Gradient Flow Approach for Pose Estimation of Quadratic Surface

Moonhong Baeg Hideki Hashimoto Fumio Harashima

University of Tokyo
Institute of Industrial Science
7-22-1, Roppongi, Minato-Ku
Tokyo, Japan

John B. Moore*

Australian National University
Dept. of System Engineering
ATC 0200
Canberra, Australia

Abstract - A key problem in robotics is the estimation of the location and orientation of objects from surface measurement data. This is termed pose estimation. Our pose estimation problem is converted to a non-linear optimization problem that minimize an error objective function between the measured surface data and one of CAD model. The authors study gradient flows on the Lie groups toward a solution of the pose estimation problem of quadratic surfaces.

In this paper, the projected gradient flow of the objective function onto the manifold $SO(3)\times\mathbb{R}^3$ is derived and converge to an equilibrium point as usual steepest decent methods. Discretizations of flow lead to recursive numerical methods for pose estimation.

1 INTRODUCTION

The problem of recognizing and locating 3-D curved object in 3-D space is important for applications of computer vision, robotics and autonomous robots. To represent 3-D curved objects, B-rep(Boudary representation), piecewise quadratic representation and curvature of surface have been popular used. This paper concentrate on optimization techenique for locating 3-D curved objcts, specially one for quadratic surface.

There are two approaches in pose estimation of 3-D objects. One is a matching method based on geometric relations between the extracted primitives and CAD model, for example points, lines, curves, normal vectors and curvatures of surface. This kinds of methods need extracting the features, derivatives of surface and preprocessing 3-D data. And also this kind of corresponing point-set matching problem was well studied[1].

The other is optimization of the error measure between CAD model and measured scatted data , which is called surface fitting[2][4], which can handle plane and quadratic surface so on. This paper treats the later.

Pose estimation of known quadratic surfaces from possibly noisy data is important in robotics, and there is room for generating improved algorithms which achieve global optima, and if possible on line.

Faugeras and Hebert [3] introduced the problem of recognizing and location rigid object in 3-D space from range measurements. In their paper representing surface information and extrating such information were discussed. Han, Snyder and Bilbro [5] applied the tree annealing technique to the pose determination of quadratic surface which is similar to our problems presented in this paper. This work is not clear in selection of a intial value and computing time problems.

Current algorithms [3][5][6][7][8][9][10] frequently converge to local minima of the performance index and are unsuited for on-line applications became of the intensive computer effort required.

The optimization of a least squares index is proposed via gradient flows. The stable equilibrium points of the gradient flows from arbitrary initial conditions appear from simulation studies to give the global optimum estimates in the generic case. However, at this stage, firm results are based on implict conditions rather than explicit ones. Even so, confidence in the optimality of the estimates in the small noise case and initializations from the two stage procedure must be high. Such methods are appropriate for on-line optimization, and when organized as recursions using second derivative information converge quadratically. These are also studied, as are related methods based on instrumental variables. And also, the proposed algorithm is tested with real surface range data.

2 PROBLEM FORMULATION

2.1 Quadratic Surface

Consider a known quadratic surface with coefficient matrix $Q = Q' \in \mathbb{R}^{4\times4}$, so that its surface has the form $\xi' Q\xi = 0$, or in obvious notation $x'Q_{11}x + 2Q_{12}'x + Q_{22} = 0$, where $x = [x^1, x^2, x^3] \in \mathbb{R}^3$, $xi = [x, 1]$,and Q is patitioned as

$$Q = \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{12}' & Q_{22} \end{bmatrix}$$

. Without loss of generality, let us consider that this object is located at the origin and that $Q_{22} < 0$. Now a translation by $p \in \mathbb{R}^3$, and a rotation by R belonging to the set of rotation matrices, denoted $SO(3)$, leads to a new quadratic surface coefficient matrix

$$A(R,p) := T'(R,p)QT(R,p) \ , \quad T(R,p) = \begin{bmatrix} R & p \\ 0 & 1 \end{bmatrix} . \quad (1)$$

satisfying

$$\xi' A(R,p)\xi = 0 \ . \quad (2)$$

The set of rotation matrices $SO(3)$ is the set of signed orthogonal matrices R satisfying $R'R = I_3$ such that $|R| = 1$. (Recall that R_1, $R_2 \in SO(3)$ implies $R_1R_2 \in SO(3)$ since $|R_1R_2| = |R_1||R_2|$, and if $|R_1| = -1$, selecting $S = \{\pm1, \pm1, \pm1\}$ such that $|S| = -1$, then $R_2 = R_1S \in SO(3)$. Also $R \in SO(3)$ implies $R' \in SO(3)$).

0-7803-3026-9/95 $4.00 © 1995 IEEE

One representation of R in terms of the Z-Y-Z Euler angles (α, β, γ) is as follows[15][16].

$$R(\alpha,\beta,\gamma) = \begin{bmatrix} c\alpha c\beta c\gamma - s\alpha s\gamma & -c\alpha c\beta s\gamma - s\alpha c\gamma & c\alpha s\beta \\ s\alpha c\beta c\gamma + c\alpha s\gamma & -s\alpha c\beta s\gamma + c\alpha c\gamma & s\alpha s\beta \\ -s\beta c\gamma & s\beta s\gamma & c\beta \end{bmatrix}$$

where $s\alpha = \sin\alpha$, $c\alpha = \cos\alpha$, $s\beta = \sin\beta$, $c\beta = \cos\beta$, $s\gamma = \sin\gamma$, $c\gamma = \cos\gamma$.
The formulas for extracting the Z-Y-Z Euler angles from the rotation matrix is found on lots of text books [16][15].

2.2 Problem Setting

Consider a quadratic object with known coefficient matrix $Q = Q' \in \mathbb{R}^{4\times4}$. In the event of noise free surface data x_i, or $\xi_i' = [x_i' \ 1]$ for $i = 1, 2, \cdots k$, then pose estimation is the estimation of $R \in SO(3)$ and $p \in \mathbb{R}^3$ such that $\xi_i' T'(R,p) Q T(R,p) \xi_i = 0$ for all i with $T(R,p)$ given by (2). In the noisy data case, an appropriate index is a least squares (or variance) index as

$$\Phi_v(R,p) = \frac{1}{2k} \sum_{i=1}^{k} {\phi_i}^2 \ , \quad \phi_i = \xi_i' T'(R,p) Q T(R,p) \xi_i, \quad (3)$$

where $R \in SO(3)$ is the rotation matrix and $p \in \mathbb{R}^3$ is the position vector. Using the definitions

$$\tilde{H}(R,p) := \frac{1}{k} \sum_{i=1}^{k} \phi_i(R,p) \xi_i \xi_i' := \begin{bmatrix} \tilde{H}_{11} & \tilde{H}_{12} \\ \tilde{H}_{12}' & \tilde{H}_{22} \end{bmatrix} \quad (4)$$

then the index can be reformulated as

$$\Phi_v(R,p) = \frac{1}{2} \sum_{i=1}^{k} tr \left(T'(R,p) Q T(R,p) \tilde{H}(R,p) \right) . \quad (5)$$

Now the minimization task is

$$\min_{R \in SO(3), p \in \mathbb{R}^3} \Phi_v(R,p) . \quad (6)$$

3 GRADIENT FLOWS

3.1 Derivation of Gradient Flows

To achieve a gradient flow exploiting techniques in [12][14], first note that $R \in SO(3), p \in \mathbb{R}^3$ is a smooth manifold with tangent space

$$T_{R,p}\left(SO(3) \times \mathbb{R}^3\right) = \left\{ R\Omega \ , \zeta \mid \Omega = -\Omega' \right\} . \quad (7)$$

We define a Riemannian metric

$$\langle (R\Omega_1, \zeta_1), (R\Omega_2, \zeta_2) \rangle := tr \left((R\Omega_1)' (R\Omega_2) + 2\zeta_1' \zeta_2 \right) \quad (8)$$

The directional derivative of Φ at R, p is

$$\begin{aligned} & D\Phi|_{R,p}(R\Omega, \zeta) \\ = \ & \langle (\nabla\Phi_R(R,p), \nabla\Phi_p(R,p)), (R\Omega, \zeta) \rangle \\ = \ & tr \left(\nabla\Phi_R'(R,p) \ R\Omega + 2\nabla\Phi_p'(R,p) \ \zeta \right) . \end{aligned} \quad (9)$$

However

$$\begin{aligned} & D\Phi_v|_{R,p}(R\Omega, \zeta) \\ = \ & tr \left(T' Q \begin{bmatrix} \Omega R & \zeta \\ 0 & 0 \end{bmatrix} \tilde{H} + \begin{bmatrix} -\Omega R' & 0 \\ \zeta' & 0 \end{bmatrix} Q T \tilde{H} \right) \\ = \ & tr \left\{ \underbrace{\left[\left(\tilde{H}_{11} R' + \tilde{H}_{12} p' \right) Q_{11} R + \tilde{H}_{12} Q_{12}' R - R' Q_{11} R \tilde{H}_{11} - R' \left(Q_{11} p + Q_{12} \right) \tilde{H}_{12}' \right]}_{\nabla\Phi_{v,R}'} \Omega \right\} \\ & + 2 \ tr \left\{ \overbrace{\left[\left(\tilde{H}_{12}' R' + \tilde{H}_{22} p' \right) Q_{11} + \tilde{H}_{22} Q_{12}' \right]}^{\nabla\Phi_{v,p}'} \zeta \right\} \\ = \ & tr \left(\nabla\Phi_{v,R}'(R,p) \ R\Omega + 2 \ \nabla\Phi_{v,p}(R,p) \ \zeta \right) \end{aligned} \quad (10)$$

from which the gradient on Φ_v can be identified to yield the gradient flow equation.

$$\begin{aligned} \begin{bmatrix} \dot{R} \\ \dot{p} \end{bmatrix} & = -\begin{bmatrix} \nabla\Phi_{v,R}(R,p) \\ \nabla\Phi_{v,p}(R,p) \end{bmatrix} \\ & = -\begin{bmatrix} R \left(R' \tilde{L}(R,p) - \tilde{L}'(R,p) R \right) \\ Q_{11} \left(R\tilde{H}_{12}(R,p) + p\tilde{H}_{22}(R,p) \right) + Q_{12} \tilde{H}_{22} \end{bmatrix} \end{aligned} \quad (11)$$

where
$$\tilde{L}(R,p) = Q_{11} \left(R\tilde{H}_{11}(R,p) + p\tilde{H}_{12}'(R,p) \right) + Q_{12} \tilde{H}_{12}'(R,p)$$

3.2 Equilibrium Conditions

Observe from(11) that at all equilibria of (11), then

$$R' \tilde{L}(R,p) = \tilde{L}'(R,p) R \ , \quad (12)$$

$$Q_{11} R \tilde{H}_{12}(R,p) + (Q_{11} p + Q_{12}) \tilde{H}_{22}(R,p) = 0 \quad (13)$$

Equivalently, simple manipulations give

$$p = -R\tilde{H}_{12}(R,p) \tilde{H}_{22}^{-1}(R,p) - Q_{11}^{-1} Q_{12} \quad (14)$$

$$[\ R' Q_{11} R \ , \ \tilde{\mathcal{H}}_{11}(R,p) \ ] = 0 \quad (15)$$

where
$\tilde{\mathcal{H}}_{11}(R,p) = \tilde{H}_{11}(R,p) - \tilde{H}_{12}(R,p) \tilde{H}_{22}^{-1}(R,p) \tilde{H}_{12}'(R,p)$, and $[A, B]$ is the Lie brackett $AB - BA$. Also, at all equilibria

$$\tilde{L} = Q_{11} R \tilde{\mathcal{H}}_{11}(R,p) . \quad (16)$$

Moreover, using the diagonalizations

$$\begin{aligned} \mathcal{H}_{11}(R,p) & = V_{\mathcal{H}}(R,p) \Lambda_{\mathcal{H}}(R,p) V_{\mathcal{H}}'(R,p) \\ Q & = V_Q \Lambda_Q V_Q' \end{aligned} \quad (17)$$

with $V_{\mathcal{H}}$, $V_Q \in SO(3)$ and $\Lambda_{\mathcal{H}}$, Λ_Q diagonal in *reverse* order, then the equilibria condition (15) holds if and only if

$$R = V_Q V_{\mathcal{H}}'(R,p) \Pi \ \in \ SO(3) \quad (18)$$

where Π is an arbitrary permutation matrix.
Of course $\tilde{H}_{12}$ and $\mathcal{H}_{11}$, $V_{\mathcal{H}}$ are R,p dependent, so (14) and (18) are implicit, rather than explicit equation for R and p. Since the diagonalizations in (17) are not unique, then R and p satisfying (14) and (18) are not unique. Under the equilibrium condition (14), the index $\Phi_v(R,p)$ can be re-organized by simple manipulation as

$$\begin{aligned}\Phi_v(R,p) &= tr\left[R'Q_{11}R\tilde{\mathcal{H}}_{11}(R,p)\right.\\ &\left.+\tilde{H}_{22}(R,p)\left(Q_{22}-Q_{12}'Q_{11}^{-1}Q_{12}\right)\right]. \quad (19)\end{aligned}$$

Observe that this index is minimized by an R,p selection satisfying (14) and (18) with $\Pi = I$. The minimal index is

$$\Phi_v^* = tr\ [\Lambda_Q\Lambda_{\mathcal{H}} + \tilde{H}_{22}(R,p)(Q_{22} - Q_{12}'Q_{11}^{-1}Q_{12})]. \quad (20)$$

3.3 Instrumental Variables Index

Least squares estimation can lead to biased estimation with surface data x_i, contaminated by white noise. Thus when $x_{i-1} \approx x_i$ (i.e. $\xi_{i-1} \approx \xi_i$), it makes sence to work with an index

$$\Phi_{IV}(R,p) = \frac{1}{k}\sum_{i=1}^{k}\bar{\phi}_i^2,\ \bar{\phi}_i = \xi_{i-1}'T'(R,p)QT(R,p)\xi_i (21)$$

The optimization of this index follows as before but where now $\bar{H}, \bar{L}$ are replaced by

$$\bar{H}(R,p) := \frac{1}{k}\sum_{i=1}^{k}\phi_i(R,p)\xi_{i-1}\xi_i; = \begin{bmatrix}\bar{H}_{11} & \bar{H}_{12}\\ \bar{H}_{12}' & \bar{H}_{22}\end{bmatrix} \quad (22)$$

$$\bar{L}(R,p) = Q_{11}(R,p)\left(R\bar{H}_{11}(R,P) + p\bar{H}_{12}'(R,p)\right) + Q_{12}\bar{H}_{12}'(R,p).$$

3.4 Discrete Gradient Flow

Recursive version of the gradient flows can be derived, paralleling the recursive schemes in [13]. Thus we propose

$$R_{k+1} = R_k\ e^{-\alpha_k\left[R_k'Q_{11}R_k\ ,\ \tilde{\mathcal{H}}_{11}(R_k,p_k)\right]} \quad (23)$$

$$\alpha_k = \frac{1}{4\|Q_{11}\|\ \|\mathcal{H}_{11}(R_k,p_k)\|} \quad (24)$$

$$\begin{aligned}p_{k+1} &= e^{-\tilde{H}_{22}(R_k,p_k)Q_{11}\delta}\ p_k\ - (Q_{11}R_k\tilde{H}_{12}(R_k,p_k)\\ &+Q_{12}\tilde{H}_{22}(R_k,p_k))\delta \quad (25)\end{aligned}$$

for suitably small $\delta > 0$.

A second approach to achieve recursive results is to seek solutions of the implicit optimal conditions (14), (18) recursively. Thus with the definitions (17), a reasonable selection is

$$\begin{aligned}R_{k+1} &= V_Q V_{\tilde{\mathcal{H}}}'(R_k,p_k)\\ p_{k+1} &= -R_{k+1}\tilde{H}_{12}(R_k,p_k)\tilde{H}_{22}^{-1}(R_k,p_k) - Q_{11}^{-1}Q_{12} \quad (26)\end{aligned}$$

No theory is given for convergence of these recursions, but simulation studies suggest that it is effective for local convergence.

4 Simulation Results

Figure 1 shows the exponential convergence of the least squares and instrumental variables optimizations. The log plot of $\Phi_v(t) - \Phi_v(\infty)$ is linear. The noise level is 0.05 and Q is

$$Q = \begin{bmatrix} 0.0069 & 0.0000 & 0.0000 & 0.0000\\ 0.0000 & 0.0400 & 0.0000 & 0.0000\\ 0.0000 & 0.0000 & 0.0156 & 0.0000\\ 0.0000 & 0.0000 & 0.0000 & -1.000\end{bmatrix}$$

Figures 2 - 4 show the corresponding pose estimation errors, $e_T = \|T - \hat{T}\|^2$, $e_R = \|R - \hat{R}\|^2$, $e_p = \|p - \hat{p}\|^2$, and shows that the pose estimation errors e_T are reduced by the gradient flows, although e_R increases !
We see from the comparison between the least squares and instrumental variables approach, the "advantage" of instrumental variables approach in the presence of noise.
Figure 5 $-$ 8 show further comparative studies for various noise levels.

Recursive version of the gradient flows are studied in Figure 9 - 10. The calculation time of recursive version is 50 or 100 times faster than for the continuous version using Matlab.

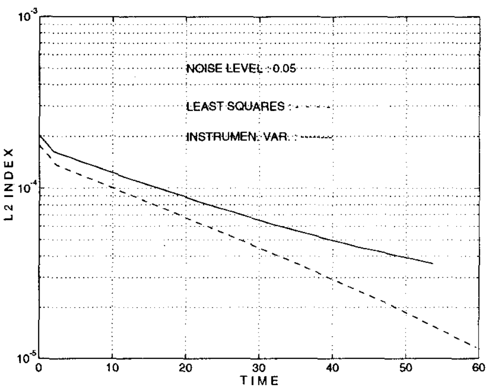

Figure 1: Exponential Convergence of $\Phi_v(t) - \Phi_\infty$

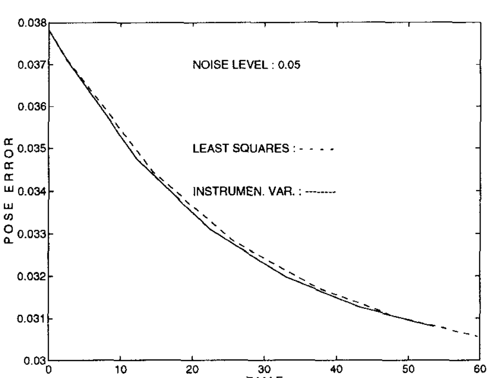

Figure 2: Norm of Transformation Matrix Error e_T

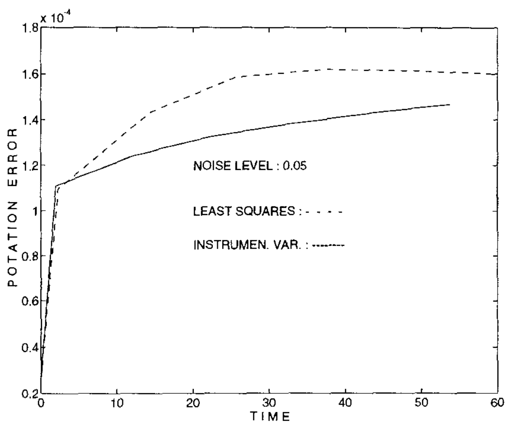

Figure 3: Norm of Rotation Matrix Error e_R

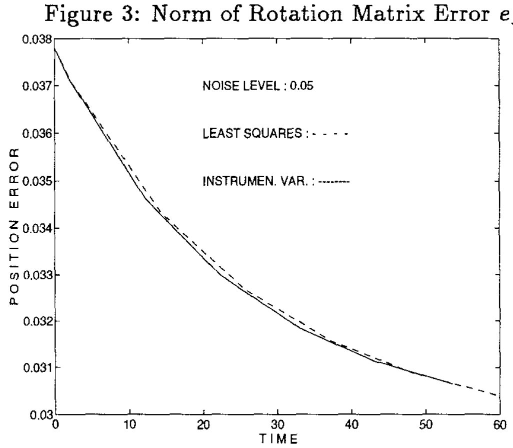

Figure 4: Norm of Position Vector Error e_p

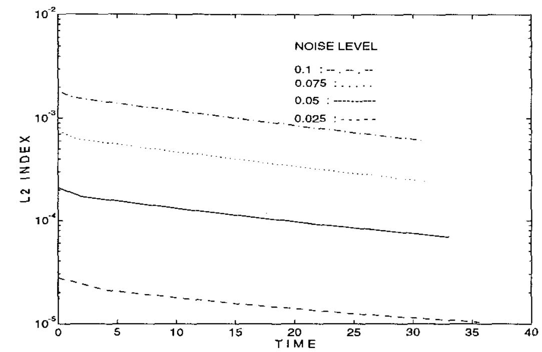

Figure 5: Exponential Convergence of $\Phi_v(R,p)$ according to Various Noise

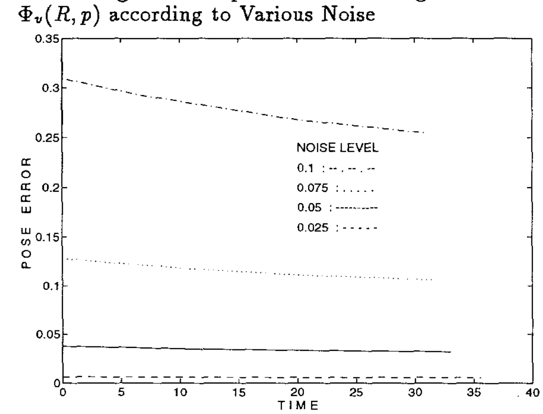

Figure 6: Norm of Transformation Matrix Error e_T according to Various Noise

5 EXPERIMENT

Now we verify the above algorithms with real 3-D range data measured by laser range finder(LRF). Our LRF has 1.5 mm maximum error which is about ten times larger than a commertial products.

Consider that an quadratic surface with known coefficient matrix from CAD model

$$0.25\ x^2 + 0.04\ y^2 + 0.25\ z^2 = 1 \tag{27}$$

is moved by Z-Y-Z Euler angles $\alpha = 0\ deg.$, $\beta = 0\ deg.$, $\gamma = 0\ deg.$ and a relocation $p = [\,-1.0\ \ 6.0\ \ -38.0\,]' \in \mathbb{R}^3$. The new quadratic surface coefficients matrix

$$A = \begin{bmatrix} -0.0007 & 0 & 0 & 0.0007 \\ 0 & -0.0001 & 0 & -0.0007 \\ 0 & 0 & -0.0007 & 0.0263 \\ 0.0007 & -0.0007 & 0.0263 & -1.0000 \end{bmatrix} \tag{28}$$

is obtained from (1).

The object reconstructed by 3D range data (15 x 15) is depicted as Figure 11.

Let us consider now pose estimation. The quadratic surface in this experiment has a freedom on Y axis as shown on Figure 12. Therefore the only Y axis is considered.

The optimization of a least squares index is performed via gradient flows initialized by the pose estimates from as a good candidator[17].

Figure 13 shows the convergence of the gradient flow with real data. Figure 14- 16 show the corresponding pose estimation errors, $e_p, e_R, \delta_{XY}, \delta_{YZ}$.

6 CONCLUSIONS

The pose estimation procedures have been presented, based on an approach improving gradient flows on manifolds. A number of variations of this optimization are presented, including recursive schemes. Equilibria conditions are studied and convergence results given. Simulation studies show the relative strengths of the methods in a typical example.

To validate our approach, we made the Laser Range Finder with 1.5 mm maximum error and estimated the pose of quadratic object by using the real range data.

References

[1] K. Arun T. S. Hwang and S. Blostein,"Least-Squares Fitting of Two 3-D point sets", IEEE Trans. on Pattern Anal. Machine Intell., pp. 698-700, Sep., 1987.

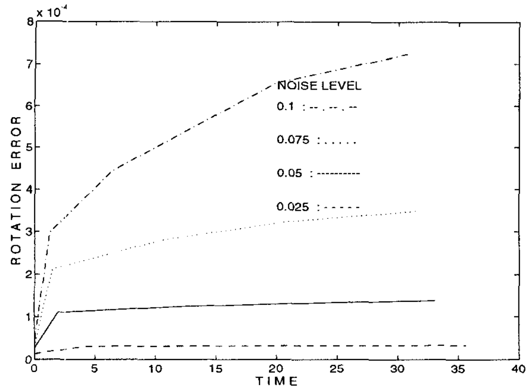

Figure 7: Norm of Rotation Matrix Error e_R according to Various Noise

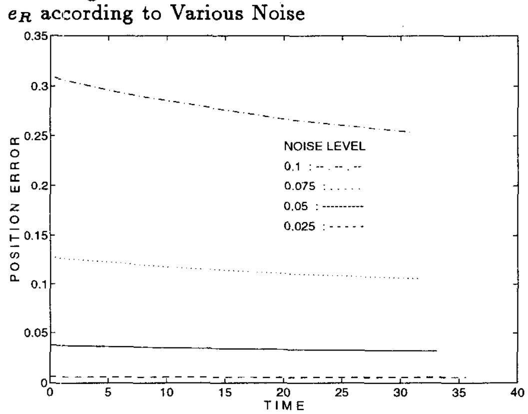

Figure 8: Norm of Position Vector Error e_p according to Various Noise

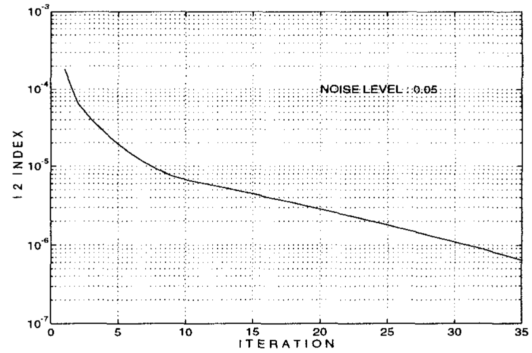

Figure 9: Recursive Flow: Exponential Convergence of $\Phi_v(R,p)$

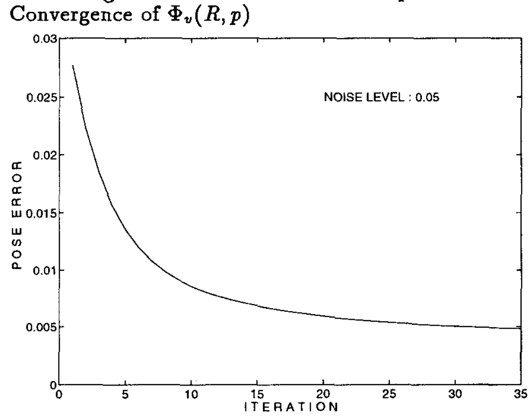

Figure 10: Recursive Flow: Norm of Transformation Matrix Error e_T

[2] P. J. Besl and N. D. Mckey,"A Method for Registration of 3-D Shapes", IEEE Trans. on Pattern Anal. Machine Intell., **Vol. 14 No. 2**, pp. 239-256, Feb., 1992.

[3] O. D. Faugeras and M. Hebert," The representation, Recognition, and Locating of 3-D Objects", IEEE Inter. J. of Robotics Research, Vol.5, No.3, 1986.

[4] T. S. Newman, P. J. Flynn and A. K. Jain,"Model Based Classification of Quadratic Surfaces", **CVGIP**: Image Understanding, **Vol. 58, No.2**, pp. 235-249, 1993.

[5] G. L. Bilbro and W. E. Snyder,"Linear Estimation of Object Pose from Local Fits to Segments", IEEE Inter. Conf. on Robotics & Automation, pp. 1747-1752, 1987.

[6] J. S. Bay,"Tactile Shap Sensing via Single- and Multi-Fingered Hands", IECON'89, pp. 290-295, 1989.

[7] D. S. Kalivas, A. A. Sawchuk,"A Region Matching Motion Estimation Algorithm", CVGIP: Image Understanding, pp. 275-288, 1991.

[8] Y. Han and W. E. Snyder and G. L. Bilbro ,"Pose Determination using Tree Annealing", IEEE Inter. Conf. of Robotics & Automation, pp. 427-432, 1990.

[9] I. S. Kweon and T. Kanade,"High Resolution Terrain Map from Multiple Sensor Data", IEEE International Workshop on Intelligent Robots and System, 1990.

[10] D. G. Lowe,"Fitting Parameterized Three-Dimensional Models to Images", IEEE PAMI, Vol. 13, No. 5, pp. 441-450, 1991.

[11] U. Helmke and J. B. Moore: Optimization and Dynamical Systems, Spriner-Verlag, Communication and Control Engineering Series, pp. 45-59, 1993.

[12] U. Helmke and J. B. Moore: Optimization and Dynamical Systems, Spriner-Verlag, Communication and Control Engineering Series, pp. 45-59, 1993.

[13] U. Helmke and J. B. Moore: Optimization and Dynamical Systems, Spriner-Verlag, Communication and Control Engineering Series, pp. 69-77, 1993.

[14] R. W. Brockett,"Least squares matching problems", Linear Algebra Appl., 122-124, pp. 761-777.

[15] K. S. Fu and R. C.Gonzalez and C.S.G. Lee ,"Robotics", McGRAW-HILL.

[16] J. J. Craig,"Introduction to Robotics Machanics and Control", Addison Wesley, 1986.

[17] M. Baeg, H. Hashimoto, F. Harashima, J. Moore,"Pose Estimation of Quadratic Surface Using Surface Fitting Technique",IROS'95, pp.204-209,1995.

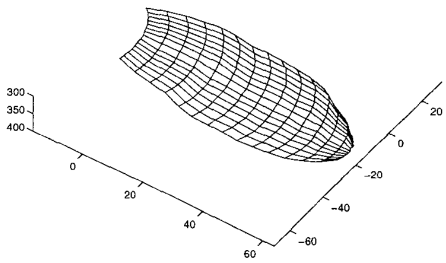

Figure 11: Reconstruction of Object

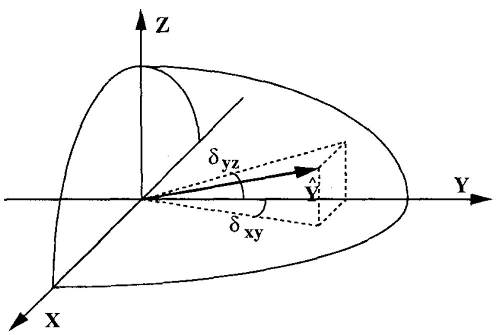

Figure 12: Degree of Freedom on Y axis

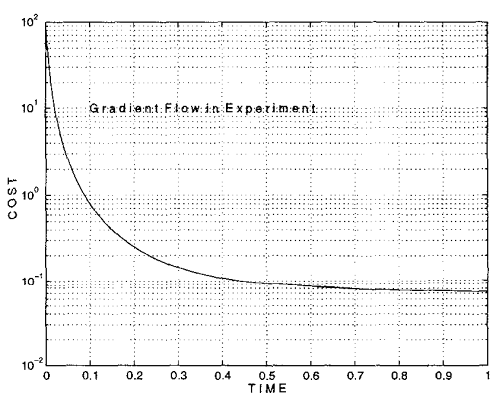

Figure 13: Gradient Flow on Experiment

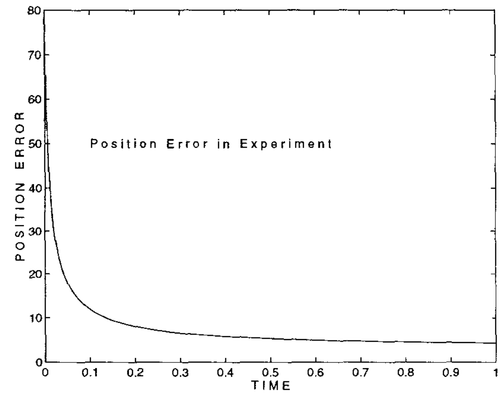

Figure 14: Position Error on Experiment

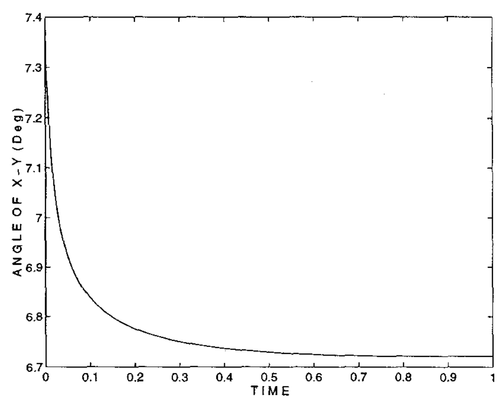

Figure 15: Angle Between Y-axis and Y-Z Plane

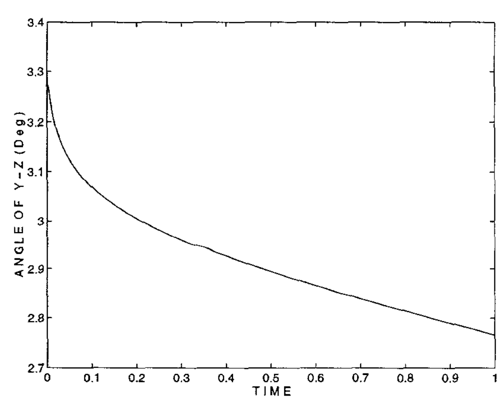

Figure 16: Angle Between Y-axis and X-Y Plane