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ON THE ESTIMATION OF INTERLEAVED PULSE TRAIN PHASES.

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ABSTRACT

Some signals are transmitted as periodic pulse trains where information is in the times of arrival of pulses. A number of pulse trains arriving over the same time interval are said to be *interleaved*. We propose a method for estimating phase and fine-tuning previously obtained frequency estimates of a known number of interleaved pulse trains using an extended Kalman filter, where discontinuities in the signal model are first smoothed. The advantage of this method is its computational efficiency.

1. INTRODUCTION

Some signals occur as periodic trains of pulses. For example, signals used in radar systems, communication systems and possibly neural systems appear in this form. Often, a number of pulse trains are received over a single channel during the same time interval resulting in an *interleaved* pulse train. It is important to be able to separate the pulses in the interleaved train in terms of their source in order to extract desired information. This process is termed *pulse train deinterleaving*. One application for pulse train deinterleaving is in radar detection [1].

Previously proposed pulse train deinterleaving methods include sequential search [2] and histogramming [2, 3], which work well in low noise environments. Another approach is first to formulate the problem as a stochastic discrete-time dynamic linear model and then deinterleave the signal using either forward dynamic programming with fixed look-ahead or a probabilistic teacher [4]. All these methods are computationally expensive. If the pulse train to be deinterleaved contains N pulses, then these methods require computational effort of order N^2 or higher. An extended Kalman filter approach to deinterleaving using a modified version of the signal model in [4] is presented in [5]. Here the computational effort required is of order N.

The deinterleaving task relies on the assumption that the different trains will have different frequencies and phases, therefore the estimation of these characteristics is perhaps a useful starting point. One computationally efficient method for this uses fast Fourier transform techniques to determine the number of trains present and their periods but does not deinterleave the trains [6]. The computational effort required is of order $N \log N$.

None of the above methods seeks to directly estimate the phases of the interleaved pulse trains. In this paper, a new signal model for use with an extended Kalman filter is proposed. As with [6], we do not deinterleave the pulse trains, but estimate their characteristics. Here we directly estimate the phases and fine-tune the frequency estimates of the interleaved pulse trains with computational effort of order N. It is assumed that the pulse trains are periodic and that the number of sources is finite and known.

2. THE SIGNAL MODEL

Consider M periodic pulse train sources. Let $f^{(i)}$ and $\theta^{(i)}$ denote the frequency and phase of the i^{th} source. Pulses then occur at times $(2\pi n + \theta^{(i)})/(2\pi f^{(i)})$ for $i \in [1, M]$ and $n = 0, 1, 2, \ldots$ The received interleaved signal consists of the superposition of the M pulse trains produced by these sources. Let $t_1, t_2, \ldots, t_N$ denote the times of arrival of N consecutive pulses. The deinterleaving problem is as follows.

The Deinterleaving Task. *Given pulses $t_1, \ldots, t_N$ and the number of sources present, M, determine which source produced each pulse.*

A first step towards solving this is phase/frequency estimation.

The Phase/Frequency Estimation Task. *Given pulses $t_1, \ldots, t_N$ and the number of sources present, M, estimate the frequencies, $f^{(i)}$ and phases, $\theta^{(i)}$, of each pulse train $i = 1, 2, \ldots, M$.*

Since our computer implementation of the estimation algorithms are in discrete time, it makes sense to work with discrete-time models which are fast sampled approximate versions of the precise continuous-time models. The signal can be described by a discrete-state, discrete-time model.

$$\begin{aligned} x_{k+1} &= x_k + v_k \quad , x_0 \\ y_k &= h_k + w_k \end{aligned} \tag{1}$$

Here k is the discrete-time index, and the received signal y_k is the *number of pulses* detected before the discrete time

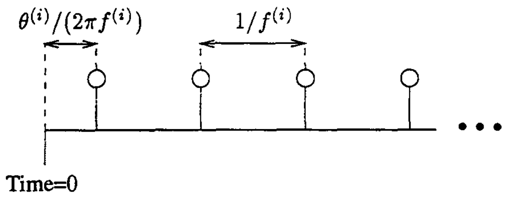

Figure 1: A single pulse train and its characteristics.

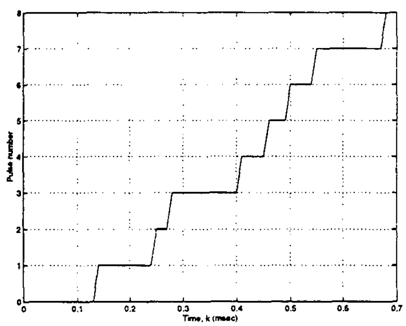

Figure 2: The received signal h_k.

step k. Also, x_k is the state variable at k, with elements the frequencies $f^{(i)}$ and phases $\theta^{(i)}$ as follows

$$x'_k = \left[f_k^{(1)}, \dots, f_k^{(M)}, \theta_k^{(1)}, \dots, \theta_k^{(M)}\right] \qquad (2)$$

where there are M pulse train sources, $f_k^{(i)}$ is the frequency and $\theta_k^{(i)}$ the phase of train i as shown in Figure 1. The terms v_k and w_k represent noise on the states and received signal.

The received signal h_k is the number of pulses that have arrived in the interleaved train at time k. A typical example of h_k for $M = 4$ is shown in Figure 2, where jumps occur when a pulse is received. Note that the jumps are not instantaneous due to the discrete nature of the model. A pulse is only known to arrive within a discrete time period rather than at an exact time. Thus h_k can be expressed in terms of the state as follows.

$$\begin{aligned} h_k^{(i)} &= f^{(i)}k - \theta^{(i)}/2\pi - r_k^{(i)} \\ h_k &= \sum_{i=1}^{M} h_k^{(i)} \end{aligned} \qquad (3)$$

where $r_k^{(i)}$ is a minimum value remainder term that ensures that $h_k^{(i)}$ is an integer. The remainder term can also be related to the state since for each pulse train at each time instant $r_k^{(i)}$ is equal to the fractional part of $f^{(i)}k - \theta^{(i)}/2\pi$.

The terms v_k, w_k in (1) represent noise on the states and received signal, respectively. Thus v_k represents drift on frequencies and jitter on phases, and w_k represents the number of false detections of pulses. For simplicity, we assume v_k, w_k are independent, zero mean and Gaussian with covariances Q_k and R_k, respectively. Here Q_k can be assumed to take the following form

$$Q_k = \begin{bmatrix} Q_k^d & 0 \\ 0 & Q_k^j \end{bmatrix} \qquad (4)$$

where Q_k^d represents the drift covariance and Q_k^j the jitter covariance.

It can be seen from (3) that h_k depends nonlinearly on the state, so the state space model (1) is nonlinear. The nonlinearities in h_k are discontinuous, so the signal model can not be used in its present form to derive an extended Kalman filter (EKF).

Remark: In [5], a discrete-event state-space model is formulated with the "discrete-time" variable k being the integer number of pulses that have arrived. This method is of order N. Here, k depends on the rate at which the interleaved train is sampled. Since the sample rate only linearly increases the order of the deinterleaver, this method is also of order N for any reasonable sample rate.

2.1. Smoothing h_k

A key proposal of this paper is to exploit the extended Kalman filter in some way for recursively estimating the states (phases and frequencies of the pulse trains) from the data y_k. This is a first step towards deinterleaving. In order to use an extended Kalman filter for deinterleaving our approach is to smooth the nonlinearities inherent in the signal h_k so that these can be linearised. This can be done, for example, by truncating a Fourier series expansion of h_k to A terms. The resultant expression for h_k is:

$$\begin{aligned} h_k^{s\,(i)} &= (f_k^{(i)}k - \theta_k^{(i)}/2\pi - 1/2) \\ &\quad + \sum_{a=1}^{A} \sin(2\pi a f_k^{(i)} k - a\theta_k^{(i)})/a\pi \\ h_k^{s} &= \sum_{i=1}^{M} h_k^{s\,(i)} \end{aligned} \qquad (5)$$

In practice, and indeed to our surprise, the first term of the Fourier series ($A = 1$ in (5a)) appears sufficient to approximate the original discontinuous signal for filtering purposes. Figure 3 shows the smoothed version of the signal in Figure 2 in this case.

The smoothed signal model is now

$$\begin{aligned} x_{k+1} &= x_k + v_k \quad, x_0 \\ y_k &= h_k^{s}(x_k) + w_k \end{aligned} \qquad (6)$$

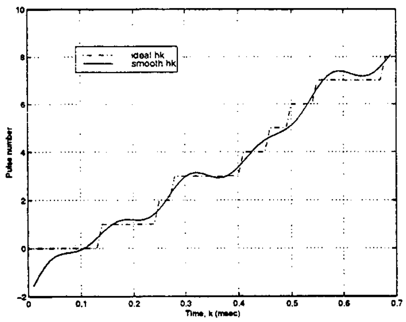

Figure 3: The smoothed signal h_k^s.

where $h_k^s(x_k)$ is given in (5).

3. THE EXTENDED KALMAN FILTER

In order to construct an extended Kalman filter from the signal model given in (6) a linearisation of $h_k^s(x_k)$ (5) is needed. This is:

$$H_k^{s\prime} = \left[\ \ldots, \partial h_k^s(x_k)/\partial f_k^{(i)}, \ldots, \partial h_k^s(x_k)/\partial \theta_k^{(i)}, \ldots\ \right] \quad (7)$$

where

$$\frac{\partial h_k^s(x_k)}{\partial f_k^{(i)}} = k + 2k\cos(2\pi f_k^{(i)}k - \theta_k^{(i)}) \quad (8)$$

$$\frac{\partial h_k^s(x_k)}{\partial \theta_k^{(i)}} = -\frac{1}{2\pi} - \frac{1}{\pi}\cos(2\pi f_k^{(i)}k - \theta_k^{(i)})$$

The extended Kalman filter equations are simplified by the constant state equation in the signal model. This yields constant time-update equations, so the filter equations are:

$$\hat{x}_{k+1/k} = \hat{x}_{k/k-1} + K_k[y_k - h_k(\hat{x}_{k/k-1})] \quad (9)$$

$$K_k = P_{k/k-1}H_k^s(H_k^{s\prime}P_{k/k-1}H_k^s + R_k)^{-1}$$

$$P_{k+1/k} = P_{k/k-1} - K_k H_k^{s\prime} P_{k/k-1} + Q_k$$

where $\hat{x}_{k+1/k}$ is the filtered estimate of x_{k+1}, K_k is the Kalman gain and $P_{k/k-1}$ is the error covariance matrix at k given measurements to $k-1$. Notice that the non-smooth estimate of the input signal, $h_k(\hat{x}_{k/k-1})$, is used in the state update equation. The initialisation here is given by $\hat{x}_{0/-1} = \hat{x}_0$ where $\hat{x}_0 = [\hat{f}_0' \ \hat{\theta}_0']'$ and $P_{0/-1} = P_0$.

4. RESULTS

Figure 4 shows a typical result for the phase estimation from the extended Kalman filter. The interleaved train input consists of 2000 pulses ($N = 2000$) with five component pulse trains ($M = 5$). The estimation results are as shown in Tables 1 and 2. These results are with an initial 10% uncertainty in the frequencies and unknown phases on the interval $[0, 2\pi)$.

As can be seen from Table 1, the frequency fine-tuning is very effective. Table 2 indicates that there can be some bias in estimating the phase (due to model inaccuracies) but that this bias is small. Processing more pulses slightly improves the phase estimation, but beyond around 2000 pulses this improvement is minimal. This method of pulse train parameter estimation can fine-tune the frequency estimates and estimate the phases of up to 8 interleaved pulse trains with an average phase estimate bias of around 2% after processing an interleaved train with 2000 pulses.

As with [5], this parameter estimation method is sensitive to the initial values chosen for the phase. Since the

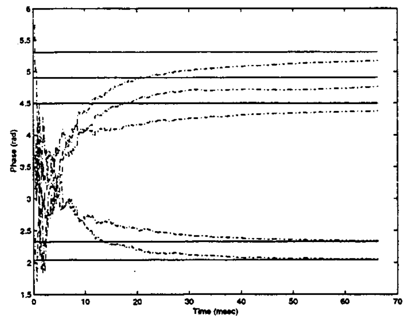

Figure 4: Actual and estimated phases after 1000 pulses

Train No.	Actual Freq. (kHz)	Estimated Freq. (kHz)	Error (%)
1	1.3238	1.3237	4.6e-3
2	1.7364	1.7364	4.4e-3
3	2.0830	2.0829	1.9e-3
4	2.7595	2.7595	0.0e-3
5	7.2244	7.2244	0.8e-3

Table 1: Comparison of frequencies.

Train No.	Actual Phase (rad)	Estimated Phase (rad)	Error (%)
1	4.4958	4.4906	0.083
2	5.3098	5.2940	0.252
3	4.9027	4.9066	0.062
4	2.0383	2.0691	0.489
5	2.3263	2.4838	2.506

Table 2: Comparison of phases.

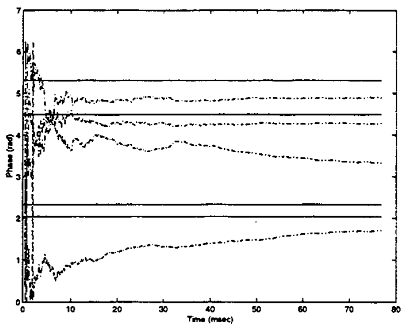

Figure 5: Actual and estimated phases in the presence of noise with known variance $\sigma^2 = 0.001$ after 1000 pulses.

pulse train phases are assumed to lie on the interval $[0, 2\pi)$, the initial estimates for the phases are randomly chosen from a uniform distribution over this interval. The initialisation for the error covariance, P_0, reflects this. A bank of filters is then used, each with a different initialisation for the phase, with the filter leading to the least average prediction error squared being chosen. Since this method is of order N, it will retain its computational advantage over most previous deinterleaving methods if the number of filters in the bank is much less than N.

Of course, one would not be surprised if a method involving of order N^2 calculations (or higher) achieved similar, or better results, but the main message from this work is that very useful results are achieved using a method requiring only of order N calculations.

Jitter noise

Noise is present in all real world signals, so it is important to test this pulse train parameter estimation method in the presence of noise. Here we examine only the case where white Gaussian noise with known variance is present. It could be possible to deal with noise of unknown variance using an *adaptive* Kalman filter, but this is beyond the scope of this paper.

Time of arrival jitter noise with variance $\sigma^2 = 0.001$ was added to the interleaved pulse train. This noise is input into the extended Kalman filter as jitter on the phase term, hence Q_k^j (see (4)) is a diagonal matrix of the noise variance. Since there is no frequency drift applied to the system, Q_k^d is zero. Figure 5 shows the effect on phase estimation of jitter noise on a four train system. The frequencies are fine tuned to within 0.1% of their true values, while the phases are estimated to within 10% after 2000 pulses are processed. This pulse train parameter estimation method is not robust to noise since the number of trains that can be estimated drops to four in the presence of noise and higher noise prevents any estimation of the pulse train parameters.

5. CONCLUSION

An important aspect of this scheme for estimating the parameters of interleaved pulse trains is its computational efficiency. If there are N pulses to be processed in the interleaved train, then computations are of order N. Since computationally efficient (order $N \log N$) fast Fourier transform methods give fairly accurate estimates of the pulse train frequencies [6], this new method is used to fine-tune the frequency estimates and yield estimates of phase.

Accurate estimates of both phase and frequency are necessary in order to deinterleave periodic pulse trains. With the method applied here the phases of up to 8 interleaved trains can be estimated to around 2% of their actual values if 2000 pulses are processed. The frequency estimates can be fine-tuned from a 10% error to virtually zero (around 0.01%) error. This method is not robust to the presence of noise, however robustness is not the central message of this work.

Of course, in hindsight the proposed method is a straightforward application of the familiar extended Kalman filter. The significance of this work is that this tool is able to achieve such useful results in a context in which *a priori* one would not expect it to perform well since it approximates discontinuities by smooth functions.

6. REFERENCES

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