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AN HMM APPROACH TO ADAPTIVE DEMODULATION OF QAM SIGNALS IN FADING CHANNELS

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SUMMARY

In this paper the techniques of extended Kalman filtering (EKF) and hidden Markov model (HMM) signal processing are combined to adaptively demodulate quadrature amplitude-modulated (QAM) signals in noisy fading channels. This HMM approach is particularly suited to signals for which the message symbols are not equally probable, as is the case with many types of coded signals. Our approach is to formulate the QAM signal by a finite–discrete state process and represent the channel model by a continuous state process. The *mixed state model* is then reformulated in terms of conditional information states using HMM theory. This leads to models which are amenable to standard EKF or related techniques. A sophisticated EKF scheme with an HMM subfilter is discussed, as well as more practical schemes coupling discrete state HMM filters and continuous state Kalman filters. The case of white noise is considered, as well as generalizations to cope with coloured noise. Simulation studies demonstrate the improvement gained over standard schemes.

KEY WORDS Hidden Markov models Quadrature amplitude modulation Kalman filtering Fading channels

1. INTRODUCTION

The problem of noisy fading channels can be the limiting factor in certain communications systems, particularly with multipath, Raleigh fading situations arising from mobile receivers or transmitters. Demodulation of signals under these noisy fading conditions requires adaptive estimation of the transmission channel characteristics. Of course, traditional matched filters (MFs), phase-locked loops (PLLs) and automatic gain controllers (AGCs) can be effective in the digital case, but they are known to be far from optimal, particularly when dealing with signals which do not have equally probable message symbols. Optimal schemes, on the other hand, are inherently infinite-dimensional and are thus impractical. Also, they may not be robust to modelling errors. The challenge is to devise suboptimal robust demodulation schemes to cope with fading signals, particularly in the case where the message symbols are not equally probable.

In tackling demodulation using recent techniques in stochastic and adaptive systems, it is worth recalling the role of the Kalman filter (KF) and extended Kalman filter (EKF). The EKF turns out to be a PLL in disguise. Examples of the use of the EKF are given in Reference 1 (target tracking (p. 53), frequency modulation (p. 200)). Schemes have recently been developed for continuous phase-modulated (CPM) signals in fading channels,[2] coupling Kalman filtering

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techniques with maximum likelihood sequence estimation. Our approach for quadrature amplitude-modulated (QAM) signals is to use EKF techniques coupled with optimal hidden Markov model (HMM) filtering. We also present more practical schemes coupling Kalman filtering and HMM filtering.

The Kalman filter is the optimal linear filter for linear finite-dimensional stochastic systems. The EKF is the KF when applied to non-linear systems and is also finite-dimensional. As with the KF, the HMM filter is a finite-dimensional optimal filter where the HMM has states belonging to a finite–discrete set. The term finite–discrete refers to the fact that the allowable states are discrete and there is a finite number of them. To date, HMM filters have been widely applied in areas such as speech processing and biological signal processing.[3,4] These applications have involved off-line processing; however, the non-linear nature of the HMM approach and the discrete state nature of the formulations seem to suggest application to problems of digital communications. An important aspect of this paper is to apply on-line processing to the HMM formulation.

A typical communications channel such as a mobile telephone channel can introduce amplitude gain and phase shifts to the transmitted signal. The traditional signal model formulation for modulated digital signals leads to a non-linear task for estimating the signal and channel distortions. This is commonly performed using a matched filter (MF) for state estimation and an analogue PLL operating in tandem with an AGC for channel estimation, as discussed in Reference 5 (Chaps 5 and 6). Recent approaches to the fading problem for QAM signals have involved the use of pilot symbol-aided schemes[6] and alterations to the QAM signal constellation.[7] These deal mainly with the transmitter in an effort to improve the bit error rates (BERs). Our approach is to apply hidden Markov modelling to the signal to incorporate symbol-to-symbol dependences and as such can be implemented in tandem with the above techniques. It should, however, be pointed out that the HMM schemes do not require any modification to the transmitter.

In this paper we couple KF/EKF techniques and HMM signal processing in an adaptive HMM approach to estimate both the signal and the time-varying transmission channel parameters on-line. We propose such adaptive HMM schemes for demodulation of QAM signals with fading channels and in high noise. The HMM filter is ideal for signals which do not have equally probable (or i.i.d.) message symbols, as is the case with coded signals for example. Coding techniques such as convolutional coding (Reference 8, p. 441) produce signals which are not i.i.d. and as such display Markov properties. Also, recent investigations in trellis coding (reported in Reference 9) seem to suggest that a similar situation arises for trellis-coded signals. Actually, in the uncoded case of equally probable message symbols the HMM filter with a maximum *a priori* estimate is in fact identical to the matched filter, which is known to be optimal for non-fading i.i.d. digital signals.

In this paper our technical approach is to work with the signal in a discrete set and associate with this signal a discrete state vector X_k. X_k is an indicator function for the signal and in this case each of the allowable values of X_k represents one of the QAM signal constellation points. Here X_k belongs to a discrete set of unit vectors. The states X_k are assumed to be first-order Markov with known transition probability matrix $\mathbf{A}$ and state values $\mathbf{Z}$. This is a reasonable assumption given that the coding scheme is known. Associated with the channel are time-varying parameters (gain, phase shift and noise colour) which are modelled as states x_k in a continuous range $x_k \in \mathbb{R}^n$. The channel parameters arise from a known linear time-invariant stochastic system. State space models are formulated involving a mixture of the states X_k and x_k and are termed *mixed state models*. These are reformulated using HMM filtering theory to achieve a non-linear representation with a state vector consisting of α_k and x_k, where

α_k is an unnormalized information state representing a discrete state conditional probability density for X_k. These reformulated models are termed *conditional information state models.* Next the EKF algorithm or some derivative scheme can be applied for state estimation of this innovations representation, thereby achieving both signal and channel estimation. The resulting adaptive HMM algorithms appear either as coupled KF and HMM filters or as a more sophisticated EKF with an HMM filter as a subfilter.

In addition to reformulating the QAM signal representation, we employ a non-standard channel representation. Rather than work directly with a linear stochastic model for channel gain and phase shift, with its intuitive appeal and considerable precedence, we propose to formulate the channel in terms of a linear stochastic model with the state being the real and imaginary components of the channel. Working in rectangular co-ordinates instead of polar co-ordinates allows us to write the models in a familiar state space form driven by Gaussian noise. This facilitates the application of the EKF scheme, which approaches optimality in the low noise case. Unfortunately the rectangular co-ordinate representation introduces coupling between the two noise sources in the model. This coupling is, however, well understood.

When the channels are time-invariant (non-fading), the EKF and derivative schemes specialize to the recursive prediction error (RPE) approach for HMM identification and estimation, the subject of our earlier work.[10] There are quite solid theoretical foundations in the RPE case, giving confidence of asymptotic optimality with quadratic convergence rates. With known channels (i.e. in the asymptotic case) the HMM is known to achieve optimal performance. When the channels are fading, however, we are in general within the context of EKF theory, which is less developed. We do not therefore seek strong theoretical convergence results here, save that we expect from known theory that in the low-noise case the EKF is near-optimal after initial transients.

This paper is organized as follows. In Section 2 we formulate the QAM signal model in the HMM framework. In Section 3 we present the HMM/EKF and HMM/KF adaptive algorithms. Coloured noise is considered in Section 4. In Section 5 simulation examples are given which demonstrate good tracking ability for fast-changing channels. Finally, our conclusions are presented in Section 6.

2. QUADRATURE AMPLITUDE MODULATION (QAM)

Digital information grouped into fixed length bit strings is frequently represented by suitably spaced points in the complex plane. Quadrature amplitude modulation (QAM) transmission schemes are based on such a representation. In this section we first present the usual QAM signal model and then propose a reformulation so as to apply hidden Markov model (HMM) and extended Kalman filtering (EKF) methods.

2.1. Signal model

Let m_k be a complex discrete time signal ($k \in \mathcal{Z}^+$) where for each k

$$m_k \in \mathbf{Z} = \{z^{(1)}, \ldots, z^{(2^N)}\}, \quad z^{(i)} \in \mathbb{C}, \quad N \in \mathcal{Z}^+ \tag{1}$$

We also define the vector

$$z = z^R + j z^I = (z^{(1)}, \ldots, z^{(2^N)})^T \in \mathbb{C}^{2^N} \tag{2}$$

For digital transmission each element of $\mathbf{Z}$ is used to represent a string of N bits. In the case of QAM each of these complex elements $z^{(i)}$ is chosen so as to generate a rectangular grid of equally spaced points in the complex space $\mathbb{C}$. A 16-state ($N = 4$) QAM signal constellation is illustrated in Figure 5. We now note that at any time k the message $m_k \in \mathbf{Z}$ is complex-valued and can be represented in either polar or rectangular form, in obvious notation, as

$$m_k = \rho_k \exp(\mathrm{j}\Upsilon_k) = m_k^{\mathrm{R}} + \mathrm{j}m_k^{\mathrm{I}} \tag{3}$$

The real and imaginary components of m_k can then be used to generate piecewise constant time signals $m(t) = m_k$ for $t = [t_k, t_{k+1})$, where t_k arises from regular sampling. The messages are then modulated and transmitted in quadrature as a QAM bandpass signal

$$s(t) = A_{\mathrm{c}}[m^{\mathrm{R}}(t)\cos(2\pi ft + \theta) + m^{\mathrm{I}}(t)\sin(2\pi ft + \theta)] \tag{4}$$

where the carrier amplitude A_{c}, frequency f and phase θ are constant. This transmission scheme is termed QAM because the signal is *quadrature* in nature, where the real and imaginary components of the message are transmitted as two *amplitudes* which *modulate* quadrature and in-phase carriers.

2.2. Channel model

The QAM signal is passed through a channel which can cause amplitude and phase shifts, as for example in fading channels owing to multiple transmission paths. The channel can be modelled by a multiplicative disturbance $g(t)$, resulting in a discrete time baseband disturbance

$$g_k = \varkappa_k \exp(\mathrm{j}\phi_k) = g_k^{\mathrm{R}} + \mathrm{j}g_k^{\mathrm{I}} \tag{5}$$

which introduces time-varying gain and phase changes to the signal. The time variations in g_k are realistically assumed to be slow in comparison with the message rate.

Channel state — Cartesian co-ordinate representation. In this co-ordinate system we work with the vector x_k associated with the real and imaginary parts of g_k:

$$x_k = \begin{pmatrix} \varkappa_k \cos \phi_k \\ \varkappa_k \sin \phi_k \end{pmatrix} = \begin{pmatrix} g_k^{\mathrm{R}} \\ g_k^{\mathrm{I}} \end{pmatrix} \tag{6}$$

Channel state — polar co-ordinate representation. An alternative to Cartesian co-ordinates in the complex plane is the more traditional polar co-ordinate representation

$$x_k^{\mathrm{P}} = \begin{pmatrix} \varkappa_k \\ \phi_k \end{pmatrix} \tag{7}$$

As mentioned previously, the Cartesian co-ordinates allow the observations to be written in a form which enables linear Kalman filtering to be applied, while the polar co-ordinates require the non-linear suboptimal PLL for phase estimation. The practical benefits of each approach are discussed later in Section 5.

Assumption on channel-fading characteristics. Consider that the dynamics of x_k from (6) are given by

$$x_{k+1} = Fx_k + v_{k+1}, \qquad v_k \sim N[0, Q_k] \tag{8}$$

for some known F (usually with $\lambda(F) < 1$, where λ indicates eigenvalues, to avoid unbounded x_k and typically with $F = fI$ for some scalar $0 \ll f < 1$). In polar co-ordinates (7) a corresponding model is

$$\begin{aligned} \varkappa_{k+1} &= f_\varkappa \varkappa_k + v_{k+1}^\varkappa, \quad \text{where } v_k^\varkappa \text{ is Rayleigh distributed } [\mu_\varkappa, \sigma_\varkappa^2] \\ \phi_{k+1} &= f_\phi \phi_k + v_{k+1}^\phi, \quad \text{where } v_k^\phi \text{ is uniformly distributed over } [0, 2\pi) \end{aligned} \tag{9}$$

and typically $0 \ll f_\varkappa < 1$ and $0 \ll f_\phi < 1$.

For both channel models we assume that the variations associated with the magnitude of the channel gain $\varkappa$ and the phase shift ϕ are independent, with variances given by $\sigma_\varkappa^2$ and σ_ϕ^2 respectively. It follows from Reference 1 (p. 53) that the covariance matrix of the Cartesian channel model noise vector v_k is given by

$$Q_k = E[v_k v_k^{\mathrm{T}}] \simeq \begin{bmatrix} \sigma_\varkappa^2 \cos^2 \phi_k + \varkappa_k^2 \sigma_\phi^2 \sin^2 \phi_k & (\sigma_\varkappa^2 - \varkappa_k^2 \sigma_\phi^2) \sin \phi_k \cos \phi_k \\ (\sigma_\varkappa^2 - \varkappa_\varkappa^2 \sigma_\phi^2) \sin \phi_k \cos \phi_k & \sigma_\varkappa^2 \sin^2 \phi_k + \varkappa_k^2 \sigma_\phi^2 \cos^2 \phi_k \end{bmatrix} \tag{10}$$

For the remainder of this paper we will work with the Cartesian channel model, since it allows us to write the system in the familiar state space form.

2.3. *Observation model*

The baseband output of the channel, corrupted by additive noise w_k, is therefore given by

$$y_k = g_k m_k + w_k \tag{11}$$

Assume that $w_k \in \mathbb{C}$ has i.i.d. real and imaginary parts w_k^{R} and w_k^{I} respectively with zero mean and Gaussian, so that $w_k^{\mathrm{R}}, w_k^{\mathrm{I}} \sim N[0, \sigma_w^2]$.

In vector notation the observations have the form

$$\begin{pmatrix} y_k^{\mathrm{R}} \\ y_k^{\mathrm{I}} \end{pmatrix} = \begin{pmatrix} m_k^{\mathrm{R}} & -m_k^{\mathrm{I}} \\ m_k^{\mathrm{I}} & m_k^{\mathrm{R}} \end{pmatrix} \begin{pmatrix} g_k^{\mathrm{R}} \\ g_k^{\mathrm{I}} \end{pmatrix} + \begin{pmatrix} w_k^{\mathrm{R}} \\ w_k^{\mathrm{I}} \end{pmatrix} \tag{12}$$

2.4. *State space signal model*

Consider the following assumption on the message sequence.

Assumption on message signal

$$m_k \text{ is a first-order homogeneous Markov process} \tag{13}$$

Remark. This assumption enables us to consider the signal in a Markov framework and thus allows Markov filtering techniques to be applied. It is a reasonable assumption on the signal, given that error-correcting coding has been employed in transmission. Coding techniques such as convolutional coding produce signals which do not have equally probable (or i.i.d.) message symbols and as such display Markov properties. Of course i.i.d. signals can be considered in this framework too, since a Markov chain with a transition probability matrix which has all elements the same gives rise to an i.i.d. process.

Let us define the vector X_k to be an indicator function associated with m_k. Thus the state space of X_k, without loss of generality, can be identified with the set of unit vectors $\mathbf{S} = \{e_1, e_2, \ldots, e_{2^N}\}$, where $e_i = (0, \ldots, 0, 1, 0, \ldots, 0)^{\mathrm{T}} \in \mathbb{R}^{2^N}$ with 1 in the ith position, so that

$$m_k = z^{\mathrm{T}} X_k \tag{14}$$

where z is as defined above. Under assumption (13) the transition probability matrix associated with m_k, in terms of X_k, is

$$\mathbf{A} = (a_{ij}), \quad 1 \leqslant i, j \leqslant 2^N, \quad \text{where } a_{ij} = P(X_{k+1} = e_j \mid X_k = e_i)$$

so that

$$E[X_{k+1} \mid X_k] = A^{\mathrm{T}} X_k$$

where $E[\cdot]$ denotes the expectation operator. Of course $a_{ij} \geqslant 0$ and $\Sigma_{j=1}^{2^N} a_{ij} = 1$ for each

i. We also denote $\{\mathscr{F}_l, l \in \mathcal{Z}^+\}$ the complete filtration generated by X, i.e. for any $k \in \mathcal{Z}^+$, $\mathscr{F}_k$ is the complete σ-field generated by X_k, $l \leqslant k$.

Lemma 1

The dynamics of X_k are given by the state equation

$$X_{k+1} = A^{\mathrm{T}} X_k + M_{k+1} \tag{15}$$

where M_{k+1} is an $(\mathbf{A}, \mathscr{F}_k)$ martingale increment in that $E[M_{k+1} \mid \mathscr{F}_k] = 0$.

Proof[11]

$$E[M_{k+1} \mid \mathscr{F}_k] = E[X_{k+1} - \mathbf{A}^{\mathrm{T}} X_k \mid X_k, \mathbf{A}] = E[X_{k+1} \mid X_k, \mathbf{A}] - \mathbf{A}^{\mathrm{T}} x_k = 0 \qquad \blacksquare$$

As noted previously, in the case of quadrature modulated signals the states represented by X_k are each characterized by a complex value $z^{(i)}$ corresponding to the unit vector $e_i \in \mathbf{S}$. These are termed the state values of the Markov chain.

The *observation process* from (12) for the Cartesian channel model can be expressed in terms of the state X_k as

$$\begin{pmatrix} y_k^{\mathrm{R}} \\ y_k^{\mathrm{I}} \end{pmatrix} = \begin{pmatrix} (z^{\mathrm{R}})^{\mathrm{T}} X_k & -(z^{\mathrm{I}})^{\mathrm{T}} X_k \\ (z^{\mathrm{I}})^{\mathrm{T}} X_k & (z^{\mathrm{R}})^{\mathrm{T}} X_k \end{pmatrix} \begin{pmatrix} g_k^{\mathrm{R}} \\ g_k^{\mathrm{I}} \end{pmatrix} + \begin{pmatrix} w_k^{\mathrm{R}} \\ w_k^{\mathrm{I}} \end{pmatrix} \tag{16}$$

or equivalently, with the appropriate definition of $h(\cdot)$, as

$$\mathbf{y}_k = h(X_k) x_k + \mathbf{w}_k, \qquad \mathbf{w}_k \sim N[0, R_k] \tag{17}$$

Note that $E[w_{k+1}^{\mathrm{R}} \mid \mathscr{F}_k \vee \mathscr{Y}_k] = 0$ and $E[w_{k+1}^{\mathrm{I}} \mid \mathscr{F}_k \vee \mathscr{Y}_k] = 0$, where $\mathscr{Y}_l$ is the σ-field generated by y_k, $k \leqslant l$. We also define $Y_k \triangleq (y_0 \ \dots \ y_k)$. It is usual to assume that w_k^{R} and w_k^{I} are independent so that the covariance matrix associated with the measurement noise vector $\mathbf{w}_k$ has the form

$$R_k = \begin{bmatrix} \sigma_{w^{\mathrm{R}}}^2 & 0 \\ 0 & \sigma_{w^{\mathrm{I}}}^2 \end{bmatrix} \tag{18}$$

It is now readily seen that

$$E[M_{k+1} \mid \mathscr{F}_k \vee \mathscr{Y}_k] = 0 \tag{19}$$

In order to demonstrate the attractiveness of the Cartesian channel model, we now use the properties of the indicator function X_k to express the observations (17) in a linear form with respect to X_k and x_k:

$$\begin{aligned} \mathbf{y}_k &= h(X_k) x_k + \mathbf{w}_k \\ &= [h(e_1)x_k,\ h(e_2)x_k, \dots, h(e_{2^N})x_k] X_k + \mathbf{w}_k \\ &= H^{\mathrm{T}} [I_{2^N} \otimes x_k] X_k + \mathbf{w}_k \end{aligned} \tag{20}$$

where $H^{\mathrm{T}} = [h(e_1), \dots, h(e_{2^N})]$ and '$\otimes$' denotes a Kronecker product. The observations (20) are now in a form which is bilinear in X_k and x_k.

We shall define the vector of parametrized probability densities (which we will loosely call symbol probabilities) as $\mathbf{b}_k = (b_k(i))$ for $b_k(i) \triangleq P[y_k \mid X_k = e_i,\ x_k]$, where

$$b_k(i) = \frac{1}{2\pi\sigma_w^2} \exp\left(- \frac{\{y_k^{\mathrm{R}} - [(z^{\mathrm{R}})^{\mathrm{T}} e_i g_k^{\mathrm{R}} - (z^{\mathrm{I}})^{\mathrm{T}} e_i g_k^{\mathrm{I}}]\}^2}{2\sigma_w^2} - \frac{\{y_k^{\mathrm{I}} - [(z^{\mathrm{I}})^{\mathrm{T}} e_i g_k^{\mathrm{R}} + (z^{\mathrm{R}})^{\mathrm{T}} e_i g_k^{\mathrm{I}}]\}^2}{2\sigma_w^2} \right) \tag{21}$$

Because w_k^R and w_k^I are white, the independence property $E[y_k \mid X_{k-1} = e_i, \mathcal{F}_{k-2}, \mathcal{Y}_{k-1}] = E[y_k \mid X_{k-1} = e_i]$ holds and is essential for formulating the problem as an HMM, remembering, however, that it is an HMM which is parametrized by the fading channel model parameter x_k.

To summarize, we now have the following lemma.

Lemma 2

Under assumption (13) and (8) the QAM signal model (1)–(11) has the following state space representations in terms of the 2^N-dimensional finite–discrete state message indicator function X_k:

$$\boxed{\begin{aligned} X_{k+1} &= \mathbf{A}^{\mathrm{T}} X_k + M_{k+1} \\ x_{k+1} &= F x_k + v_{k+1} \\ \mathbf{y}_k &= H^{\mathrm{T}} [I_{2^N} \otimes x_k] X_k + \mathbf{w}_k \end{aligned}} \tag{22}$$

with x_k the continuous state associated with the fading channel characteristics. ■

The system is now in a bilinear form with respect to X_k and x_k.

Remarks

1. If x_k is known, then the model specializes to an HMM denoted $\lambda = (\mathbf{A}, \mathbf{Z}, \underline{\pi}, \sigma_w^2, x_k)$, where $\underline{\pi} = (\pi_i)$, defined from $\pi_i = P(X_1 = e_i)$, is the initial state probability vector for the Markov chain.
2. If X_k is known, then the model specializes to a linear state space model.
3. By way of comparison, for the polar co-ordinate channel representation (7) the observation process can only be expressed in terms of a linear operator on the channel gain with a non-linear operator on the phase. Thus if X_k and ϕ_k, or $\varkappa_k$ and ϕ_k, are known, then the model specializes to a linear state space model, but not if X_k and $\varkappa_k$ are known and ϕ_k is unknown.
4. In Figure 6 we present the output constellation, with signal-to-noise ratio SNR = 6 dB, from a channel with sinusoidal characteristics given by

$$\varkappa(t) = 1 + 0{\cdot}5 \sin(3\pi t/1000), \qquad \phi(t) = 0{\cdot}75\pi \cos(10\pi t/1000)$$

   The plots show 1000 data points at each of the constellation points for times $k = 200$ and 450 and give an indication of how the channel affects the QAM signal constellation.

2.5. Conditional information state signal model

Let $\hat{X}_{k\mid\mathscr{X}}$ denote the conditional filtered state estimate of X_k at time k given the channel parameters $\mathscr{X}_k = \{x_0, \ldots, x_k\}$, i.e.

$$\hat{X}_{k\mid\mathscr{X}} \triangleq E[X_k \mid \mathcal{Y}_k, \mathscr{X}_k] \tag{23}$$

Let us define $\underline{1}$ to be the column vector containing all ones and the 'forward' variable $\alpha_{k\mid\mathscr{X}}$ is such that the ith element $\alpha_{k\mid\mathscr{X}}(i) \triangleq P(Y_k, X_k = e_i \mid \mathscr{X}_k)$. Observe that $\hat{X}_{k\mid\mathscr{X}}$ can be expressed in terms of $\alpha_{k\mid\mathscr{X}}$ by

$$\hat{X}_{k\mid\mathscr{X}} = \langle \alpha_{k\mid\mathscr{X}}, \underline{1} \rangle^{-1} \alpha_{k\mid\mathscr{X}} \tag{24}$$

Here $\alpha_{k|\mathscr{X}}$ is conveniently computed using the 'forward' recursion[3]

$$\alpha_{k+1|\mathscr{X}} = \mathbf{B}(y_{k+1}, x_{k+1})\mathbf{A}^{\mathrm{T}}\alpha_{k|\mathscr{X}} \tag{25}$$

where $\mathbf{B}(y_{k+1}, x_{k+1}) = \mathrm{diag}(b_{k+1}(1), \ldots, b_{k+1}(2^N))$ and $b_k(i)$ is as defined in (21).

We use the term 'information state' for $\alpha_{k|\mathscr{X}}$ since it provides information about the state X_k. We now seek to express the observations y_k in terms of the unnormalized conditional information state $\alpha_{k|\mathscr{X}}$.

Lemma 3

The conditional measurements $\mathbf{y}_{k|\mathscr{X}}$ are defined by

$$\mathbf{y}_{k|\mathscr{X}} = H^{\mathrm{T}}[I_{2^N} \otimes x_k]\langle \alpha_{k-1|\mathscr{X}}, \underline{1}\rangle^{-1}\mathbf{A}^{\mathrm{T}}\alpha_{k-1|\mathscr{X}} + n_{k|\mathscr{X}} \tag{26}$$

where $\alpha_{k|\mathscr{X}}$ is as defined in (25) and $n_{k|\mathscr{X}}$ is an $(\mathscr{X}_k, \mathscr{Y}_{k-1})$ martingale increment. In addition, the covariance matrix of the conditional noise term $n_{k|\mathscr{X}}$ is given by

$$R_n = \sigma_w^2 I + H^{\mathrm{T}}[I_{2^N} \otimes x_k](\hat{X}^{\mathrm{D}}_{k|\mathscr{X}} - \hat{X}_{k|\mathscr{X}}\hat{X}^{\mathrm{T}}_{k|\mathscr{X}})[I_{2^N} \otimes x_k]^{\mathrm{T}}H \tag{27}$$

where $\hat{X}^{\mathrm{D}}_k$ is the matrix which has diagonal elements which are the elements of $\hat{X}_k$.

Proof. Following standard arguments, since $\alpha_{k|\mathscr{X}}$ is measurable with respect to $\{\mathscr{X}_k, \mathscr{Y}_k\}$, $E[w^{\mathrm{R}}_{k+1} \mid \mathscr{Y}_k] = 0$, $E[w^{\mathrm{I}}_{k+1} \mid \mathscr{Y}_k] = 0$ and $E[M_{k+1} \mid \mathscr{Y}_k] = 0$, then

$$\begin{aligned} E[n_{k|\mathscr{X}} \mid \mathscr{X}_k, \mathscr{Y}_{k-1}] &= E[H^{\mathrm{T}}[I_{2^N} \otimes x_k]X_k + \mathbf{w}_k \\ &\quad - H^{\mathrm{T}}[I_{2^N} \otimes x_k]\langle \alpha_{k-1|\mathscr{X}}, \underline{1}\rangle^{-1}\mathbf{A}^{\mathrm{T}}\alpha_{k-1|\mathscr{X}} \mid \mathscr{X}_k, \mathscr{Y}_{k-1}] \\ &= H^{\mathrm{T}}[I_{2^N} \otimes x_k](\mathbf{A}^{\mathrm{T}}\hat{X}_{k-1|\mathscr{X}} - \langle \alpha_{k-1|\mathscr{X}}, \underline{1}\rangle^{-1}\mathbf{A}^{\mathrm{T}}\alpha_{k-1|\mathscr{X}}) = 0 \end{aligned}$$

Also,

$$\begin{aligned} R_n = E[n_k^2 \mid \mathscr{X}_k, \mathscr{Y}_{k-1}] &= E\left[\left[w_k + H^{\mathrm{T}}[I_{2^N} \otimes x_k]\left(X_k - \frac{\mathbf{A}^{\mathrm{T}}\alpha_{k-1}}{\langle \alpha_{k-1}, \underline{1}\rangle}\right)\right]^2 \mid \mathscr{X}_k, \mathscr{Y}_{k-1}\right] \\ &= E[w_k^2 \mid \mathscr{X}_k, \mathscr{Y}_{k-1}] \\ &\quad + E\left[H^{\mathrm{T}}[I_{2^N} \otimes x_k]\left(X_k - \frac{\mathbf{A}^{\mathrm{T}}\alpha_{k-1}}{\langle \alpha_{k-1}, \underline{1}\rangle}\right)\left(X_k - \frac{\mathbf{A}^{\mathrm{T}}\alpha_{k-1}}{\langle \alpha_{k-1}, \underline{1}\rangle}\right)^{\mathrm{T}}[I_{2^N} \otimes x_k]^{\mathrm{T}}H \mid \mathscr{X}_k, \mathscr{Y}_{k-1}\right] \\ &= \sigma_w^2 I + H^{\mathrm{T}}[I_{2^N} \otimes x_k]E[(X_k - \hat{X}_{k|\mathscr{X}})(X_k - \hat{X}_{k|\mathscr{X}})^{\mathrm{T}} \mid \mathscr{X}_k, \mathscr{Y}_{k-1}][I_{2^N} \otimes x_k]^{\mathrm{T}}H \\ &= \sigma_w^2 I + H^{\mathrm{T}}[I_{2^N} \otimes x_k](\hat{X}^{\mathrm{D}}_{k|\mathscr{X}} - \hat{X}_{k|\mathscr{X}}\hat{X}^{\mathrm{T}}_{k|\mathscr{X}})[I_{2^N} \otimes x_k]^{\mathrm{T}}H \end{aligned}$$

∎

In summary we have the following lemma.

Lemma 4

The state space representation (22) can be reformulated to give the following conditional information state signal models with states $\alpha_{k|\mathscr{X}}$:

$$\boxed{\begin{aligned} \alpha_{k+1|\mathscr{X}} &= \mathbf{B}(y_{k+1}, x_{k+1})\mathbf{A}^{\mathrm{T}}\alpha_{k|\mathscr{X}} \\ x_{k+1} &= Fx_k + v_k \\ \mathbf{y}_{k|\mathscr{X}} &= H^{\mathrm{T}}[I_{2^N} \otimes x_k]\langle \alpha_{k-1|\mathscr{X}}, 1\rangle^{-1}\mathbf{A}^{\mathrm{T}}\alpha_{k-1|\mathscr{X}} + n_{k|\mathscr{X}} \end{aligned}} \tag{28}$$

Remarks

1. When $F \equiv I$ and $v \equiv 0$, then x_k is constant. Under these conditions the problem of channel state estimation reduces to one of parameter identification, and recursive prediction error techniques can be used as in Reference 10. However, an EKF or some derivative scheme is required for parameter tracking when x_k is not constant, as in Section 3.
2. By way of comparison, in the reformulated information state signal model for the polar coordinate channel representation (for which the observations are non-linear in terms of the channel phase parameter) it makes sense to consider the case where ϕ_k is restricted to a discrete set, or approximated as such, and is assumed to be Markov with indicator function $X_k^\phi \in \{e_1, e_2, \ldots\}$. Then a conditional filtered estimate of X_k^ϕ can be generated by the same means as used for the conditional filtered estimate of X_k, (24). The reformulated information state signal model for the polar channel model case is now given, in obvious notation, by

$$\begin{aligned} \alpha_{k+1} &= \mathbf{B}^{\mathrm{P}}(y_{k+1}, \varkappa_{k+1}, \alpha_{k+1}^{\phi})\mathbf{A}^{\mathrm{T}}\alpha_k \\ \alpha_{k+1}^{\phi} &= \mathbf{B}^{\phi}(y_{k+1}, \varkappa_{k+1}, \alpha_{k+1})(\mathbf{A}^{\phi})^{\mathrm{T}}\alpha_k^{\phi} \\ \varkappa_{k+1} &= f_x \varkappa_k + v_k^{\varkappa} \\ \mathbf{y}_k &= H_{\mathrm{P}}^{\mathrm{T}}(\langle \alpha_{k-1}, \underline{1}\rangle^{-1}\mathbf{A}^{\mathrm{T}}\alpha_{k-1})(\varkappa_k)H_{\phi}^{\mathrm{T}}(\langle \alpha_{k-1}^{\phi}, \underline{1}\rangle^{-1}(\mathbf{A}^{\phi})^{\mathrm{T}}\alpha_{k-1}^{\alpha}) + n_k \end{aligned} \tag{29}$$

where $H_{\mathrm{P}}^{\mathrm{T}} = [h_{\mathrm{P}}(e_1) \ \ldots \ h_{\mathrm{P}}(e_{2^N})]$, $H_{\phi}^{\mathrm{T}} = [h_\phi(e_1) \ \ldots \ h_\phi(e_{L_\phi})]$, and

$$h_{\mathrm{P}}(\cdot) = \langle z_\rho, \cdot\rangle \exp(\mathrm{j}\langle z_\gamma, \cdot\rangle), \qquad h_\phi(\cdot) = \exp(\mathrm{j}\langle z_\phi, \cdot\rangle) \tag{30}$$

Here z_ϕ is the vector containing the discrete values of ϕ, z_ρ and z_Υ are vectors containing the magnitudes and phases respectively of the QAM signal constellation, L_ϕ is the number of discrete values of ϕ and $X_k^\phi \in \mathscr{S} = \{e_1, \ldots, e_{L_\phi}\}$ is the indicator function associated with ϕ_k so that when $\phi_k = z_i^{(i)}$, $X_k^\phi = e_i$.

Also,

$$\mathbf{B}^{\mathrm{P}}(y_{k+1}, \varkappa_{k+1}, \alpha_{k+1}^{\phi}) = \mathrm{diag}(b_{k+1}^{\mathrm{P}}(1), \ldots, b_{k+1}^{\mathrm{P}}(2^N))$$

for

$$b_{k+1}^{\mathrm{P}}(i) \triangleq b^{\mathrm{P}}(y_{k+1}, e_i, \varkappa_{k+1}, \alpha_{k+1}^{\phi}),$$

where

$$b_k^{\mathrm{P}}(i) = \frac{1}{2\pi\sigma_w^2}\exp\left(\frac{-[y_k - h_{\mathrm{P}}(e_i)(\varkappa_k)H_\phi^{\mathrm{T}}\alpha_k^\phi]^2}{2\sigma_w^2}\right) \tag{31}$$

and

$$\mathbf{B}^{\phi}(y_{k+1}, \alpha_{k+1}, \varkappa_{k+1}) = \mathrm{diag}(b_{k+1}^{\phi}(1), \ldots, b_{k+1}^{\phi}(L_\phi))$$

for

$$b_{k+1}^{\phi}(i) \triangleq b^{\phi}(y_{k+1}, e_i, \alpha_{k+1}, \varkappa_{k+1}),$$

where

$$b_k^{\phi}(i) = \frac{1}{2\pi\sigma_w^2}\exp\left(\frac{-[y_k - H_{\mathrm{P}}^{\mathrm{T}}\alpha_k(\varkappa_k)h_\phi(e_i)]^2}{2\sigma_w^2}\right) \tag{32}$$

The key property which facilitates estimator construction is that now, with the discrete state assumption on ϕ in this polar co-ordinate system for the channel, the measurements are trilinear in X_k, $\varkappa$ and X_k^ϕ. However, as was shown before, in the case of the Cartesian representation of the channel, (28), the discrete state assumption on ϕ is not necessary to achieve linearity in the measurements.

3. ADAPTIVE HMM ALGORITHMS

Two adaptive HMM schemes are presented here. The first is referred to as the HMM/EKF scheme and is a full non-linear scheme for the information state signal model (28) with the augmented vector $(\alpha_k, x_k)^{\mathrm{T}}$. The second scheme is referred to as the HMM/KF scheme and consists of a simplification assumption which results in a KF for channel estimation, coupled with an HMM filter for signal state estimation.

Adaptive HMM algorithm with EKF

Let $\underline{x}_k = (\alpha_k, x_k)^{\mathrm{T}}$. Then (28) can be written as

$$\begin{aligned} \underline{x}_{k+1} &= f_k(\underline{x}_k) + g_k(\underline{x}_k)v_k \\ y_k &= h_k(\underline{x}_k) + n_k \end{aligned} \tag{33}$$

where the non-linear functions are given by

$$f_k(\underline{x}_k) = \begin{pmatrix} \mathbf{B}(x_k)\mathbf{A}^{\mathrm{T}}\alpha_k \\ Fx_k \end{pmatrix}, \qquad g_k(\underline{x}_k) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \qquad h_k(\underline{x}_k) = h(\alpha_{k|k-1})x_k$$

Given that the non-linearities are smooth functions, they can be expanded in Taylor series about the conditional means $\hat{\underline{x}}_{k|k}$ and $\hat{\underline{x}}_{k|k-1}$. With an assumption that higher-order terms can be ignored, as will be the case when $\hat{\underline{x}}_{k|k}$ is close to $\underline{x}_k$, (33) can be written as

$$\begin{aligned} \underline{x}_{k+1} &= F_k\underline{x}_k + G_kv_k + u_k \\ y_k &= H_k^{\mathrm{T}}\underline{x}_k + n_k + z_k \end{aligned} \tag{34}$$

where

$F_k = \partial f_k/\partial \underline{x}_k$, $G_k = \partial g_k/\partial \underline{x}_k$, $H_k = \partial h_k/\partial \underline{x}_k$, $u_k = f_k(\hat{\underline{x}}_{k|k}) - F_k\hat{\underline{x}}_{k|k}$ and $z_k = (\hat{\underline{x}}_{k|k-1}) - H_k^{\mathrm{T}}\hat{\underline{x}}_{k|k-1}$.

The EKF equations for (33) are the KF equations for (34), now summarized:

$$\hat{\underline{x}}_{k|k} = \hat{\underline{x}}_{k|k-1} + K_k[y_k - h_k(\hat{\underline{x}}_{k|k-1})] \tag{35}$$

$$\hat{\underline{x}}_{k+1|k} = f_k(\hat{\underline{x}}_{k|k}) \tag{36}$$

$$K_k = \Sigma_{k|k-1}h_k(H_k^{\mathrm{T}}\Sigma_{k|k-1}H_k + R_k)^{-1} \tag{37}$$

$$\Sigma_{k|k} = \Sigma_{k|k-1} - \Sigma_{k|k-1}H_k(H_k^{\mathrm{T}}\Sigma_{k|k-1}H_k + R_k)^{-1}H_k^{\mathrm{T}}\Sigma_{k|k-1} \tag{38}$$

$$\Sigma_{k+1|k} = F_k\Sigma_{k|k}F_k^{\mathrm{T}} + G_kQ_kG_k^{\mathrm{T}} \tag{39}$$

where (37) gives the Kalman gain and (38) and (39) are the Riccati equations. Figure 1 shows a block diagram for this adaptive HMM scheme when switch 1 is closed and switch 2 is in the top position. If switch 1 were in the open position, then the HMM/KF scheme given below would result. Further assumptions could be made for simplification if the maximum *a priori* estimate of α_k were used, indicated by having switch 2 in the lower position. This approach would be similar to using the matched filter, where only the most likely message symbol is used and not the full information state.

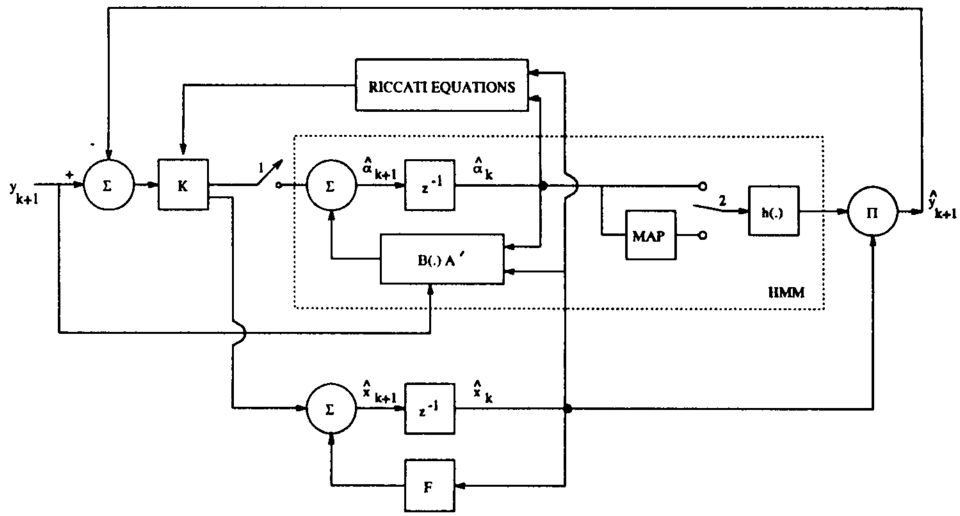

Figure 1. EKF/HMM scheme for adaptive HMM filter

Remarks

1. This HMM/EKF scheme suffers from the fact that through (35) the update for α_k requires a further projection to ensure positivity of each element. This adds undesired non-linearities to the model and provides further incentive to consider the HMM/KF scheme presented below, where this problem does not arise.
2. The filter here is in fact a smoothed filter in the sense that $f_k(\underline{x}_k)$ is actually $f_{k+1}(\underline{x}_k)$ owing to the dependence of $\mathbf{B}(\underline{x}_k)$ on y_{k+1}. This again provides incentive to consider the HMM/KF scheme presented below, where this problem does not arise.

HMM/KF schemes for adaptive HMM filter

This scheme can be viewed as a derivative of the HMM/EKF scheme above by setting the Kalman gain term associated with the $\hat{\alpha}_k$ update to zero. The rationale for this is that in the case where the channel parameters are constant, this term in fact does go to zero asymptotically. Indeed, setting it to zero under constant parameter conditions results in the HMM/RPE scheme of Reference 10 for which there are strong theoretical foundations. If the channel is only slowly varying, then it is expected that the components of the Kalman gain associated with the $\hat{\alpha}_k$ update will be asymptotically small. There is then a temptation and some rationale to neglect these terms for the simplicity of the resulting scheme which we now describe in more detail. The resulting scheme can be viewed as a coupled conditional HMM filter together with a conditional Kalman filter as follows.

The HMM estimator for the signal information state α_k conditioned on the channel estimate sequence $\{\hat{x}_k\}$ is given by

$$\hat{\alpha}_{k+1|\hat{x}_k} = \mathbf{B}(y_{k+1}, \hat{x}_k)\mathbf{A}^{\mathrm{T}}\hat{\alpha}_{k|\hat{x}_{k-1}} \tag{40}$$

$$\hat{X}_{k|\hat{x}_{k-1}} = \langle \alpha_{k|\hat{x}_{k-1}}, \underline{1}\rangle^{-1}\alpha_{k|\hat{x}_{k-1}} \tag{41}$$

The Kalman filter equations for the channel parameter x_k conditioned on the indicator state estimates $\hat{X}_k$ are

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k(y_k - H_k^{\mathrm{T}}\hat{x}_{k|k-1}) \tag{42}$$

$$\hat{x}_{k+1|k} = F\hat{x}_{k|k} \tag{43}$$

$$K_k = \Sigma_{k|k-1}H_k(H_k^{\mathrm{T}}\Sigma_{k|k-1}H_k + R_k)^{-1} \tag{44}$$

$$\Sigma_{k|k} = \Sigma_{k|k-1} - \Sigma_{k|k-1}(H_k^{\mathrm{T}}\Sigma_{k|k-1}H_k + R_k)^{-1}H_k^{\mathrm{T}}\Sigma_{k|k-1} \tag{45}$$

$$\Sigma_{k+1|k} = F\Sigma_{k|k}F^{\mathrm{T}} + Q_k \tag{46}$$

where

$$H_k^{\mathrm{T}} = \partial(H^{\mathrm{T}}[I_{2^N} \otimes x_k]\hat{X}_k)/\partial x_k \tag{47}$$

R is the covariance matrix of the noise on the observations $\mathbf{w}$ given in (18), Q is the covariance matrix of v given in (10) and Σ is the covariance matrix of the channel parameter estimate $\hat{x}$ (x is defined in (6)). Figure 2 shows the scheme in block form.

A further suboptimal KF/HMM scheme can be generated by using the state space signal model (22) and estimating the KF conditioned on a maximum *a priori* probability estimate $\hat{X}_k^{\mathrm{MAP}}$. Here

$$H_k^{\mathrm{T}} = \partial(H^{\mathrm{T}}[I_{2^N} \otimes x_k]\hat{X}_k^{\mathrm{MAP}})/\partial x_k \tag{48}$$

Figure 3 shows this scheme in block form. In fact, hybrid versions can be derived by setting the small-valued, i.e. low-probability, elements of $\hat{X}_k$ to zero and renormalizing.

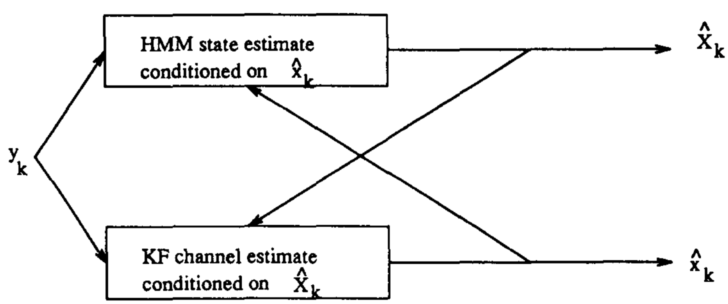

Figure 2. EKF/HMM adaptive HMM scheme

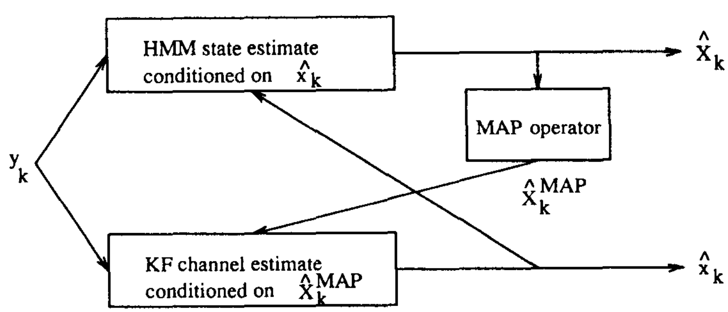

Figure 3. KF/HMM adaptive HMM scheme with MAP approximation

4. COLOURED NOISE CASE

In the coloured noise case it is reasonable to work with the following signal model involving a moving average of white noise:

$$\mathbf{y}_k = h(\cdot)x_k + \mathbf{w}_k + c_1\mathbf{w}_{k-1} + \cdots + c_n\mathbf{w}_{k-n} \tag{49}$$

The task of estimating the noise coefficients c_i, is now carried out by augmenting the state vector x_k by a vector $x_k^w = (w_{k-1}, w_{k-2}, \ldots)^{\mathrm{T}}$. We give an example for the case $n = 3$. The state vector x_k^w is the vector of noise values, $x_k^w = (w_{k-1}, w_{k-2}, w_{k-3})^{\mathrm{T}}$, and the vector of noise coefficients is $\theta = (c_1, c_2, c_3)^{\mathrm{T}}$.

$$\begin{pmatrix} x_{k+1} \\ x_{k+1}^w \end{pmatrix} = \begin{pmatrix} F & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} x_k \\ x_k^w \end{pmatrix} + \begin{pmatrix} v_k \\ w_k \\ 0 \\ 0 \end{pmatrix} \tag{50}$$

$$y_k = [h(\cdot)\ \theta^{\mathrm{T}}] \begin{pmatrix} x_k \\ x_k^w \end{pmatrix} + w_k \tag{51}$$

The ascribed estimation task can now be solved with an EKF or derivative KF where the state vector is now the augmented vector (α_k, x_k, x_k^w). If θ is unknown, it can be adaptively estimated using standard RPE/EKF ideas.

5. IMPLEMENTATION CONSIDERATIONS AND SIMULATIONS

In this section we present results which demonstrate the ability of the adaptive KF/HMM scheme to demodulate QAM signals in noisy fading channels. For comparisons we use the standard MF/AGC/PLL scheme which is diagrammatically represented in Figure 4 (similar to the LMS algorithm presented in Reference 12). The signal we consider is a 16-state QAM signal (Figures 5 and 6) with a strong dependence from one message symbol to the next (as is the case

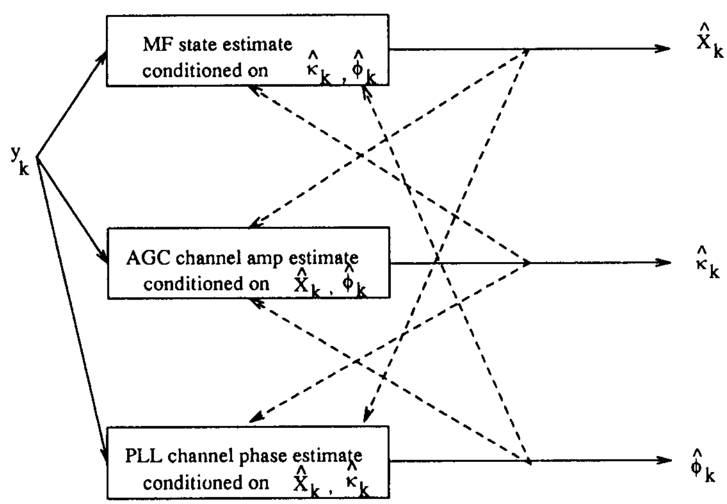

Figure 4. MF/AGC/PLL standard scheme

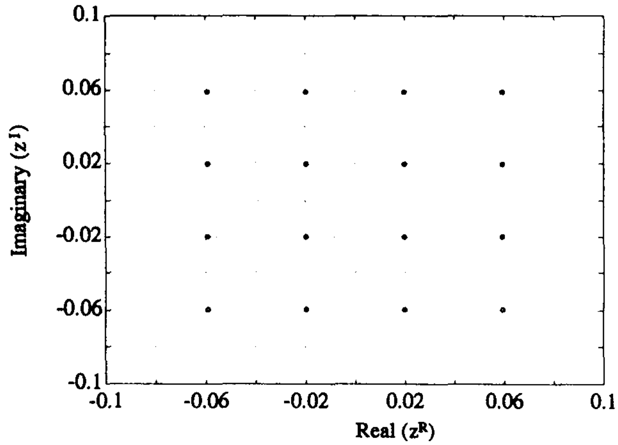

Figure 5. Sixteen-state QAM signal constellation

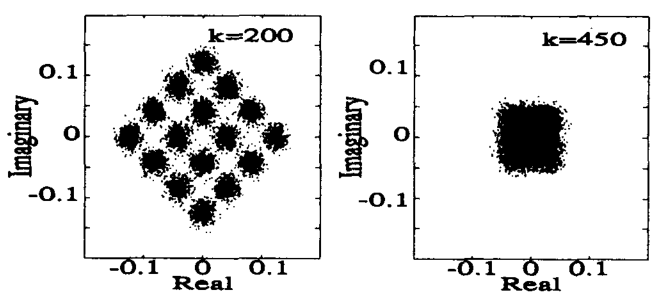

Figure 6. Sixteen-state QAM signal constellation output from channel

with some convolutional codes or if oversampling were to be used). The channel characteristics are given by a lowpass-filtered (LPF) white Gaussian noise stochastic process. The variance of the Gaussian process is 1 for amplitude variations and 5 for phase variations. The bandwidth of the LPF is W_c times the bit rate (W_c is different in each example). An example of a real-valued channel is shown in Figure 7 for $W_c = 0 \cdot 1$. These variations are very fast in the case of FAX and modem applications but are more reasonable in applications involving mobile communications and indoor communication channels.[13,14]

Two main points can be gained from the following examples. The first is that under these non-equally probable message symbol conditions the HMM filter is a major improvement over the MF. The second point is that the Cartesian and polar co-ordinate systems can each have their advantages, depending on the channel conditions. Computationally the MF is of course less taxing, but for mobile communications under the conditions (16-QAM, $19 \cdot 2\ \mathrm{kB\,s^{-1}}$, $f_c = 1800$ MHz, car travelling at $100\ \mathrm{km\,h^{-1}}$ and with one channel update every 120 samples) the processing power required for the HMM/KF approach is only 10 MFlops, which is reasonable with current DSP technology. Therefore the approach presented in this paper is computationally feasible and is seen to outperform the traditional scheme for the case of non-equally probable messages while being identical to the traditional scheme in the equally probable message case.

Example 1

In this example we demonstrate the ability of the HMM/KF adaptive algorithm to demodulate a 16-QAM signal in the presence of a real-valued stochastic channel. The signal parameter values are $a_{ii}=0{\cdot}95$, $(z^{(i)})^{R} = \pm 0{\cdot}01976 \pm 0{\cdot}03952$, $(z^{(i)})^{I} = \pm 0{\cdot}01976 \pm 0{\cdot}03952$. The results for this example are displayed in Figure 8, where signals of length 50 000 data points have been used to generate bit error rate (BER) values. The simulations assume that 90° phase-invariant coding is used. A comparison is given with the conventional MF/AGC/PLL system (of course the PLL is not required since the channel is real-valued). It can be seen that our HMM/KF scheme provides distinct advantages over the traditional scheme. As noted before, the case of $W_c = 0{\cdot}1$ is one of severe fading and it is seen that even under such conditions the HMM/KF scheme performs well.

Example 2

In this example we demonstrate that the HMM approach is identical to the MF approach in the case of equally probable message signals. The discrepancies which can be seen between

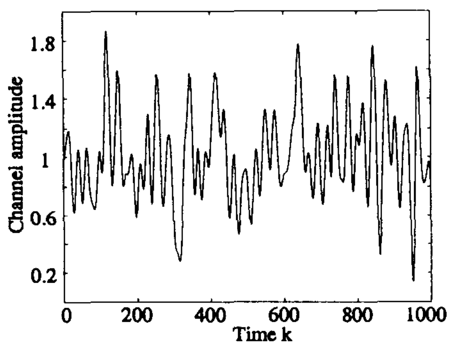

Figure 7. Stochastic channel gain, $W_c = 0{\cdot}1$

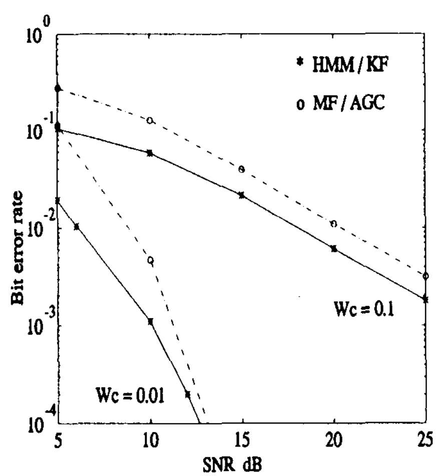

Figure 8. BER versus SNR for real-valued channels

the two schemes in the results of Figure 9 are due to our Cartesian approach compared with the polar approach of the traditional scheme. It seems that under these channel conditions in the high-SNR case the polar scheme is better than the Cartesian approach. Such a comparison is the subject of the next two examples.

Example 3

In this example we demonstrate the ability of the HMM/KF adaptive algorithm to demodulate a 16-QAM signal in the presence of a complex-valued stochastic channel. The signal characteristics are the same as for Example 1. The results for this example are displayed in Figure 10. It can be seen again that our approach has significantly better performance than

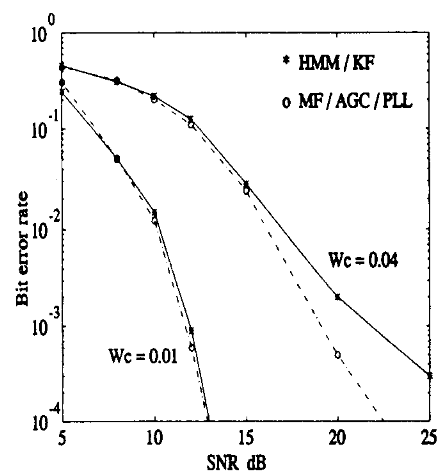

Figure 9. BER versus SNR for complex-valued channels with equally probable message symbols

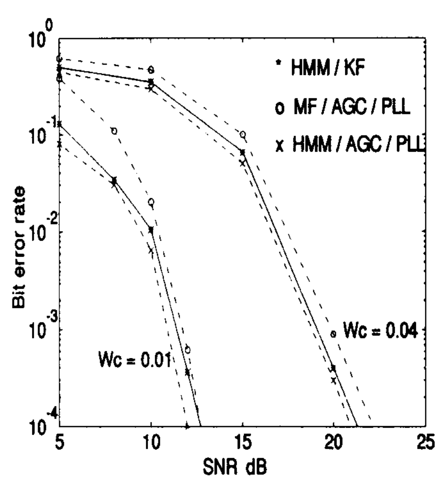

Figure 10. BER versus SNR for complex-valued channels

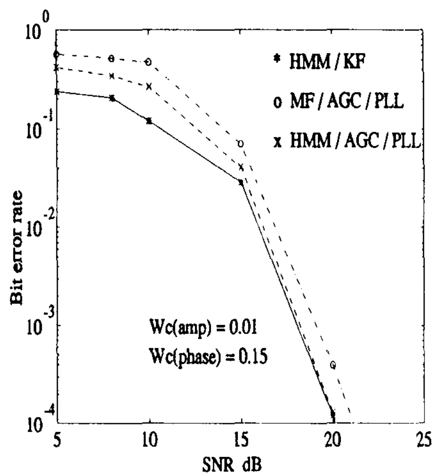

Figure 11. BER versus SNR for complex-valued channels

the traditional scheme involving the MF. Here we also present the results of our adaptive HMM approach when formulated in the polar representation. For this case we have implemented an AGC/PLL scheme for the channel parameter. It can be seen that under these conditions the non-linearities in the PLL approach are not detrimental and in fact the HMM/AGC/PLL approach performs better than the HMM/KF scheme.

Example 4

In this example we investigate the relative benefits of the Cartesian channel parametrization versus the polar representation. The signal characteristics are the same as for Example 1 and the results are displayed in Figure 11. In the previous example the channel phase shift varied more slowly than in this example. It can be seen that under the more stringent conditions presented here, the non-linearities in the PLL approach are detrimental and the HMM/KF approach performs better than the HMM/AGC/PLL scheme.

From these examples it can be easily seen that the HMM approach is more suited than the MF for signals with non-equally probable message symbols. Also, depending on the channel characteristics, the Cartesian co-ordinate representation can provide improvements over the traditional polar representation. Such improvements are most apparent under conditions of rapidly varying phase where the non-linearities associated with the PLL are detrimental to performance.

6. CONCLUSIONS

In this paper we have derived adaptive HMM on-line state and parameter estimation schemes for QAM signals in fading communications channels. A key element of our approach, which appears to be quite powerful, is to work with mixed finite–discrete and continuous range state models. These are reformulated via HMM filtering theory as conditional information state models. The resulting adaptive algorithms blend EKF and HMM techniques. They are based on optimal techniques but are inevitably suboptimal. Simulation studies are presented which show the ability to effectively track time-varying channel parameters for QAM signals.

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