

Risk-sensitive Dual Control

Subhrakanti Dey, *Student Member, IEEE* John B. Moore, *Fellow, IEEE*

Department of Systems Engineering,
Research School of Information Sciences and Engineering,
Australian National University, Canberra ACT 0200, Australia
Tel: +61 6 249 2456 Fax: +61 6 279 8088 E-mail: subhra@syseng.anu.edu.au

Abstract

In this paper, we develop new results concerning the risk-sensitive dual control problem for output feedback nonlinear systems, with unknown time-varying parameters. A dynamic programming equation solution is given to an optimal risk-sensitive dual control problem penalizing outputs, rather than the states, for a reasonably general class of nonlinear signal models. This equation, in contrast to earlier formulations in the literature, clearly shows the dual aspects of the risk-sensitive controller regarding control and estimation. The extensive computational burden for solving this equation motivates our study of risk-sensitive versions for one-step horizon cost indices and suboptimal risk-sensitive dual control. The idea of a more generalized optimal risk-sensitive dual controller is briefly introduced.

1 Introduction

The concept of dual control is generally attributed to Fel'dbaum [1]. In the case of a partially observable system, it has been shown [2] [3] that the dynamic programming equation solution to the optimal control problem is computationally more difficult than for the complete information case. The additional computational effort is attributed to the fact that in the case of a system with unknown (possibly time-varying) parameters, the task of the control actions is twofold- probing for achieving information concerning the states, and feedback of this information to achieve control objectives. Probing for state estimation needs more aggressive control than for the case when the states are known, and hence good control and good estimation are conflicting objectives. The optimal control in the case of partially observable systems achieves a trade-off between these two conflicting demands.

In this paper, we study the risk-sensitive version of the dual control problem. Although [8] actually addresses the risk-sensitive optimal control problem for partially observable systems and achieves a dynamic programming equation by applying change of probability measure technique, it is difficult to interpret the dual aspects of risk-sensitive control from these results. By considering a cost function which penalizes the system output, we achieve a dynamic programming equation in Section 2, which achieves the

same objectives, without resorting to the measure change technique of [8]. In Section 3, we consider the risk-sensitive version of the one-step horizon control problem (also known as the cautious control problem [2]) which has not been considered in literature for systems with unknown time-varying parameters. In Section 4, we present a suboptimal risk-sensitive dual controller, the risk-neutral version of which has been considered in [7]. Finally, the idea of a more generalized optimal risk-sensitive dual controller is briefly introduced in Section 5. We do not present the proofs of the theorems stated in this paper due to lack of space, but they are readily available with the authors.

2 Risk-sensitive Dual Control

In this section, we introduce the risk-sensitive dual control problem for a certain class of nonlinear systems. We describe the signal model, introduce the cost criterion and give a dynamic programming equation solution to the optimal control problem assuming separability between estimation and control.

2.1 Signal Model

We consider the following discrete-time stochastic nonlinear state space model defined on a probability space (Ω, \mathcal{F}, P) :

$$\begin{aligned}x_{k+1} &= A_k(x_k) + B_k(u_k) + w_{k+1} \\ y_k &= C_k(x_k) + v_k\end{aligned}\quad (1)$$

where $x_k, w_k \in \mathbb{R}^n$, $y_k, v_k \in \mathbb{R}^p$, $u_k \in \mathbb{R}^m$. Here, x_k denotes the augmented state of the system including the unknown system parameters, u_k denotes the control input, y_k denotes the measurement, w_k and v_k are the process noise and the measurement noise respectively. The vectors A_k , B_k and C_k are nonlinear functions in general. We assume that $w_k, k \in \mathbb{N}$ has a density function ψ_k and $v_k, k \in \mathbb{N}$ has a strictly positive density function ϕ_k . The initial state x_0 or its density is assumed to be known and w_k is independent of v_k .

Remark 1 The following results can be obtained for more general nonlinear signal models without much difficulty,

although special restrictions might apply to the functional nature of the nonlinear relationship. Such restrictions have been reported for risk-neutral nonlinear filtering in [11] and for risk-sensitive nonlinear filtering in [9]. Similarly, these results can be easily extended to hidden Markov models with finite-discrete states.

2.2 Cost Criterion

Define $Y_k \triangleq (y_0, y_1, \dots, y_k)$, the σ -field generated by Y_k as \mathcal{Y}_k^0 and the corresponding complete filtration by \mathcal{Y}_k . Also define $U_{m,n}$ to be the set of the admissible controls u_k in the interval $m \leq k \leq n$, where u_k is \mathcal{Y}_k measurable. The risk-sensitive cost criterion for the dual control problem is given as, for $u \in U_{k-1, T-1}$,

$$J(u) = E \left[\exp \left\{ \theta \left(\sum_{i=k}^T L(y_i, u_{i-1}, r_i) \right) \right\} \right] \quad (2)$$

The problem objective is to find $u^* \in U_{k-1, T-1}$ such that

$$u^* = \underset{u \in U_{k-1, T-1}}{\operatorname{argmin}} E \left[\exp \left\{ \theta \left(\sum_{i=k}^T L(y_i, u_{i-1}, r_i) \right) \right\} \right] \quad (3)$$

Here, $r_i \in \mathbb{R}^p$, $i \in \mathbb{N}$ is the reference output that is supposed to be tracked by y_i . We also assume that $L \in C(\mathbb{R}^p \times \mathbb{R}^m \times \mathbb{R}^p)$ is non-negative, bounded and uniformly continuous. $\theta (> 0)$ is the risk-sensitive parameter.

Using a fundamental result of stochastic control [2], the problem objective is to find u^* such that

$$u^* = \underset{u \in U_{k-1, T-1}}{\operatorname{argmin}} E \left[\exp \left\{ \theta \left(\sum_{i=k}^T L(y_i, u_{i-1}, r_i) \right) \right\} \right] \quad (4)$$

Remark 2 The cost criterion could have been expressed in terms of the state x_i , rather than the output y_i , as

$$J(u) = E \left[\exp \left\{ \theta \left(\sum_{i=k}^{T-1} L(x_i, u_{i-1}) + \Phi(x_T) \right) \right\} \right] \quad (5)$$

where $L \in C(\mathbb{R}^n \times \mathbb{R}^m)$ is non-negative, bounded and uniformly continuous in (x, u) and $\Phi \in C(\mathbb{R}^n)$ is non-negative, bounded, and uniformly continuous. This risk-sensitive control problem has been solved in [8] using change of probability measure techniques. But the dual aspects of the control are not so evident from the dynamic programming equation obtained in [8] and so this case is not studied here any further.

2.3 Dynamic Programming

We have separability between estimation and control as in [8]. The estimation problem is solved by evaluating the information state, which in this case is a conditional probability density function of the state given the observations.

Definition 3 Define the information state $\alpha_{k|k-1}(x)$ such that

$$\alpha_{k|k-1}(x) dx = E [I(x_k \in dx) | \mathcal{Y}_{k-1}] \quad (6)$$

Remark 4 It is obvious that for a general nonlinear signal model, $\alpha_{k|k-1}(x)$ will be given by an infinite-dimensional recursion. Also, $\alpha_{k|k-1}(x)$ can be approximated by the Gaussian sum approach [11].

Definition 5 Let us define the value function $V(\alpha_{k|k-1}, k)$ such that

$$V(\alpha_{k|k-1}, k) = \inf_{u \in U_{k-1, T-1}} E \left[\exp \left\{ \theta \left(\sum_{i=k}^T L(y_i, u_{i-1}, r_i) \right) \right\} \right] \quad (7)$$

Remark 6 We assume here that $\exp \left\{ \theta \left(\sum_{i=k}^T L(y_i, u_{i-1}, r_i) \right) \right\}$ is integrable.

Theorem 7 The value function $V(\alpha_{k|k-1}, k)$ satisfies the following recursive dynamic programming equation

$$V(\alpha_{k|k-1}, k) = \inf_{u_{k-1}} E \left[\exp \left\{ \theta (L(y_k, u_{k-1}, r_k)) \right\} V(\alpha_{k+1|k}, K+1) | \mathcal{Y}_{k-1} \right] \quad (8)$$

$$V(\alpha_{T|T-1}, T) = \inf_{u_{T-1}} \int_{\mathbb{R}^p} \int_{\mathbb{R}^n} \exp \left\{ \theta (L(c_T(x) + v, u_{T-1}, r_T)) \right\} \alpha_{T|T-1}(x) \phi_T(v) dx dv \quad (9)$$

Remark 8 Note that considering the cost criterion (2) instead of (5) results in the dynamic programming equation (8) which clearly shows the dual nature of the risk-sensitive control, which is explained below. Also, change of probability measure techniques are not required to achieve this dynamic programming equation.

2.4 Dual Aspects of Risk-sensitive Control

The dynamic programming equation (8) involves computing the expectation of the product of two terms. The first term denotes the immediate risk-sensitive control cost.

The second term is a function of $\alpha_{k+1|k}(x)$ which itself is a function of Y_k and u_{k-1}, \dots, u_0 . This implies that u_{k-1} not only affects the immediate risk-sensitive control cost but also influences the future information state, or in other words, the estimation procedure. It has been shown similarly in [2] for a linear single-input, single-output model with unknown time-varying parameters (where the information state is finite-dimensional), the optimal control affects the immediate control cost as well as influences the future parameter estimates and their accuracy. In analogy to this risk-neutral dual control problem, the optimal control problem under consideration in this paper is indeed a risk-sensitive dual control problem.

Remark 9 The solution of (8) involves, at each step, discretization of the value function in the variables of the information state, evaluation of the expectation and minimizing with respect to u_{k-1} subject to the constraint that the information state takes a particular combination of values, for each such combination. Of course, for the general nonlinear signal model, this becomes analytically impossible due to the infinite dimensionality of the information state, unless one approximates the information state by a finite-dimensional representation such as a Gaussian sum representation. Even when the information state is finite-dimensional, it has been shown [2] that the solution to the corresponding dynamic programming equation for the risk-neutral dual control problem cannot be achieved analytically. Numerical solutions to a handful of simple problems exist, but are computationally very expensive because the computational complexity increases exponentially with the dimension of the information state.

3 Risk-sensitive Cautious Control

The risk-neutral version of the finite-horizon multi-step optimization task considered in the previous section has been found to be analytically impossible to solve. Even numerical solutions are computationally expensive. This difficulty has led researchers to consider alternative cost functions. In the risk-neutral case, a single-step analytically solvable cost-criterion (see pg 21, [2]) results in an optimizing controller which is known to be a *myopic controller*. It is also known as a *cautious controller* because, in comparison with the *certainty equivalence controller*, it “hedges” by decreasing the feedback gain when the parameter estimates are uncertain and have large variances. Unfortunately, this optimizing control does not introduce any probing feature into the algorithm, and hence does not have the dual aspects. We present the risk-sensitive version of the cautious control problem in this section, followed by the analytical expression for the optimizing control for a single-input single-output ARX model with unknown

time-varying parameters. We also illustrate with an example that the solution to the risk-neutral cautious control is not optimal for the risk-sensitive cautious control cost criterion, which is appropriate for systems with plant or noise uncertainties. This optimization task has been also considered in [10] but not for systems with unknown time-varying parameters.

3.1 Cost-criterion

The risk-sensitive cautious control cost-criterion is given as (by putting $T=k$ in (4))

$$J_c(u_{k-1}) = E \left[\exp \left\{ \theta \left(L(y_k, u_{k-1}, r_k) \right) \right\} \mid \mathcal{Y}_{k-1} \right] \quad (10)$$

and the problem objective is to find a u_{k-1}^* such that

$$u_{k-1}^* = \underset{u_{k-1} \in U_{k-1, k-1}}{\operatorname{argmin}} J_c(u_{k-1}) \quad (11)$$

where the necessary assumptions are same as in the previous section.

3.2 Risk-sensitive cautious control for an SISO ARX model

We consider the risk-sensitive cautious control problem for a single-input, single-output, minimum phase, ARX model where the parameters are unknown and time-varying. We present the signal model and then present the solution to the problem. Note that since this is a single-step optimization problem, there is no dynamic programming involved here.

Signal model

Consider the discrete-time SISO ARX model

$$y_k + a_k^1 y_{k-1} + \dots + a_k^n y_{k-n} = b_k^1 u_{k-1} + \dots + b_k^n u_{k-n} + v_k \quad (12)$$

where y_k, u_k, v_k are output, input and measurement noise respectively at the k -th time instant. The noise sequence $\{v_k\}, k \in \mathbb{N}$ is assumed to be Gaussian distributed with a density $\phi_v \sim N(0, \sigma_v^2)$. v_k is also assumed to be independent of $y_i, i \in \{1, 2, \dots, k-1\}$ and $a_i^j, b_i^j, i \in \{1, 2, \dots, k\}, j \in \{1, 2, \dots, n\}$. It is further assumed that $b_k^1 \neq 0 \forall k$.

The state of the system is denoted by $x_k = [b_k^1 \ b_k^2 \ \dots \ b_k^n \ a_k^1 \ \dots \ a_k^n]'$ and the state dynamics is given by

$$x_{k+1} = A_k x_k + w_k \quad (13)$$

where A_k is a known matrix and $\{w_k\}$ is a sequence of *i.i.d* random vectors distributed with a density function $\phi_w \sim N(0, \Sigma_w), \forall k \in \mathbb{N}$.

With this state description, the output dynamics is given by

$$y_k = \psi'_{k-1} x_k + v_k \quad (14)$$

where

$$\psi'_{k-1} = [u_{k-1} \dots u_{k-n} - y_{k-1} \dots - y_{k-n}]$$

The initial state x_0 or its distribution is assumed to be known.

Cost-criterion

The single-step optimization index is chosen to be an exponential of a quadratic in this case, so that the problem objective is to find

$$u_{k-1}^* = \underset{u_{k-1} \in U_{k-1, k-1}}{\operatorname{argmin}} J_c(u_{k-1}) \quad (15)$$

where

$$J_c(u_{k-1}) = E \left[\exp \left\{ \frac{\theta}{2} (y_k - r_k)^2 \right\} \mid \mathcal{Y}_{k-1} \right]$$

Remark 10 It should be noted that the minimum phase assumption is not restrictive. Non-minimum phase systems can be treated by including a term penalizing the control cost in the cost index described above.

Remark 11 SISO systems are treated to maintain notational simplicity and to develop a theory that is intuitively appealing. Multiple-input, multiple-output (MIMO) systems can be treated easily by considering a suitable cost index.

Optimal cautious control

Estimation:

As mentioned earlier, the estimation problem is separated from the control by applying the separation principle. It can be easily shown [2] that $\alpha_{k|k-1}(x)$ is Gaussian with mean $\hat{x}_{k|k-1}$ and variance $P_{k|k-1}$ satisfying the recursions

$$\hat{x}_{k+1|k} = A_k \hat{x}_{k|k-1} + K_k (y_k - \psi'_{k-1} \hat{x}_{k|k-1}) \quad (16)$$

$$P_{k+1|k} = (A_k - K_k \psi'_{k-1}) P_{k|k-1} A_k' + \Sigma_w \quad (17)$$

$$K_k = A_k P_{k|k-1} \psi_{k-1} (\sigma_v^2 + \psi'_{k-1} P_{k|k-1} \psi_{k-1})^{-1} \quad (18)$$

with $x_{0|-1}$, $P_{0|-1}$ known. Hence, the conditional density of y_k given \mathcal{Y}_{k-1} is Gaussian with mean \hat{y}_k and variance $\sigma_{y_k}^2$ given by

$$\hat{y}_k = \psi'_{k-1} \hat{x}_{k|k-1} \quad (19)$$

$$\sigma_{y_k}^2 = \sigma_v^2 + \psi'_{k-1} P_{k|k-1} \psi_{k-1}$$

Control:

Theorem 12 The optimizing risk-sensitive cautious control is given by

$$u_{k-1}^* = \underset{u_{k-1} \in U_{k-1, k-1}}{\operatorname{argmin}} \left[\frac{1}{\sqrt{1 - \theta \sigma_{y_k}^2}} \exp \left\{ \frac{\theta}{2(1 - \theta \sigma_{y_k}^2)} \right. \right. \\ \left. \left. \times (\psi'_{k-1} \hat{x}_{k|k-1} - r_k)^2 \right\} \right] \quad (20)$$

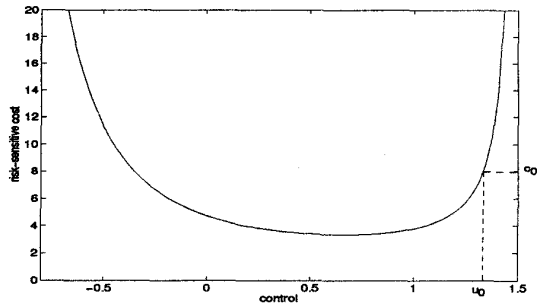
Note 13 It is assumed here that $\theta < \frac{1}{\sigma_{y_k}^2}$, $\forall k$. One way to guarantee this is to choose a high enough $P_{0|-1}$ and choose $\theta < \frac{1}{\sigma_{y_0}^2}$ because it is assumed that $\sigma_{y_k}^2 \leq \sigma_{y_0}^2$, $\forall k > 0$.

Remark 14 It is fairly easy to show that as $\theta \rightarrow 0$, this optimizing control approaches the solution given in [2] for the risk-neutral cautious control. This is in agreement with similar observations made in [8], [9] where it has been shown that the risk-neutral case can be always obtained as a special case of the risk-sensitive problem as $\theta \rightarrow 0$.

Example

Consider an integrator in discrete-time with a time-varying gain given by

$$b_{k+1} = A_k b_k + w_k \\ y_k = y_{k-1} + b_k u_{k-1} + v_k \quad (21)$$



c_0 = optimal cost for risk-neutral cautious control
 u_0 = optimal control value for risk-neutral cautious control

Figure 1: Risk-sensitive cautious control

We assume $w_k \sim N(0, \sigma_w^2)$, $v_k \sim N(0, \sigma_v^2)$ and A_k to be known. Figure 1 shows the plot of the cost of the risk-sensitive cautious control described by (20) versus the control variable u_{k-1} and the corresponding (i.e using the same realization for y_{k-1} , same value for r_k and the same distribution for $b_{k|k-1}$) optimal risk-neutral cost denoted by c_0 for the optimal control u_0 [2]. It is seen that the

optimal risk-neutral cost is higher than the optimal risk-sensitive cost. θ was chosen to be 3.7 for this particular example. It explains that risk-neutral optimization problem is not the best approach when there are uncertainties involved in the model dynamics, taken care of by choosing θ suitably in the risk-sensitive optimization problem.

4 Robust (risk-sensitive) suboptimal dual controller

In Section 2, we derived a dynamic programming equation (8) which has to be solved in order to achieve the solution to the multi-step optimization criteria (2) or the optimal risk-sensitive dual control problem. Remark 9 explains why such a solution even for simple linear systems could pose extreme computational difficulties. Due to similar difficulties encountered in the risk-neutral optimal dual control problem, researchers have considered other suboptimal strategies which could substantially simplify the computational procedure. Since the cautious controller is not a dual controller, adding perturbation signals to the cautious controller has been considered in [4]. In [5], constrained one step minimization techniques have been considered, the constraint being on the minimum value of the control signal or on the variance of the parameter estimates. Several works (e.g., [6]) have considered different extensions of the single-step cost criterion (i.e. the cost criterion for the cautious control problem) in the risk-neutral case.

In this section, we consider a similar extension of the single-step risk-sensitive cost criterion (10). The corresponding extension in the risk-neutral case has been studied in [7]. We first present a generalized extended cost-criterion for a risk-sensitive suboptimal dual controller, followed by a specific cost-criterion for the SISO minimum phase ARX model described in Section 3. We then present an analytic solution for the control that optimizes this specific cost-criterion.

4.1 Cost-criterion

Consider a generalized cost-criterion for a risk-sensitive suboptimal dual controller for the system (1) given by

$$J_{sub}(u_{k-1}) = E \left[\exp \left\{ \theta_1 \left(L(y_k, u_{k-1}, r_k) + \theta_2 f(x_k, \hat{x}_k) \right) \right\} \mid \mathcal{Y}_{k-1} \right] \quad (22)$$

where θ_1, θ_2 are risk-sensitive parameters and $f : \mathbb{R}^n \times \mathbb{R}^n \rightarrow \mathbb{R}$ is a convex function reflecting a measure of the estimation error energy, so that both the control and estimation cost are penalized.

Now let us consider the following cost criterion for the

SISO ARX model described in Sec. 3.2, given by

$$J_{sub}^{SISO}(u_{k-1}) = E \left[\exp \left\{ \theta_1 \left((y_k - r_k)^2 + \lambda e_k^2 \right) \right\} \mid \mathcal{Y}_{k-1} \right] \quad (23)$$

where $e_k = y_k - \psi'_{k-1} \hat{x}_{k|k-1}$ and $\lambda = \frac{\theta_2}{\theta_1}$.

Therefore, the problem objective is to find u_{k-1}^* such that

$$u_{k-1}^* = \underset{u_{k-1} \in U_{k-1,k-1}}{\operatorname{argmin}} J_{sub}^{SISO}(u_{k-1}) \quad (24)$$

4.2 Robust suboptimal dual controller for an SISO ARX model

Consider the SISO ARX model described by (13), (14). Separability of estimation and control applies as before and the estimation is carried out in the same way as in Sec. 3. The following theorem gives the result for the risk-sensitive suboptimal dual controller for the SISO ARX model (12).

Theorem 15 *The risk-sensitive suboptimal dual control that optimizes the cost criterion (23) is given by*

$$u_{k-1}^* = \underset{u_{k-1} \in U_{k-1,k-1}}{\operatorname{argmin}} \beta_k \exp \left[\frac{\theta_1 (1 - \lambda \theta_1 \sigma_{y_k}^2)}{2(1 - \theta_1 (1 + \lambda) \sigma_{y_k}^2)} \times (\psi'_{k-1} \hat{x}_k - r_k)^2 \right] \quad (25)$$

where $\beta_k = \frac{1}{\sqrt{(1 - \theta_1 (1 + \lambda) \sigma_{y_k}^2)}}$ and $\sigma_{y_k}^2, \hat{x}_k$ are as defined in Sec. 3.

Remark 16 This optimization has to be done numerically since ψ_{k-1} and hence $\sigma_{y_k}^2$ and β_k are functions of u_{k-1} .

Remark 17 We assume $\theta_1 < \frac{1}{(1 + \lambda) \sigma_{y_k}^2}$, $\forall k \in \mathbb{N}$. The choice of θ_1 and λ is dependent on the trade-off between good control and good estimation.

Remark 18 Simulation studies [12] have shown that the risk-sensitive suboptimal dual controller yields a lower cost than its risk-neutral counterpart in uncertain noise environments. We do not present them in this paper due to limited space.

5 A generalized robust dual controller

In the existing dual control literature, the risk-neutral dual control problem has been always treated as an optimal control problem, where the optimizing control not only minimizes the immediate control cost, but also introduces a learning feature ensuring good estimation in future. In our paper so far, we have considered risk-sensitive versions of such dual control problems. In this section, we

introduce a more generalized optimal risk-sensitive dual control problem, where the risk-sensitive cost incorporates a term involving the estimation error energy along with the control cost. We present the cost-criterion and then give a brief outline of the solution and its significance.

5.1 Cost-criterion

Consider the signal model (1). The generalized optimal risk-sensitive dual control cost-criterion is given by

$$J_g(u, \hat{X}_{k,T}) = E[\exp\{\theta_c (\sum_{i=k}^{T-1} L(x_i, u_{i-1}) + \Phi(x_T)) + \theta_e \sum_{i=k}^T (x_i - \hat{x}_i)' Q_i (x_i - \hat{x}_i)\} | \mathcal{Y}_{k-1}] \quad (26)$$

where $u \in U_{k-1, T-1}$, $\hat{X}_{k,T} \triangleq (\hat{x}_k, \dots, \hat{x}_T)$ and $Q_i \geq 0$. $\theta_c, \theta_e > 0$ are risk-sensitive parameters.

Therefore the problem objective is to find $u^* \in U_{k-1, T-1}$ and $\hat{X}_{k,T}$ such that

$$J_g(u^*, \hat{X}_{k,T}) = \inf_{u \in U_{k-1, T-1}, \xi_{k,T}} E[\exp\{\theta_c (\sum_{i=k}^{T-1} L(x_i, u_{i-1}) + \Phi(x_T)) + \theta_e \sum_{i=k}^T (x_i - \xi_i)' Q_i (x_i - \xi_i)\} | \mathcal{Y}_{k-1}]$$

where $\xi_{k,T} = (\xi_k, \dots, \xi_T)$.

Note 19 The relevant assumptions about L and Φ are given in Remark 2.

5.2 Optimal Solution

It is obvious that this optimization task is much harder than the ones considered in this paper so far, since the optimization has to be done over two variables u_{k-1}, \hat{x}_k at time k . By considering $\hat{x}_i, \forall i \in \{k, \dots, T\}$, to be another set of control variables, the optimization task can be looked upon as an optimal control problem, similar to the one considered in [8]. Hence, to solve this generalized risk-sensitive dual control problem, we have to apply the change of probability measure techniques to obtain a dynamic programming equation similar to the one obtained in [8].

It should be noted here that the estimates $\hat{X}_{k,T}$ obtained as the solution of the optimization task, along with the control $u^* \in U_{k-1, T-1}$ are, in general nonlinear functions of the information state, which in this case is no longer just a conditional probability density function [8]. Although $\hat{X}_{k,T}$ does not affect the future evolution of the system, it can be viewed as a robust estimate of the state and/or the unknown parameter vector of the system, which will be

appropriate in the presence of plant and noise uncertainties. Also having two risk-sensitive parameters θ_c, θ_e gives the provision of having a trade-off between good control and good estimation.

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