Improved Demodulation of Sampled FM Signals in High Noise

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Abstract—Simulation results are presented which are very convincing in favor of FM demodulators driven by in-phase and quadrature-phase signals. Application of the extended Kalman filtering algorithms to the appropriate signal model directly yields demodulators in which the error covariance equations are uncoupled from the state estimate equations. These demodulators perform a little better than others derived using different nonlinear filtering techniques and the necessary approximations to achieve "decoupling." More significantly, these demodulators using in-phase and quadrature-phase sampled signals are readily augmented to achieve demodulation with delay by the application of fixed-lag smoothing algorithms. Simulations highlight the attractive trade-offs between demodulation complexity and performance results for this class of demodulator.

INTRODUCTION

With recent advances in digital system technology, it appears attractive to employ digital FM demodulators in some applications. In particular, in-phase and quadrature phase signal ideas explored in [1-3] can now be handled conveniently using present day digital technology. More significantly fixed-lag smoothing ideas [4-6] to achieve demodulation with delay are most conveniently realized in discrete time.

In [1, 2], Mallinckrodt, et al. and Bucy, et al. have noted the equivalence between phase-locked loops and continuous-time extended Kalman filters for phase demodulation. Their phase demodulation procedure involves first the removal of the known carrier by heterodyning down to base band, producing both in-phase and quadrature components. Application of the extended Kalman filter (EKF) algorithm to such a base band model then results in the continuous-time state estimate equation being decoupled from the continuous-time error covariance equation. Our own discrete-time simulation of these demodulators suggests that they give performance characteristics superior to earlier schemes in [3, 5, 7]. There is a difficulty, however, for the continuous-time demodulators to further improve performance using fixed-lag smoothing ideas.

In [3], the advantages of in-phase and quadrature-phase sampled signals are explored. The demodulators are derived using discrete invariant embedding to a two point boundary value problem. However, the resulting matrix equation needed for the evaluation of the processor gain is still coupled to the processor equation and low noise assumptions are required to achieve a decoupling of these.

Fixed-lag smoothing ideas are applied to achieve FM demodulators with delay in [5-7]. The performance of demodulators with delay is always an improvement on demodulators without delay, although in high noise (near threshold) the performance improvement may be negligible. It should also be noted that demodulators with delay are difficult to design in continuous-time since suboptimal techniques must be employed and certain approximations are not at all straightforward to optimize. It is preferable to work with discrete-time signals for the ready design and implementation of fixed-lag smoothers.

In this work, we derive discrete-time demodulators from quadrature and in-phase sampling of the received analog signal (at IF frequencies) as studied in [3]. It turns out that various versions of the discrete-time extended Kalman filter, when applied to the appropriate discrete-time model, directly yield demodulators in "uncoupled" form. The discrete-time filter equations are very like those used for simulation studies in [1, 2] and it is not surprising that the performance of our discrete-time demodulators is comparable to that of the continuous-time versions of [1, 2]. The demodulators of this note incorporate ideas from [1-3] to achieve properties not achieved in [1-3]. In particular, the performance of the discrete-time demodulators derived in this note can be readily improved by additional computation in discrete-time to achieve fixed-lag smoothed estimates using the ideas of [4-6]. Moreover, the resulting FM demodulators with delay achieve worthwhile performance improvement even at threshold noise levels.

SIGNAL MODEL

For simplicity, we shall restrict attention to the first-order Butterworth message spectrum and assume oscillator instability to be negligible. In [3, 5, 7], the system equations for such a model are in the following form:

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$$\dot{x}(t) = \begin{bmatrix} \dot{\lambda}(t) \\ \dot{\theta}(t) \end{bmatrix} = F \begin{bmatrix} \lambda(t) \\ \theta(t) \end{bmatrix} + Gu(t); \qquad F = \begin{bmatrix} -\alpha & 0 \\ \gamma & 0 \end{bmatrix},$$

$$G = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad h(t) = \sqrt{2} \sin\left[\omega_c t + \theta(t)\right]. \tag{1}$$

The measurement equation depends on the type of sampling as follows.

Scalar Sampling [5, 7]

$$z(t_k) = h(t_k) + v(t_k), \qquad k = 0, 1, 2, \cdots. \tag{2}$$

Quadrature and In-Phase Sampling [3]

$$z(t_k) = \sqrt{2} \begin{bmatrix} \sin\theta(t_k) \\ \cos\theta(t_k) \end{bmatrix} + \begin{bmatrix} v_1(t_k) \\ v_2(t_k) \end{bmatrix}. \tag{3}$$

DIGITAL DEMODULATION SCHEMES

In [5, 7], the extended Kalman filter algorithm and higher order versions of this are applied to the signal model with *scalar sampling*. It is found that the error covariance equation for any of such filters is coupled to the processor equation and thus cannot be calculated off-line as in linear filtering. By expanding terms involving coupling in a Fourier series and neglecting terms with higher order harmonic content, as noted in [7], the error covariance equation can be decoupled from the processor equation. However, simulation results in [7] indicate that the performance of such uncoupled estimators is not as good as that of the coupled estimators.

We propose to apply the extended Kalman filter algorithms together with the fixed-lag smoothing equations of [5, 6] to the signal model with *quadrature and in-phase sampling*. Because the system dynamics is linear, there is obviously no coupling between the state and error covariance equations for one step ahead predicton. Fortuitously, the measurement updating equations for the extended Kalman filter algorithm are also uncoupled. Since these are readily derived as in [1, 2] they are not repeated here. The smoothing equations are however presented to indicate their complexity.

Smoothing Equations

$$\hat{x}_i(t_k \mid t_k) = \hat{x}_i(t_k \mid t_{k-1}) + P_i(t_k \mid t_{k-1}) P^{-1}(t_k \mid t_{k-1}) \cdot [\hat{x}(t_k \mid t_k) - \hat{x}(t_k \mid t_{k-1})] \tag{4}$$

$$P_i(t_k \mid t_k) = P_i(t_k \mid t_{k-1}) - P_i(t_k \mid t_{k-1}) \cdot [I - P^{-1}(t_k \mid t_{k-1}) P(t_k \mid t_k)] \tag{5}$$

$$P_{ii}(t_k \mid t_k) = P_{ii}(t_k \mid t_{k-1}) - P_i(t_k \mid t_{k-1}) P^{-1}(t_k \mid t_{k-1}) \cdot [I - P(t_k \mid t_k) P^{-1}(t_k \mid t_{k-1})] P_i(t_k \mid t_k) \tag{6}$$

where $x_i(t_k) \equiv x(t_{k-i})$, $P_i(t \mid t_k) \equiv E\{[x_i(t) - \hat{x}_i(t \mid t_k)] \cdot [x(t) - \hat{x}(t \mid t_k)]'\}$ and $p_{ii}(t \mid t_k) \equiv E\{[x_i(t) - \hat{x}_i(t \mid t_k)] \cdot [x_i(t) - \hat{x}_i(t \mid t_k)]'\}$.

SIMULATION RESULTS FOR ZERO-LAG DEMODULATORS

Simulations are performed with the model (1), (3) with $\lambda(t_0)$ a Gaussian random variable of zero mean and unity variance, and $\theta(t_0)$ uniformly distributed in $[-\pi, \pi)$. We set $\gamma = 1$, $q = 2\alpha$ so that $\lim_{t\to\infty} E\{\lambda^2(t)\} = 1$, the root-mean-square bandwidth of the FM baseband spectrum = 1 radian, and the bandwidth expansion ratio $\beta = 1/\alpha$. We select $t_k = kT$. $T = 2\pi/16$ seconds to permit adequately fast sampling of the FM baseband spectrum.

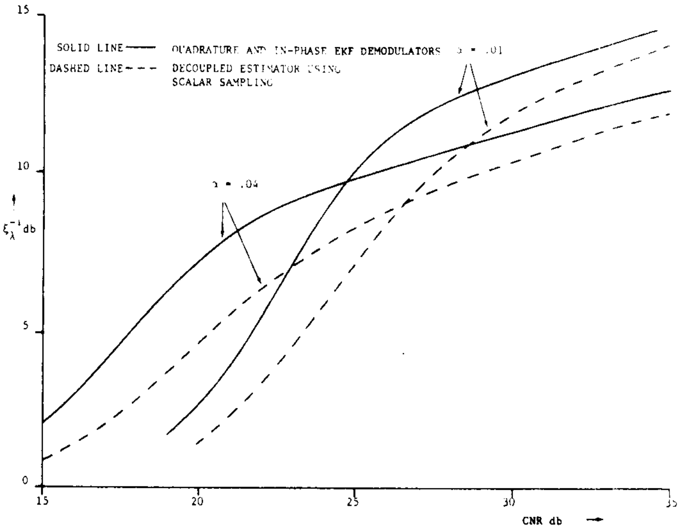

Fig. 1. Comparison of FM demodulators.

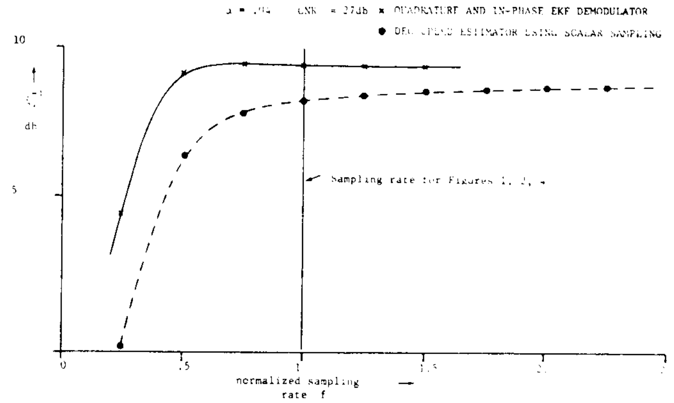

Fig. 2. Performance variation with sampling rate.

Simulations show that the performance of the extended Kalman filter is almost the same as that of the more complicated modified second order filters.

Our demodulation for quadrature and in-phase sampling provides about 2 dB threshold extension when compared with uncoupled demodulation of [7] using scalar sampling, as indicated in Figure 1 where ξ_λ^{-1} is the inverse of the evaluated mean square message error and CNR $= 2/(\alpha rT)$ is the carrier to noise ratio in the message bandwidth.

As an illustration of the variation of performance with sampling rate, Figure 2 shows plots of ξ_λ^{-1} versus f the normalized sampling rate for $\alpha = .04$ and CNR = 27 dB. f is evaluated as T_0/T_v when T_v is the variable sampling time interval and $T_0 = 2\pi/16$ (the sampling time interval used in the other simulation results is presented in Figure 1, 3).

SIMULATION RESULTS FOR FINITE-LAG DEMODULATORS

Figure 4 presents digital computer simulation results for fixed-lag demodulators around the threshold region. It can be seen that the fixed-lag demodulators always perform better

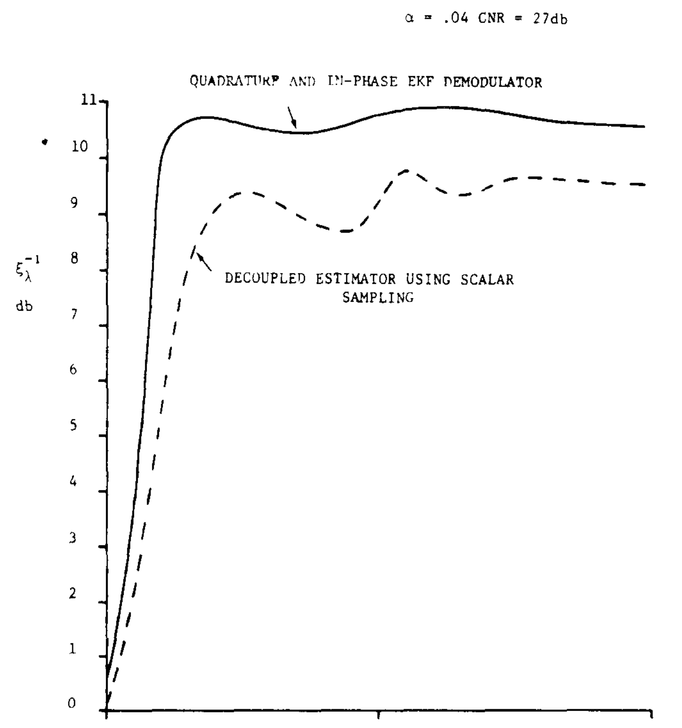

Fig. 3. Transient response.

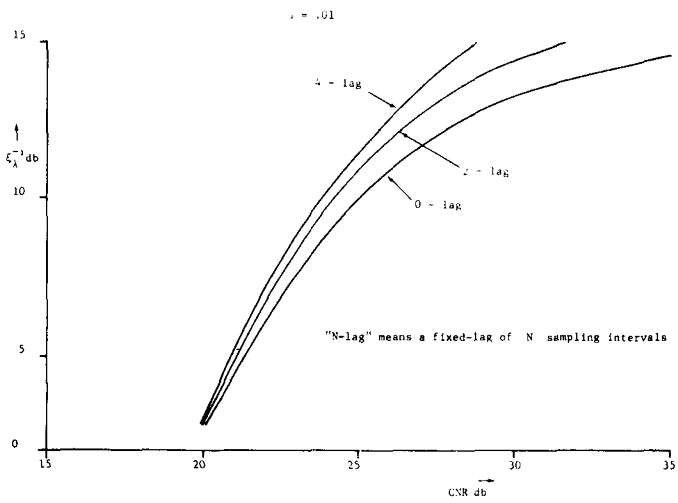

Fig. 4. Performance of fixed-lag FM demodulators.

than the demodulator with zero lag. The improvement of inverse mean-square message error ranges from a fraction of a dB in the high noise region to several dB in the low noise region.

CONCLUDING REMARKS

Our simulations have demonstrated the power of using in-phase and quadrature phase signals, and of using fixed-lag smoothing for FM demodulation. The discrete-time versions driven by in-phase and quadrature phase sampled signals and derived by application of extended Kalman filter theory to the appropriate signal model are clearly very attractive demodulators from the complexity versus performance point of view. The fact that the error covariance equations are automatically uncoupled from this filtering equations gives these schemes a slight advantage over other schemes where all else is equal. To achieve even further improvement in performance appears to require banks of such demodulators as in [8, 9].

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