semantics of prepositions like *left, right, above, below, before, behind, contains* and so on, and at first glance not on very artificial concepts like the RCC-relations. However, the first author of this paper was able to show the causal power of topological knowledge in a number of "language-free" memory experiments and grouping tasks (Knauff, 1997). The starting point of the empirical investigations was very similar to the research reported in the present paper.

While we focus on topological knowledge here, our previous research was concerned with the conceptual adequacy of the interval relations introduced by Allen (1983), which were soon transferred to the spatial domain. In the experiments subjects had to recognize or recall previously learned spatial arrangements that were defined with respect to Allen's interval relations. In the recognition experiment subjects got instances of other relations as distractors, whereas in the recall experiment they had to generate (draw) the previously learned spatial relations after getting a cue-stimulus. Dependent measures were false alarms, reaction times and incorrectly generated arrangements. Taken together, the most important result of these experiments was that subjects remember the topological aspects of the learned arrangements very well, whereas significantly more mistakes are made according to the ordering information (Knauff, 1997; for a brief account see: Knauff et al., in press)[1].

In our opinion the results support the assumption that topological spatial concepts are represented in our mind and have a causal power in a number of cognitive processes, even if there is no direct translation into natural language expressions. Taking these results, together with findings of other research groups (for example the empirical work of Mark et al., 1995), it seemed to be a promising research issue to investigate the conceptual representation of spatial knowledge and look firstly at the usability or "conceptual adequacy" of well known formal models of topological relations, such as the RCC-theory and Egenhofer's approach.

1. For readers who know Allen's interval relations, here a short example: when subjects had learned the relation "s" ("starts"), more than 20% of the mistakes (false alarms) were due to the relation "f" ("finishes"), if the relation "o" (overlaps from the left") was learned nearly the same relative frequency of mistakes were made on the "oi" relation ("overlaps from the right") and so on.

Topological relations in spatial information theory

In recent years two different approaches dealing with topological information were developed independently. One of them was proposed by Egenhofer (1991) in the area of geographical information systems, the other by Randell, Cui, and Cohn (1992), working in the area of qualitative spatial reasoning. Both approaches describe binary topological relationships between spatial regions, where spatial regions are subsets of the topological space, i.e. we do not distinguish between objects and the space they occupy.

Egenhofer (1991) defines topological relations using the interior, the boundary, and the complement of point sets. Given two point sets X and Y, relations between them are expressed using the 9 possible intersections of the interior, boundary, and complement of X with the interior, boundary, and complement of Y, which is therefore called "9-intersection". When each of the intersections is only viewed as either empty or non-empty, 2^9 relations are possible. Provided that only regular closed non-empty point sets of the same dimension are used with the additional topological constraints that the interior, the boundary, and the complement of each set is connected, only eight different relations are realizable (Egenhofer 1991).

These relations include *"disjoint", "meet", "equal", "inside", "coveredBy", "contains", "covers",* and *"overlap"*. They are *jointly exhaustive* and *pairwise disjoint* (JEPD), i.e. exactly one relation holds between any two regions.

Two dimensional examples of these relations are given in Figure 1. When the restrictions on regions and intersections are lowered, much more relations can be defined, among other things relations between sets of different dimensions (Egenhofer & Herring, 1994). In the *Region Connection Calculus* (RCC) of Randell, Cui & Cohn (1992), relations are defined in terms of the primitive binary relation C, where C(X,Y) is interpreted as *"the closure of region X shares a common point with the closure of region Y"* or simply *"X connects Y"*.

With this interpretation there is no distinction between open, semi-open, and closed regions, however all regions must be of the same dimension, non-empty and regular. Regions don't have to be internally connected, i.e. they may consist of different non-connected parts.

Written in first-order logic, other relations can be defined as follows and the eight relations *DC (DisConnected), EC (Externally Connected), PO (Partial Overlap), EQ (EQual), TPP (Tangential Proper Part), NTPP (Non-Tangential Proper Part),* and their converse relations TPP^{-1} and $NTPP^{-1}$ form a set of JEPD relations called RCC-8.

$$DC(x, y) \equiv \neg C(x, y)$$
$$P(x, y) \equiv \forall z\,[C(z, x) \rightarrow C(z, y)]$$
$$PP(x, y) \equiv P(x, y) \wedge \neg P(y, x)$$
$$x = y \equiv P(x, y) \wedge P(y, x)$$
$$O(x, y) \equiv \exists z\,[P(z, x) \wedge P(z, y)]$$
$$PO(x, y) \equiv O(x, y) \wedge \neg P(x, y) \wedge \neg P(y, x)$$
$$DR(x, y) \equiv \neg O(x, y)$$
$$TPP(x, y) \equiv PP(x, y) \wedge \exists z\,[EC(z, x) \wedge EC(z, y)]$$
$$EC(x, y) \equiv C(x, y) \wedge \neg O(x, y)$$
$$NTPP(x, y) \equiv PP(x, y) \wedge \neg \exists z\,[EC(z, x) \wedge EC(z, y)]$$
$$P^{-1}(x, y) \equiv P(y, x)$$
$$PP^{-1}(x, y) \equiv PP(y, x)$$
$$TPP^{-1}(x, y) \equiv TPP(y, x)$$
$$NTPP^{-1}(x, y) \equiv NTPP(y, x)$$

Apart from the more general definition of regions in RCC, the RCC-8 relations and Egenhofer's eight relations actually describe the same set of topological relations (see Figure 1). This is especially the case in our experiments where regions are (two-dimensional) circles .

With the eight different relations defined above, the boundary of regions is an important distinction criterion. One of the questions that led to the empirical investigation described in this paper is whether or not the boundary of regions is also an important criterion for human spatial cognition. If not, calculi where the boundary of regions is not taken into account could be cognitively more adequate.

One of these calculi is RCC-5, a set of five JEPD relations which is also defined upon the relation C. The five relations are *DR (DiscRete), PO (Partial Overlap), EQ (EQual), PP (Proper Part),* and its converse PP^{-1}. They emerge from the RCC-8 relations by not distinguishing between *TPP* and *NTPP,* between TPP^{-1} and $NTPP^{-1}$, and between *DC* and *EC* (see Figure 1). These relations are obtained when the boundary of regions is completely ignored, i.e. only the interior of regions is considered.

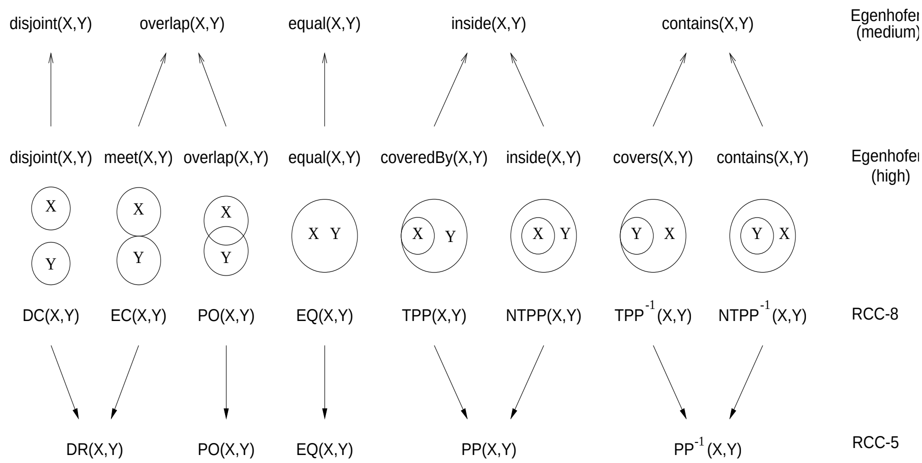

Figure 1: Two dimensional examples for the eight relations of Egenhofer and RCC-8, the correlation of high and medium resolution and RCC-8 and RCC-5.

Another possible set of JEPD relations where the boundary also has no special role is obtained when the boundary of a region is treated as any other part of the region, i.e. only the closure of regions is considered. This set contains five relations and can also be obtained by combining relations of RCC-8. The difference of this set to RCC-5 is that *EC* and *PO* are combined to a new relation instead of *EC* and *DC*. Grigni, Papadias & Papadimitriou (1995) used this set of five relations and called it *"medium resolution case"* whereas the set of eight relations was called *"high resolution case"* (see Figure 1). It will be interesting to see whether the results of our empirical investigations speak more in favor of RCC-5 or the medium resolution set.

Empirical investigation

From experimental psychology we know a large number of empirical methods to investigate the representation of conceptual knowledge, but here is not enough place to discuss them. A brief account about some of these methods translated to the domain of spatial knowledge is given by Mark et al. (1995). In the following empirical investigation we have chosen the grouping task paradigm, which is traditionally one of the famous methods to investigate conceptual knowledge in psychology. The main idea of such tasks is that conceptual knowledge plays the central role in assessing the similarity of given stimuli: stimuli are assessed as similar if they are instances of the same concepts, or are assessed as dissimilar, if they are instances of different concepts. Of course, this description is an idealization because in a lot of cases concept-independent information, such as visual aspects, can be used in grouping tasks. Beside this the underlying kind of concept representation, such as feature-based or prototype-based representations, plays an important role. However, if other aspects of presentation are controlled, like in our experiment, grouping experiments can give us important insights into the internal structure of conceptual knowledge.

Method and Procedure

20 students (10 female, 10 male) of the University of Freiburg had to accomplish a grouping task that consisted of 96 items showing a configuration of a red and a blue circle. After solving some practice trials to get acquainted with the procedure they then saw the color screen as depicted in Figure 2. The 96 items of spatial configurations of the red and blue circle consisted of 14 DC, 11 EC, 12 PO, 10 EQ, 15 TPP, 11 NTPP, 11 TPP^{-1}, and 12 $NTPP^{-1}$ instances. The number of occurrences was chosen in a random fashion around the number 12 ± 3 in order to avoid a bias towards RCC-8 grouping by using equally frequent groups. All subjects had to judge the same items because the aggregation of data had to be done across subjects per item. Instances for each relation were generated by a program that produced instances of the RCC-8 relations randomly with respect to metrical and ordering information.

After the grouping phase of the empirical investigation, subjects had to describe their groupings by natural language description in German. We did this to obtain further information on the criteria that guided subjects' grouping behavior, and we applied this (unexpected) phase at the end of the empirical session in order to avoid influences of verbalizations on the pure grouping behavior itself.

Results

Subjects needed from 14 to 35 minutes with an average of 20 minutes to group items and to describe their groupings. Aggregating grouping answers over all 20 subjects, we obtained a 96x96 matrix of Hamming distances between items that were the basis of a subsequent cluster analysis. Figure 3 shows the dendrogram — combining items and clusters first that are nearest to each other from left to right — of the cluster analysis with the method of average linkage between groups. In order to avoid interpreting clustering results that are only the consequence of different clustering methods, we computed cluster analyses with different clustering methods, but couldn't find any differences to the results as obtained by the above mentioned method—clustering results were invariant across different clustering methods.

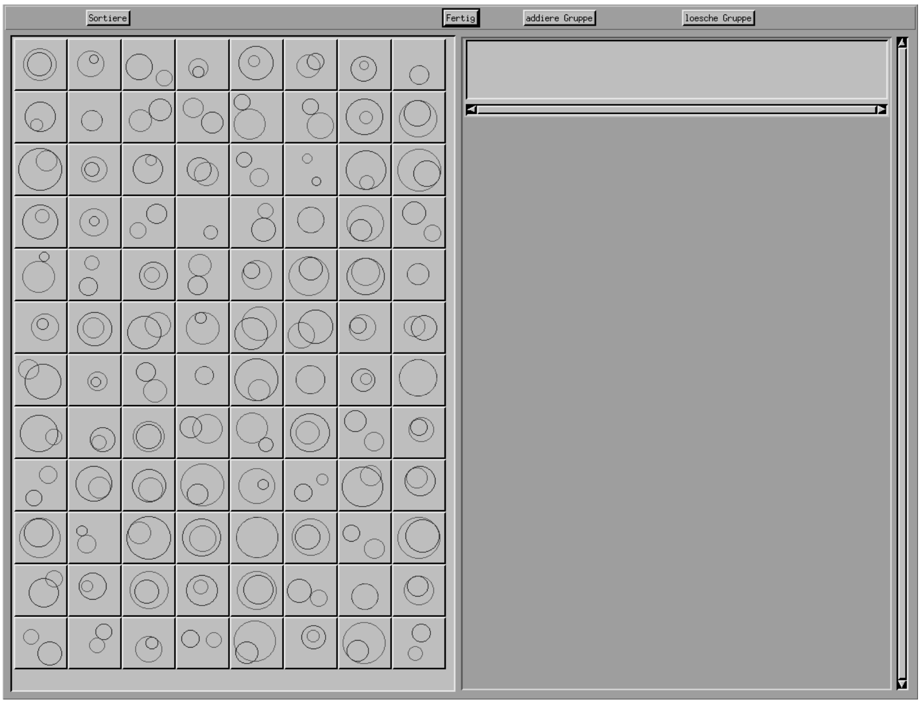

Figure 2: Screendump of the monitor at the beginning of the investigation. Subjects had to move the pictures on the left into groups at the right side of the screen. The two buttons at the right gave the possiblitiy to add as much group as needed and to remove unused groups. The EQ is not good to see in the grey screendump because in the original pictures it was presented as a red-blue broken circular line (e.g., right upper corner).

The first important result is that all instances of the RCC-8 relations were clustered at once. Therefore, we left out the “dendrites” of single instances and only display clustering on higher levels to test for compatibility of the empirical results with the RCC-5 and medium resolution case of Egenhofer’s conceptualization. As can be easily seen, there is no evidence that items were grouped together according to one of this sets of relations; so one can conclude that RCC-5 and the medium resolution set of Egenhofer’s relations can be rejected as conceptually inadequate on empirical grounds.

The clustering of TPP and NTPP with their inverses can be attributed to disregarding the relationship of a reference object (RO) with a to-be-localized object (LO), and that subjects judged the configuration as a whole where one object had another object as a tangential proper part or non-tangential proper part, respectively.

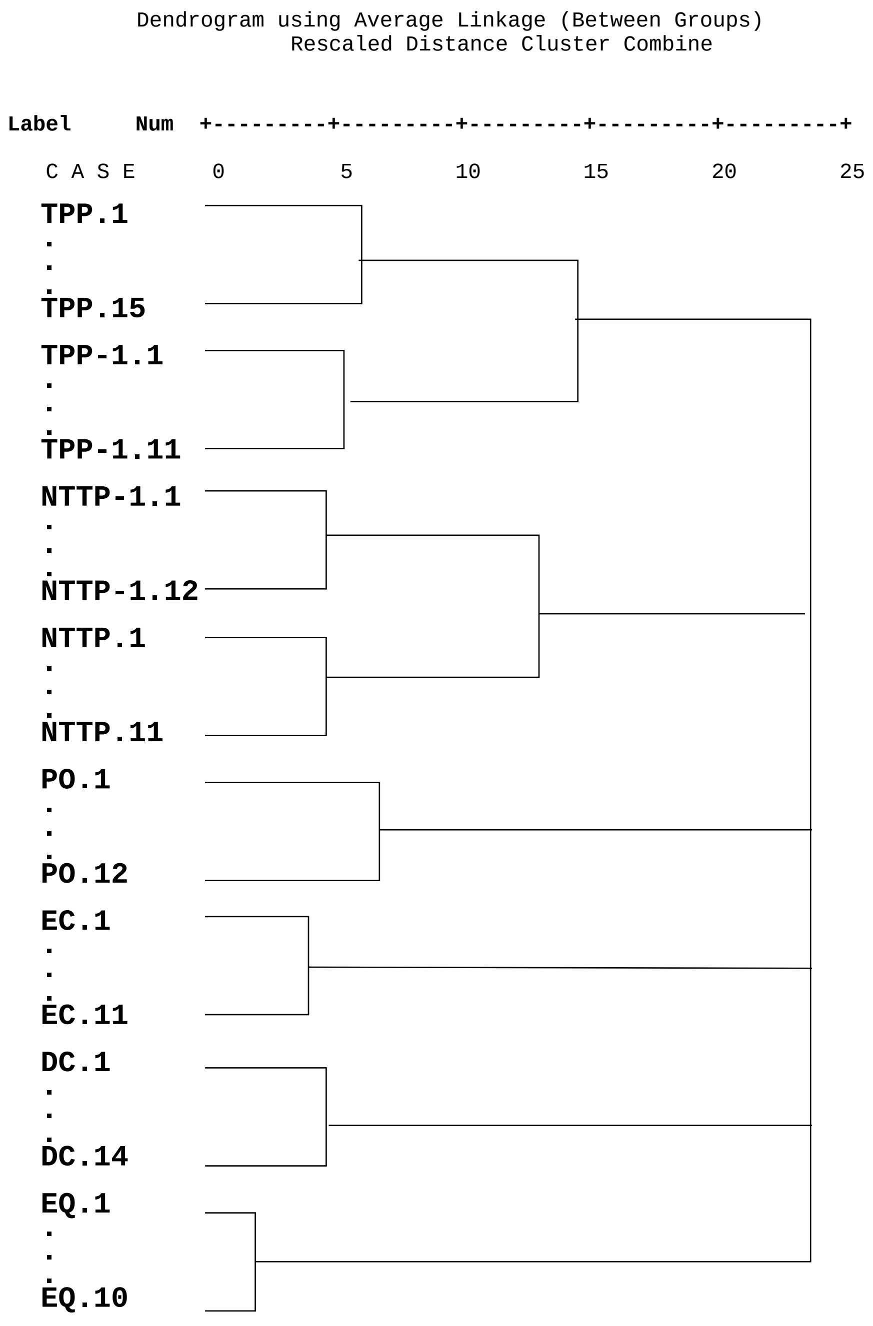

Figure 3: Dendrogram of the cluster analysis of 96 items based on Hamming distances between items.

Furthermore, the verbalizations of subjects' groupings were analyzed by categorizing them whether they included purely topological (T), orientational (O), and metrical information (M), or combinations of them. In Table 1 we present the results of the categorization of subjects' verbalizations, as completed by two raters and showing nearly perfect concordance (97.18%).

Table 1: Categorization of subjects' verbalization of their groupings

Informational content	Frequency*
Topology (T)	62.1%
Orientation (O)	0%
Metric (M)	0%
T + O	14.1%
T + M	19.2%
O + M	0%
T + O + M	0%
Other	4.6%

*. n=390 (195 verbalizations were judged by two raters)

Table 1 clearly shows that the informational content of most verbalizations was due to topological information alone or in combination with other information (62.1% + 14% + 19.2% of 390 judgements, i.e. 95.4%). Additionally, some verbalizations also incorporate orientation information (14.1% of the judgement). Verbalizations containing metrical information are concerned with the shape of the geometrical objects used and not with the spatial relation of the two objects and is always combined with topological information.

Discussion and future work

Cognitively adequate models of spatial knowledge are essential for several fields of geographic information systems (GIS), robotics, computational vision and linguistics on the one hand and for a number of subfields of cognitive science on the other. The starting point of the investigation reported here was that to our mind the term “cognitive adequacy” is often used very inflationarily. Whether or not a formal approach to spatial relations is a cognitively adequate model of human spatial knowledge is in fact an empirical question and can be answered only on the basis of empirical results.

What we have found in an empirical grouping task is that (i) topological relationships are a relevant and dominating factor in judging spatial configurations, (ii) that the topological relations as specified in RCC-8 seem to form a relevant level of conceptualizing spatial configurations, and (iii) that neither RCC-5 nor the medium resolution variant of Egenhofer’s relations can be found on a coarser level of granularity. Even if it becomes clear that the two five relation sets are for the most part cognitively irrelevant, it is an interesting observation that items of the relations DC and EC were grouped together more often than EC and PO. Thus, if one wants to consider one of the five relation sets at all, RCC-5 seems to be the better choice. The last observation was (iv) that exploratory results seem to indicate that ordering information and qualitative metrical information were also considered in grouping spatial configurations. This is a relevant point in our results because one important task in qualitative spatial reasoning is to extend topological calculi as presented in this paper for other non-topological concepts like distance, direction, or metrical aspects. If one wants to make this extension in such a way that the resulting calculi can still be called cognitively adequate, our empirical results provide useful hints in which direction to extend topological calculi.

In general, the results indicated that the RCC-8 relations, resp. Egenhofer’s definition of topological relations, are actually the most promising starting point for further psychological investigations on human conceptual topological knowledge. However, further evidence will be needed before a detailed modeling of human conceptual knowledge is possible. A series of ensuing experiments will be concerned with the question of in which way other classes of concepts, in particular ordering, and metrical relations interact. Besides this, we are planning to do further empirical investigations of the same kind where the regions we present are not restricted to circles. Apart from allowing arbitrary shape, it seems to be very interesting to use regions with holes (Egenhofer, Clementini & Di Felice, 1994) as relations between these regions can be expressed using RCC-8, although the assignment of these relations to pairs of regions doesn't seem to be straightforward.

Last but not least we are planning to conduct experiments on the inferential adequacy of the RCC-theory, as we have done previously for Allen’s interval calculus (Knauff et al. 1995; Rauh, Schlieder & Knauff, 1997). An interesting question in this context is whether the computational properties of these calculi (Renz & Nebel, 1997) can be found again in the empirical investigations.

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