# ADAPTIVE HMM FILTERS FOR SIGNALS IN NOISY FADING CHANNELS

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## ABSTRACT

In this paper, Kalman filtering (KF) and hidden Markov model (HMM) signal processing techniques are coupled to demodulate signals transmitted through noisy fading channels. The demodulation scheme presented can be applied to both digital M-ary differential phase shift keyed (MDPSK) and analog frequency modulated (FM) signals. Adaptive state and parameter estimation algorithms are devised based on the assumption that the transmission channel introduces time-varying gain and phase changes, modelled by a stochastic linear system, and has additive Gaussian noise. Our technique is to use an HMM filter, for signal estimation, coupled with a KF, for channel parameter tracking. The approach taken can easily be generalised for other transmission schemes, such as continuous phase modulated (CPM) signals.

## 1. INTRODUCTION

The capacity of communications systems is often limited by fading in the transmission channel. In this paper we address the problem of demodulating frequency modulated signals in multi-path Rayleigh fading channels. The task is twofold, involving both state (or message) estimation and parameter (or channel) tracking. In the case of digital differential phase shift keyed (DPSK) signals, the matched filter (MF) is known to be the optimal state estimator, assuming the message is an independent data sequence [1] (p.267). For analog frequency modulated (FM) signals, a phase locked loop (PLL) is commonly used for state estimation. In both cases, the parameters to be tracked are the amplitude gain and the phase shift of the transmission channel. The traditional scheme employs a PLL for channel phase shift tracking, and an automatic gain control (AGC) for channel amplitude tracking. Such schemes are discussed in [2](Ch.5,6).

In this paper we formulate the M-ary DPSK (MDPSK) and FM signal models into a hidden Markov model (HMM) framework. We then apply a finite-dimensional HMM filter for the signal state estimate, coupled with a continuous state Kalman filter (KF) for the channel parameter estimate. We term these filters *conditional coupled filters*. Recently schemes have been developed for demodulation of continuous phase modulated (CPM) signals in Rayleigh fading channels [3]. These techniques couple Kalman filtering with maximum likelihood (ML) sequence estimation, however they do not accommodate phase shifts in the channel. The technique presented in this paper can be applied to CPM transmission schemes (in particular, minimum shift keyed (MSK) signals) to allow for complex valued channels. In addition, frequency shift keyed (FSK) and phase shift keyed (PSK) signals can seen as a special case of the signal model presented in this paper. In fact the PSK model has the same form as the quadrature amplitude modulation (QAM) model, which was the subject of earlier work [4].

An important aspect of the HMM approach is that, unlike the MF, it does not require the assumption (in the digital case) that the signal be an independent data sequence. We use the term independent to signify that no dependence exists from one sample to the next. By removing the independence assumption we allow for signal coding, which can introduce dependences between the message bits. In fact the HMM specialises to the MF in the particular case of an independent sequence.

The HMM filter is a finite-dimensional optimal filter where the HMM has states which belong to a finite-discrete set. The term finite-discrete implies that the elements of the set are discrete (for example discrete frequency values) and there are a finite number of them. To date, such filters have been widely applied in areas such as speech processing and biological signal processing [5],[6]. In this paper we make use of the fact that the non-linear finite-discrete nature of the HMM approach can be applied to the MDPSK and FM demodulation problem. A key to this approach is the use of the optimal HMM on-line filter, as used in [7]. This is a new approach to HMM signal processing which enables sequential algorithms to be developed.

The algorithms presented in this paper are indeed more computationally intensive than standard MF/AGC/PLL schemes. However the advantages come from being able to allow for dependences in the message signal, and also the fact that the cartesian coordinate representation of the channel parameter, allows for a linear KF in place of the non-linear PLL.

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## 2. PROBLEM FORMULATION

In order to implement a demodulation scheme taking advantage of an adaptive HMM approach, it is necessary to have a discrete frequency space. In the case of digital MDPSK, the discretization is given by the set of allowable discrete frequency shifts. For analog FM, the frequency space must be quantised. We term this quantised signal QFM. The quantisation and digital sampling rate are design parameters which introduce sub-optimality to the FM receiver, however, if the quantisation is fine enough, and the sampling rate fast enough, then the loss in performance due to digitisation will be outweighed by the performance gain (over more standard schemes) from the demodulation scheme presented here.

### 2.1. Signal Model

We will present the algorithm by formulating the signal in a state space form. The signal state (in this case the frequency) belongs to a finite-discrete set.

Let $f_k$ be a real valued discrete-state discrete-time process, where for each $k$,

$$f_k \in \mathbf{Z}_f = \{z_f^{(1)}, \dots, z_f^{(L_f)}\} \ , \ z_f^{(i)} = (i/L_f)\pi \in \mathbb{R}$$

where $L_f \in \mathcal{Z}^+$ and $\mathcal{Z}^+$ is the set of positive integers. We also denote the vector of discrete frequencies $z_f = (z_f^{(1)}, \dots, z_f^{(L_f)})' \in \mathbb{R}^{L_f}$.

For MDPSK/QFM, the transmitted signal can be represented in base band by

$$s_k = A_c \exp[\mathrm{j}\,\theta_k] \ , \ \theta_k = (\theta_{k-1} + f_k)_{2\pi} \qquad (1)$$

where the carrier amplitude, $A_c$, is a known constant and $(.)_{2\pi}$ denotes modulo $2\pi$ addition.

### 2.2. Channel Model

The signal is passed through a channel which can cause amplitude attenuation and phase shifts, as for example in fading channels due to multiple transmission paths. The channel can be modelled by a multiplicative disturbance, $g_k$, which introduces time-varying gain and phase changes to the signal.

$$g_k = \kappa_k \exp[\mathrm{j}\,\phi_k] = g_k^R + \mathrm{j} g_k^I$$

Unlike the standard polar co-ordinate approach, we work with the vector $x_k$ associated with the real and imaginary parts of the channel $g_k$.

$$x_k = \begin{pmatrix} \kappa_k \cos\phi_k \\ \kappa_k \sin\phi_k \end{pmatrix} = \begin{pmatrix} g_k^R \\ g_k^I \end{pmatrix} \qquad (2)$$

This representation allows the observations to be written bi-linearly in the state and the channel parameter, as will be seen later, so that conditional coupled Kalman filters can be applied. The cartesian channel representation is of most benefit under conditions of rapid channel phase shifts. In such situations the non-linear PLL, used in standard schemes, often fails to track the phase, while the linear KF is able to track the real and imaginary components effectively.

*Assumption on Channel Fading Characteristics :* Consider that the dynamics of $x_k$, from (2), are given by the following linear time invariant stochastic system.

$$x_{k+1} = F\,x_k + v_{k+1} \quad , \quad v_k = N[0, Q_k] \qquad (3)$$

for some known $F$, (usually with $\lambda(F) < 1$, where $\lambda$ indicates eigen-values, to avoid unbounded $x_k$, and typically with $F = fI$ for some scalar $0 << f < 1$). The amplitude and phase of the channel are assumed to vary independently, therefore $Q_k$ (the covariance matrix) contains terms which couple the real and imaginary parts of $x_k$ (see [4]).

### 2.3. Observation Model

The baseband output of the channel, corrupted by additive noise $w_k$, is given in discrete-time, by

$$y_k = g_k\, s_k + w_k \qquad (4)$$

Assume that $w_k \in \mathbb{C}$ has i.i.d. real and imaginary parts, $w_k^R$ and $w_k^I$ respectively, with zero mean and Gaussian, so that $w_k^R, w_k^I \sim N[0, \sigma_w^2]$. We also define the vector $Y_k \triangleq (y_0, \dots, y_k)$.

In vector notation the observations have the form

$$\begin{pmatrix} y_k^R \\ y_k^I \end{pmatrix} = \begin{pmatrix} A_c\cos\theta_k & -A_c\sin\theta_k \\ A_c\sin\theta_k & A_c\cos\theta_k \end{pmatrix} \begin{pmatrix} g_k^R \\ g_k^I \end{pmatrix} + \begin{pmatrix} w_k^R \\ w_k^I \end{pmatrix} \qquad (5)$$

### 2.4. State Space Signal Model

We now formulate the signal model of Section 2.1 into a state space form in order to represent the channel in a way that will allow the application of coupled conditional filters. To do this it is necessary to make the following assumption.

*Assumption on Message Signal*

$$f_k \text{ is a first order homogeneous Markov process} \qquad (6)$$

**Remark :** In the case of MDPSK signals, the assumption is valid, given that error correcting coding has been employed in transmission. Coding techniques such as convolutional coding [1] (p.441), produce signals which are not independent and as such display Markov properties. Of course independent signals can be considered in this framework too, since a Markov chain with a transition probability matrix which has all elements the same, gives rise to an independent process. Assumption (6) also holds for other digital transmission schemes, for example the Markov properties of MSK signals are discussed in [8] (p.438). For the case of QFM, the assumption is valid if the Markov transition probability matrix used, is a diagonally dominated-Toeplitz-circulant matrix. This implies that the sampling rate is such that the frequency does not change too much from one sample to the next. □

Let us now associate a discrete state indicator vector, $X_k^f$, with the signal state $f_k$. $X_k^f$ belongs to a finite-discrete set of unit vectors. That is, $X_k^f \in \{e_1^f, e_2^f, \dots, e_{L_f}^f\}$ where $e_i^f = (0, \dots, 0, 1, 0, \dots, 0)' \in \mathbb{R}^{L_f}$ with 1 in the $i^{th}$ position. Therefore $f_k = z_f' X_k^f$. In other words, $X_k^f = e_i^f$ when $f_k = z_f^{(i)}$. It is assumed that the message sequence has a known transition probability matrix $\mathbf{A}^f$, and known state values, $\mathbf{Z}_f$. Writing the message in terms of indicator vectors allows us to formulate state space models involving a mixture of the states $X_k^f$ and $x_k$. We now have the following equation based on assumption (6).

$$X_{k+1}^f = (\mathbf{A}^f)' X_k^f + M_{k+1} \tag{7}$$

where $M_k$ is a Martingale increment. In addition, we define an indicator vector associated with the phase, by $X_k^\theta \in \{e_1^\theta, \ldots, e_{L_\theta}^\theta\}$ and the vector of discrete phase values $z_\theta = (z_\theta^{(1)}, \ldots, z_\theta^{(L_\theta)})'$ where $z_\theta^{(i)} = 2\pi i / L_\theta$. It can be shown that if $L_\theta = 2nL_f$, $n \in \mathcal{Z}$, then $\theta_k \in \{z_\theta^{(i)}\}$.

Now given (1), $X_{k+1}^\theta$ is a "rotation" on $X_k^\theta$ by an amount determined from $X_{k+1}^f$. In particular,

$$X_{k+1}^\theta = [\mathbf{A}^\theta(X_{k+1}^f)]' X_k^\theta \tag{8}$$

where $\mathbf{A}^\theta(.)$ is a transition probability matrix given by

$$\mathbf{A}^\theta(X_{k+1}^f)' = S^{r_{k+1}} \ , \ r_k = [1, 2, \ldots, L_f] X_k^f \tag{9}$$

and $S$ is the rotation operator

$$S = \begin{pmatrix} 0 & 0 & \ldots & 0 & 1 \\ 1 & 0 & \ldots & 0 & 0 \\ 0 & 1 & \ldots & 0 & 0 \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \ldots & 1 & 0 \end{pmatrix} \tag{10}$$

The *observation process* (5) can now be expressed in terms of the state $X_k^\theta$.

$$\begin{pmatrix} y_k^R \\ y_k^I \end{pmatrix} = \begin{pmatrix} A_c \cos[z_\theta' X_k^\theta] & -A_c \sin[z_\theta' X_k^\theta] \\ A_c \sin[z_\theta' X_k^\theta] & A_c \cos[z_\theta' X_k^\theta] \end{pmatrix} \begin{pmatrix} g_k^R \\ g_k^I \end{pmatrix} + \begin{pmatrix} w_k^R \\ w_k^I \end{pmatrix} \tag{11}$$

or equivalently with the appropriate definition of $h_\theta(.)$,

$$\begin{aligned} \mathbf{y}_k &= h_\theta(X_k^\theta) x_k + \mathbf{w}_k \quad , \quad \mathbf{w}_k = N[0, R_k] \\ &= [h_\theta(e_1^\theta) x_k \ , \ h_\theta(e_2^\theta) x_k \ , \ldots , \ h_\theta(e_{L_\theta}^\theta) x_k] X_k^\theta + \mathbf{w}_k \\ &= H_\theta' [I_{L_\theta} \otimes x_k] X_k^\theta + \mathbf{w}_k \end{aligned} \tag{12}$$

where the augmented matrix $H_\theta' = [h_\theta(e_1^\theta), \ldots, h_\theta(e_{L_\theta}^\theta)]$. Here we see that the cartesian co-ordinates for the channel model allow the observations to be written in a form which is bi-linear in $X_k^\theta$ and $x_k$.

In summary, we now have the following signal model

$$\boxed{\begin{aligned} X_{k+1}^f &= (\mathbf{A}^f)' X_k^f + M_{k+1} \\ X_{k+1}^\theta &= [\mathbf{A}^\theta(X_{k+1}^f)]' X_k^\theta \\ x_{k+1} &= F x_k + v_{k+1} \\ \mathbf{y}_k &= H_\theta' [I_{L_\theta} \otimes x_k] X_k^\theta + \mathbf{w}_k \end{aligned}} \tag{13}$$

## 3. ADAPTIVE FILTER ALGORITHM

To develope an adaptive filter algorithm for the signal model (13) it is necessary to estimate $X_k^f$ and $X_k^\theta$, for each $k$, and use these estimates to track the channel parameter variations $x_k$. In section 3.1 we present HMM *information state* filters which have linear recursive update equations (we denote $\alpha_k$ to be the *information state* representing a discrete un-normalised conditional probability density for the state [9] (p.79)). We then use $\alpha_k$ to generate normalised state estimates of $X_k^f$ and $X_k^\theta$. In section 3.2 we apply conditional Kalman filtering to track $x_k$. The resulting adaptive HMM algorithms appear as conditional coupled KF and HMM filters.

### 3.1. Conditional HMM Information States

Let $\hat{X}_{k|\mathcal{X},\mathcal{X}^\theta}^f$ and $\hat{X}_{k|\mathcal{X},\mathcal{X}^f}^\theta$ denote the conditional filtered normalised state estimates of $X_k^f$ and $X_k^\theta$ respectively. Here, $\mathcal{X}_k = \{x_0, \ldots, x_k\}$, $\mathcal{X}_k^\theta = \{X_0^\theta, \ldots, X_k^\theta\}$, $\mathcal{X}_k^f = \{X_0^f, \ldots, X_k^f\}$ and $\mathcal{Y}_k = \{y_0, \ldots, y_k\}$. By definition

$$\hat{X}_{k|\mathcal{X},\mathcal{X}^\theta}^f \triangleq E[X_k^f \mid \mathcal{Y}_k, \mathcal{X}_k, \mathcal{X}_{k-1}^\theta] \tag{14}$$

$$\hat{X}_{k|\mathcal{X},\mathcal{X}^f}^\theta \triangleq E[X_k^\theta \mid \mathcal{Y}_k, \mathcal{X}_k, \mathcal{X}_k^f] \tag{15}$$

Let us define $\underline{1}$ to be the column vector containing all ones. We also let the information states, $\alpha_k^f$ and $\alpha_k^\theta$, be such that their $i^{\text{th}}$ elements are given respectively by

$$\alpha_k^f(i) \triangleq P(Y_k, X_k^f = e_i^f \mid \mathcal{X}_k, \mathcal{X}_{k-1}^\theta) \tag{16}$$

$$\alpha_k^\theta(i) \triangleq P(Y_k, X_k^\theta = e_i^\theta \mid \mathcal{X}_k, \mathcal{X}_k^f) \tag{17}$$

Observe that

$$\hat{X}_{k|\mathcal{X},\mathcal{X}^\theta}^f = \langle \alpha_k^f, \underline{1} \rangle^{-1} \alpha_k^f \tag{18}$$

$$\hat{X}_{k|\mathcal{X},\mathcal{X}^f}^\theta = \langle \alpha_k^\theta, \underline{1} \rangle^{-1} \alpha_k^\theta \tag{19}$$

for which we have the following "forward" recursions based on algorithms presented in [5].

$$\boxed{\begin{aligned} \alpha_{k+1}^f &= \mathbf{B}^f(y_{k+1}, x_{k+1}, X_k^\theta)(\mathbf{A}^f)' \alpha_k^f \\ \alpha_{k+1}^\theta &= \mathbf{B}^\theta(y_{k+1}, x_{k+1}) \mathcal{A}^\theta [X_{k+1}^f \otimes I_{L_\theta}] \alpha_k^\theta \end{aligned}} \tag{20}$$

Here, $\mathcal{A}^\theta = [[\mathbf{A}^\theta(e_1^f)]' \ldots [\mathbf{A}^\theta(e_{L_f}^f)]']$. Also $\mathbf{B}^f$ and $\mathbf{B}^\theta$ are observation symbol probability distribution matrices [5]. $\mathbf{B}^f(y_{k+1}, x_{k+1}, X_k^\theta) = \text{diag}[\, b_{k+1}^f(1), \ldots, b_{k+1}^f(L_f)\,]$ and $\mathbf{B}^\theta(y_{k+1}, x_{k+1}) = \text{diag}[\, b_{k+1}^\theta(1), \ldots, b_{k+1}^\theta(L_\theta)\,]$ where

$$\begin{aligned} b_{k+1}^f(i) &\triangleq P[y_{k+1} \mid X_{k+1}^f = e_i^f, x_{k+1}, X_k^\theta] \\ b_{k+1}^\theta(i) &\triangleq P[y_{k+1} \mid X_{k+1}^\theta = e_i^\theta, x_{k+1}] \end{aligned}$$

We now have recursive filters for the state indicator vector estimates, conditioned on each other and the channel parameter sequence. Each of the filters in (20) would be optimal if the information on which they are conditioned was known precisely.

### 3.2. Conditional KF Channel Estimate

Due to the fact that the observations are bi-linear in the indicator vector $X_k^\theta$ and the channel parameter, $x_k$, it is possible to use a conditional linear KF for estimation of the time-varying channel.

The Kalman filter equation for the channel parameter, $x_k$, conditioned on the indicator vectors, $X_k^f$ and $X_k^\theta$, is

$$\boxed{\hat{x}_{k|k} = F \hat{x}_{k|k-1} + K_k [y_k - H_k' \hat{x}_{k|k-1}]} \tag{21}$$

where the gain, $K_k$, and covariance, $\Sigma_k$, are defined in [10] (p.44). Also, $H'_k = h_\theta(X^\theta_k)$ as defined in (12).

We now have a recursive filter for the channel parameter, $x_k$, conditioned on the signal indicator states. If the true indicator state sequences were known, then this filter would be optimal.

### 3.3. The Coupled Algorithm

The practical conditional coupled algorithm which is ultimately implemented, is generated by implementing the filters of equations (20) and (21), and conditioning each filter on the estimates generated by the others.

## 4. SIMULATION STUDIES

Simulation studies demonstrate the ability to estimate an MDPSK signal, and track time-varying channel parameters. The deterministic channel shown, has a very high ratio of channel parameter variation, to information bit rate. These HMM/KF schemes compare favourably to previous schemes presented, especially in the case where dependencies exist in the signal, as discussed previously. The simulation shown is for an MDPSK system with M=16 (ie. 16 allowable phase shifts in the transmitted signal). The figures show that in these conditions of rapidly changing channels and low signal to noise ratio (SNR), the HMM/KF scheme effectivly tracks the channel gain, while the phase can experience clicking. More practical channels produce better results, this simulation is presented to show an extreme case.

## 5. CONCLUSIONS

In this paper we have presented a conditionally coupled HMM/KF algorithm for demodulation of MDPSK and QFM signals in noisy fading channels. The technique makes use of information states, and is particularly suited to cases where dependences exist in the message signal.

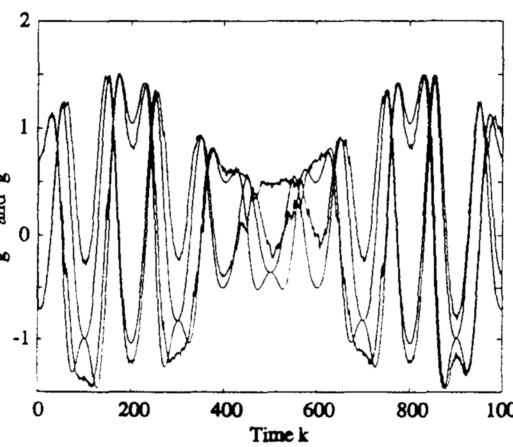


Figure 1. $\hat{x}_k$ for $SNR = 2.4dB$

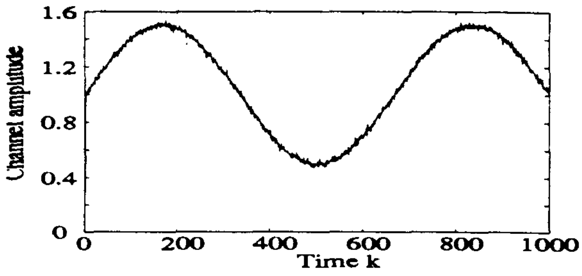


Figure 2. $\hat{\kappa}_k = |\hat{x}_k|$

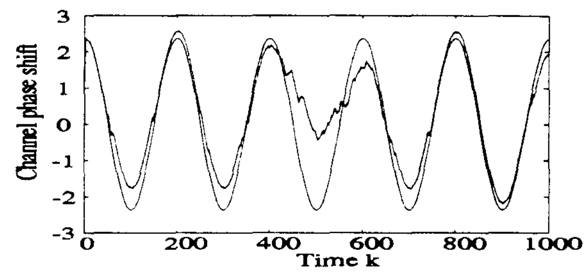


Figure 3. $\hat{\phi}_k = \arg(\hat{x}_k)$

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