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Enhancing Optimal Controllers via Techniques from Robust and Adaptive Control *

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ABSTRACT: Optimal control strategies for both nonlinear and linear plants and indices are notoriously sensitive to modelling errors and external noise disturbances. In this paper, a general framework to enhance robustness of an optimal control law is presented, with emphasis on the nonlinear case. The framework allows a blending of off-line nonlinear optimal control, on-line linear robust feedback control for regulation about the optimal trajectory, and on-line adaptive techniques to enhance performance / robustness. The adaptive-Q techniques are those developed in previous work based on the Youla-Kucera parametrization for the class of all-stabilizing two-degree-of-freedom controllers. Some general fundamental stability properties are developed which are new, at least for the nonlinear plant and linear robust controller case. Also, performance enhancement results in the presence of unmodelled linear dynamics based on an averaging analysis. are reviewed. A convergence analysis based on averaging theory appears possible in principle for any specific nonlinear system, but is beyond the scope of the present paper. Certain model-reference adaptive control algorithms come out as special cases. A nonlinear optimal control problem is studied to illustrate the efficacy of the techniques, and the possibility of further performance enhancement based on functional learning is noted.

1 Introduction

Optimal nonlinear deterministic control methods are considered very elegant in theory, but lack robustness in practice. In the optimal control approach, a mathematical model of the process is first formulated based on the fundamental laws in operation or via identification techniques. Next, a performance index is derived which reflects the various cost factors associated with the implementation of any control signal. Then, off-line calculations lead to an optimal control law u^* via one of the various methods of optimal control. In theory then, applying such a control law to the physical process should result in optimal performance. However, the process is rarely modelled accurately, and frequently is subject to stochastic disturbances. Consequently, the application of the "optimal" control signal u^* results in poor performance, in that the process output y differs from y^*, the output of the idealized process model.

One approach to achieve improved performance could be to include robustness measures in the cost function, so that for plants "near" the nominal model and "small" disturbances poor performance is avoided. This approach turns out to be difficult to develop in practice.

A standard approach to enhance open-loop optimal control performance is to measure on-line the difference between the ideal optimal process output trajectory y^* and the actual process output y. This difference signal, δy, depends on the difference, δu, between the optimal control u^* for the nominal model and any actual control signal u applied. For nominal plants with suitably smooth nonlinearities, small differences δu, δy, a linearization of the process allows an approximate linear dynamic model for relating δy to δu. With this model, optimal linear regulator theory can be applied to calculate δu in terms of δy which is measurable, so as to regulate δy to zero. Indeed, the linearization can extend to yield an associated quadratic performance index consistent with the original nonlinear index so that linear optimal control (LQG) theory can be applied to achieve optimal regulation of δy under the linearization assumptions. Robust regulater designs based on optimal theory, perhaps via loop-transmission recovery (LTR), could be expected to lead to performance improvement over a wider range of perturbations on the nominal plant model.

Even with the application of linearization and feedback regulation to enhance optimal control strategies, there can still be problems with external disturbances and modelling errors. The linearization itself may be a poor approximation when there are large perturbations from the optimal trajectory.

In this paper, it is proposed to apply robust and adaptive techniques to assist in regulation of the actual plant so that it behaves as closely as possible to the nominal (idealized) model. An adaptive control technique which is designed to enhance performance of a stabilizing regulator for a nominal time-varying linear plant model is presented in an earlier work [1], building on the time-invariant case proposed in [2] and further studied in [3]. Here, this technique is applied in conjunction with an open-loop nonlinear optimal controller and standard linear optimal feedback regulator (LQG) approach, with the view to enhancing performance of the optimal controller when applied to a plant, not the idealized model. Loop recovery (LTR) techniques are also studied to enhance robustness of the optimal regulator designs. Some analysis results are presented giving stability properties of the optimal/adaptive scheme. These generalize known linear system stability plant results to the case of mixed linear system and nonlinear systems as here. Mention is made of performance enhancement properties in the presence of unmodelled dynamics developed for the linear case based on an averaging analysis, although generalizing to a specific nonlinear case appears possible, such an analysis is beyond the scope of the present paper. Simulation results demonstrate the effectiveness of the various control strategies, and the possibility of further performance enhancement based on functional learning is noted. In Section 2, the algorithms of [1] are viewed in the context of non-linear optimal control. In Section 3, some analysis results are developed relevant to the nonlinear control situation, and in Section 4, simulation studies are presented. Conclusions are drawn in Section 5.

2 Self-Tuning Optimal Nonlinear Control

Signal Model/Optimal Control Let us consider some actual plant, in operator notation G, with a nominal model G_0, given in state space form

$$G_0: \ dx/dt = f(x,u,t), \ \ y = h(x,t) \tag{2.1}$$

with an associated performance index $I = \frac{1}{T}\int_0^T l(x,u,t)\ dt$. Let us denote the optimal control signal u^*, assuming that this exists and can be calculated under the usual assumptions on the nonlinear functions f, h, l. The optimal state / output trajectories for nominal model are denoted x^*/y^*. We will focus on the case when the time horizon T becomes infinite.

Linearize a Signal Model / Linear Optimal Regulation. Let us consider linearized versions of (2.1) about the optimal trajectory, obtained using the usual series expansion approach with $A = \frac{\partial f}{\partial x}|_{\substack{x=x^* \\ u=u^*}}, B = \frac{\partial f}{\partial u}|_{\substack{x=x^* \\ u=u^*}}, C = \frac{\partial h}{\partial x}|_{x=x^*}$ as

$$\Delta G_0 : d(\delta x)/dt = A\delta x + B\delta u, \delta y = C\delta x, \delta x(0) = \delta x_0 \tag{2.2}$$

$$\begin{aligned}\Delta I &= \frac{1}{T}\int_0^T [(\delta x)'Q_c\delta x + 2(\delta x)'S_c\delta u + (\delta u)'R_c\delta u]dt, \\ &= \frac{1}{T}\int_0^T e'e\ dt\end{aligned} \tag{2.3}$$

With ΔG the operator denoting the actual system with input $\delta u = u - u^*$, state $\delta x = x^*$ and output $\delta y = y - y^*$, then ΔG_0 denotes a linearized version of ΔG. Also e is a vector calculated in terms of δx, δu, Q_c, S_c, R_c as

$$e = \begin{bmatrix} Q_c & S_c \\ S_c' & R_c \end{bmatrix}^{\frac{1}{2}} \begin{bmatrix} \delta x \\ \delta u \end{bmatrix}, Q_c \geq 0,$$
$$Q_c - S_c R_c^{-1} S_c \geq 0, \quad R_c > 0 \tag{2.4}$$

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The associated linear optimal (LQG) regulator of the linearized model (2.2) (in a suitably stochastic environment not spelt out here), for the index (2.3) (or rather its expected value) is given the operator notation K_0 where

$$K_0: \quad d(\delta\hat{x})/dt = A\delta\hat{x} + B\delta u - H\delta r, \quad \delta u = F\delta\hat{x}, \ \delta r = \delta y - C\delta\hat{x} \tag{2.5}$$

Here H, F are time-varying matrices formed via standard LQG theory, and δr is the estimator residuals. [Ito equations are avoided in this presentation.]

Actually, the important aspect of the LQG design for our purposes is that under the relevant uniform stabilizability and uniform detectability assumptions, the (time-varying) gains H, F exist, and are given from the solution of two Riccati equations (with no finite escape time). Moreover, for the limiting case when the time horizon T becomes infinite, the controller K_0 stabilizes ΔG_0. Here stability means that all possible bounded inputs to the closed loop consisting of K_0 feeding back on ΔG_0 result in bounded loop signals (outputs).

Robust Feedback Controller It is well known that the LQG controller (2.5) for the linearized plants (2.2), although optimal for the nominal linear time-varying plant for the assumed noise environment, may be far from optimal in other than the nominal noise environments, or in the presence of structured or unstructured perturbations on (2.2). Stability may be lost even for small variations from the nominal plant.

Methods to enhance LQG regulator robustness exist, such as modifying Q_c, S_c, R_c (usually $S_c \equiv 0$) selections, or assumed noise environments, as when loop recovery is used. Such techniques could well serve to strengthen the robustness properties of the optimal/ adaptive schemes studied subsequently. In order to proceed, we here merely assume the existence of a controller (2.5) stabilizing ΔG_0, although our objective is to achieve a controller which both stabilizes ΔG, and achieves a low value of index ΔI when applied to ΔG.

The Class of all Stabilizing Controllers for ΔG_0. Based on the work of [1], the class of all stabilizing controllers $K(Q)$ for ΔG_0 is as depicted in Figure 2.1(a). Here Q is an arbitrary causal bounded-input, bounded-output operator, parametrizing the class of all stabilizing controllers. If Q is linear, rational, proper, and stable then it parametrizes the class of all linear, rational, proper, stabilizing controllers for ΔG_0. A re-arrangement is depicted in Figure 2.1(b) where the subsystem J_K is readily extracted from Figure 2.1(b) as

$$\begin{aligned} J_K \ : \ d\delta\hat{x}/dt &= (A+BF)\delta\hat{x} + Bs - H\delta r \\ \delta u &= F\delta\hat{x} + s, \quad \delta r = \delta y - C\delta\hat{x} \end{aligned} \tag{2.6}$$

Of course in obvious notation$[J_K]_{11} = K_0$.

Adaptive Q. Our proposal is to implement a controller $K(Q)$ for some adaptive Q, but applied to ΔG and not ΔG_0. The intention is for Q to be chosen to ensure that $K(Q)$ stabilizes G and achieves good performance in terms of the index ΔI. Thus consider the arrangement of Figure 2.2 where the block P is actually the arrangement depicted in Figure 2.1 but effectively characterized by ΔG and L operators.

A refinement on this proposal is to consider a two-degree-of-freedom controller scheme based on the work of [2]. This is depicted in Figure 2.3. It can be derived from a one-degree-of-freedom controller arrangement for the augmented plant $\begin{bmatrix} 0 & G^T \end{bmatrix}^T$, re-organized as a two-degree-of-freedom arrangement for G. The objective is to select Q_1, Q_2 causal, bounded-input, bounded-output operators on line so that the response e is minimized in an L_2 sense.

In order to present a least squares algorithm for selection of Q, as in the schemes of [1], some preprocessing of the signals e, δu, δy is required.

Prefiltering To design the appropriate prefilters, it is convenient to introduce coprime factorisations for ΔG_0 and K_0 such that

$$\Delta G_0 = N_0 M_0^{-1} = \tilde{M}_0^{-1}\tilde{N}_0, \quad K_0 = U_0 V_0^{-1} = \tilde{V}_0^{-1}\tilde{U}_0 \tag{2.7}$$

satisfy the double Bezout

$$\begin{bmatrix} \tilde{V}_0 & -\tilde{U}_0 \\ -\tilde{N}_0 & \tilde{M}_0 \end{bmatrix} \begin{bmatrix} M_0 & U_0 \\ N_0 & V_0 \end{bmatrix} = I_2$$

$$\begin{bmatrix} M_0 & U_0 \\ N_0 & V_0 \end{bmatrix} \begin{bmatrix} \tilde{V}_0 & -\tilde{U}_0 \\ -\tilde{N}_0 & \tilde{M}_0 \end{bmatrix} = I_2 \tag{2.8}$$

where the factors $N_0, M_0, N_0, V_0, \tilde{M}_0, \tilde{N}_0, \tilde{U}_0, \tilde{V}_0$ are stable and causal operators. We consider those defined in [1], using the shorthand notation which would represent ΔG_0 of (2.2) as $\left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right]_{\delta x_0}$ with

$$\begin{bmatrix} M_0 & U_0 \\ N_0 & V_0 \end{bmatrix} = \left[\begin{array}{c|cc} A+BF & B & -H \\ \hline F & I & 0 \\ C+DF & D & I \end{array}\right]_{\delta x_0}$$

$$\begin{bmatrix} \tilde{V}_0 & -\tilde{U}_0 \\ -\tilde{N}_0 & \tilde{M}_0 \end{bmatrix} = \left[\begin{array}{c|cc} A+HC & -(B+HD) & H \\ \hline F & I & 0 \\ C & -D & I \end{array}\right]_{\delta x_0} \tag{2.9}$$

Thus, $M_0 = \left[\begin{array}{c|c} A+BF & B \\ \hline F & I \end{array}\right]_{\delta x_0}$, etc.

In this notation J_K of (2.6) has the form

$$J_K = \begin{bmatrix} K_0 & \tilde{V}_0^{-1} \\ V_0^{-1} & -V_0^{-1}N_0 \end{bmatrix} \tag{2.10}$$

Now define P_{12} as the operator between δu and e under nominal plant assumptions. Thus since (2.2),(2.4) hold, then in operator notation

$$P_{12} = \begin{bmatrix} Q_c & S_c \\ S_c' & R_c \end{bmatrix}^{\frac{1}{2}} \begin{bmatrix} \Delta G_x \\ I \end{bmatrix} \tag{2.11}$$

where $\Delta G_x : \frac{d(\delta x)}{dt} = A\delta x + B\delta u$ With the above definitions, and using operator notation, we define filtered variables

$$\xi = \begin{bmatrix} P_{12}M_0 \ u^* \\ P_{12}M_0 \ \delta r \end{bmatrix}, \quad \zeta = e - P_{12}M_0 s \tag{2.12}$$

Least Squares Q Selection Let us define a discrete-time version of Q in Z-transforms as

$$Q_1(z^{-1}) = \frac{\gamma_0 + \gamma_1 z^{-1} + \cdots \gamma_p z^{-p}}{1 + \alpha_1 z^{-1} + \cdots \alpha_n z^{-n}}, \quad Q_2(z^{-1}) = \frac{\beta_0 + \beta_1 z^{-1} + \cdots \beta_m z^{-m}}{1 + \alpha_1 z^{-1} + \cdots \alpha_n z^{-n}}$$

$$Q(z^{-1}) = [Q_1(z^{-1}) \ Q_2(z^{-1})], \quad \theta' = [\alpha_1 \cdots \alpha_n \beta_0 \cdots \beta_m \gamma_0 \cdots \gamma_p]$$

The following state (regression) vector in discrete time is

$$\phi_k' = [-s_{k-1} \cdots - s_{k-n} \ \delta r_k \cdots \delta r_{k-m} \ \omega_k \cdots \omega_{k-p}] \tag{2.13}$$

The dimensions n, m, p are set from an implementation convenience/ performance trade-off. In the adaptive-Q case, the parameters are time-varying resulting from least squares calculations given below. We assume a unit delay in calculations. Thus θ is replaced by $\hat{\theta}_{k-1}$ and the filter with operator $Q_k = [Q_{1k} \ Q_{2k}]$ is implemented with parameters (time-varying in general) as

$$s_k = \hat{\theta}_{k-1}'\phi_k, \quad \hat{\theta}_k' = [\hat{\alpha}_{1k} \cdots \hat{\alpha}_{nk}\hat{\beta}_{0k} \cdots \hat{\beta}_{mk}\hat{\gamma}_{0k} \cdots \hat{\gamma}_{pk}] \tag{2.14}$$

We seek selections of $\hat{\theta}_k$ so that the adaptive controller minimizes the L_2 norm of the response e_k. Using theory in [2], with suitable initializing we have the adaptive-Q arrangement of Figure 2.3 with equations

$$\begin{aligned} \hat{\theta}_k &= \hat{\theta}_{k-1} + \hat{P}_k\hat{\phi}_k\hat{e}_{k/k-1}, \quad \hat{e}_{k/k-1} = \zeta_k - \hat{\phi}_k'\hat{\theta}_{k-1} \\ e_{k/k} &= \xi_k - \hat{\phi}_k'\hat{\theta}_k \\ \hat{P}_k &= (\sum_1^k \hat{\phi}_i\hat{\phi}_i')^{-1} \\ &= \hat{P}_{k-1} - \hat{P}_{k-1}\hat{\phi}_k(I + \hat{\phi}_k'\hat{P}_{k-1}\hat{\phi}_k)^{-1}\hat{\phi}_k\hat{P}_{k-1} \\ \hat{\phi}_k' &= [(\hat{e}_{k-1/k-1} - \zeta_{k-1})(\hat{e}_{k-n/k-n} - \zeta_{k-n}) - \\ &\qquad \xi_{2,k} \cdots - \xi_{2,k-m} \cdots - \xi_{1,k} \cdots - \xi_{1,k-m}] \end{aligned} \tag{2.15}$$

Summary of Proposed Direct Adaptive Scheme. The complete adaptive-Q scheme is a combination of Figures 2.3,2.4 with key equations (2.9),(2.15).

Remarks

1. The algorithms (2.15) should be modified to ensure that $\hat{\theta}_k$ is projected into a restricted domain, such as $||Q_k|| < \epsilon$ for some fixed ϵ. Such projections can be guided by the theory discussed in the next section.

2. To achieve convergence of $\hat{\theta}_k$, then $\hat{P}_k$ must approach zero, or equivalently, $\hat{\phi}_k$ must be persistantly exciting in some sense. However, parameter convergence is not strictly necessary to achieve performance enhancement. With more general algorithms which involve resetting or forgetting, then care must be taken to avoid ill-conditioning of $\hat{P}_k$, perhaps via unstable excitation in the system.

3. It turns out that appropriate scaling can be crucial to achieve the best possible performance enhancement. Scaling gains can be included, to scale r and/or e with no effect on the supporting theory, other than defining projection domains as in Remark 1 above. Likewise, the "scaling" can be generalized to stable dynamic filters for r and/or e with no effect on the supporting theory. In this frequency shaped designs can be effected.

4. Our presentation so far has been for continuous time ΔG and J_K but discrete-time updates of parameters $\hat{\theta}_k$ and then Q_k, based on samplings of r and e. Likewise, our subsequent simulation results are mixed continuous time/discrete time results. Theory, as noted below gives performance enhancement only at the discrete-time sampling instants, so that as in all mixed continuous/discrete system studies, care may be taken to achieve a suitably fast sampling rate. Of course, we could have worked exclusively in discrete-time or continuous time.

5. The scheme described above can be specialized to the cases when Q_1, Q_2 are finite impulse response filters by setting $n = 0$. The Q are stable for all bounded $\hat{\theta}_k$. Also either Q_1 or Q_2 can be set to zero to simplify the processing, although possibly at the expense of performance.

6. In the case that Q_1 is moving average and Q_2 is zero, then our scheme becomes very simple, being a moving average filter Q_1 in series with the closed loop system $\{\Delta G, K_0\}$. In this case then, if Q_1 is stable, guaranteed when the gains $\hat{\theta}_k$ are bounded, and $\{\Delta G, K_0\}$ is stable, then there is obvious stability of the adaptive scheme.

7. When the linearized plant model ΔG_0 is stable, and one selects trivial values $F, H = 0$ so that $K_0 = 0$, then the arrangement of Figure 2.3 simplifies to a familier model-reference adaptive control arrangement depicted in Figure 2.5.

8. In the case that Q_1 is set to zero, then there is no adaptive feedforward control action.

9. The operators $\Delta G_0, J_K$ are in fact functions of the optimal trajectory x^*, or under suitable generalizations of $x^*, \delta x$. It would make sense to have the operator Q also as a function of x^* (or $x^*, \delta x$). Then this adaptive-Q approach becomes a learning-Q approach as studied in a companion paper [4].

3 Convergence Properties

In this section we focus on stability results as a first step to achieving convergence results for our system. We first analyze a parametrization of the plant ΔG with input δu and output δy in terms of the co-prime factorizations of the linearized version ΔG_0, and stabilizing linear controller K_0, and establish that this parametrization covers the class of well-posed closed-loop systems under study. Next, stability of the scheme is studied in terms of such parametrizations and then expected convergence properties are noted based on this characterization and known convergence theories in the linear case.

Nonlinear System Fractional Maps As in the previous section, let us consider the right and left coprime factorizations for the nominal linearized plant and controller of [5]. These operators are expressed as functions of the desired optimal trajectory, x^*, but since x^* is time dependent, then for any specific trajectory $x^*(.)$ the operators are merely linear time-varying operators, and can be treated as such. We define $\Delta G(x^*)$ as the (nonlinear) system with input δu and output δy. Note that $\Delta G_0(x^*)$ is a linearization of $\Delta G(x^*)$. When the notation $\Delta G_0, \Delta G$ is used, the x^* dependence, or equivalently ,time dependence is understood. Also, a unity gain feedback loop with open loop operator W_{ol} is said to be well-posed when $(I + W_{ol})^{-1}$ exists. Recall that for a nonlinear operator S, then, in general $S(A+B) \neq SA + SB$, or equivalently superposition does not hold, and care must be taken in the composition of nonlinear operators. Otherwise, manipulation rules for nonlinear operators follow those more familiar ones for linear operators.

Theorem 3.1 *(Right fractional map forms) Consider that $\{\Delta G_0, K_0\}$ is well posed and stabilizing with left and right coprime factorizations for $\Delta G_0, K_0$ as in (2.7) and the double Bezout (2.8) holding. Then any nonlinear plant with ΔG such that $\{\Delta G, K_0\}$ is a well-posed closed-loop system can be expressed in terms of a (nonlinear) operator S in right fractional map forms :*

$$\begin{aligned} \Delta G &= N(S)M^{-1}(S) = (N_0 + V_0 S)(M_0 + U_0 S)^{-1} && (3.1)\\ &= \Delta G_0 + \tilde{M}_0^{-1} S (I + M_0^{-1} U_0 S)^{-1} M_0^{-1} && (3.2)\end{aligned}$$

Also, closed-loop system operators are given from

$$\begin{bmatrix} I & -K_0 \\ -\Delta G & I \end{bmatrix}^{-1} = \begin{bmatrix} I & -K_0 \\ -\Delta G_0 & I \end{bmatrix}^{-1} + \begin{bmatrix} U_0 & M_0 \\ V_0 & N_0 \end{bmatrix} \begin{bmatrix} S & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \tilde{V}_0 & \tilde{U}_0 \\ \tilde{N}_0 & \tilde{M}_0 \end{bmatrix} \quad (3.3)$$

Moreover, the maps (3.1), (3.2) have the block diagram representations of Figures 3.1 (a) and (b) where

$$J_G = \begin{bmatrix} -M_0^{-1} U_0 & M_0^{-1} \\ \tilde{M}_0^{-1} & \Delta G_0 \end{bmatrix} \quad (3.4)$$

The solutions of (3.1), (3.2) are unique , given from the right fractional maps in terms of ΔG, or $(\Delta G - \Delta G_0)$ as

$$\begin{aligned} S &= (-\tilde{N}_0 + \tilde{M}_0 \Delta G)(\tilde{V}_0 - \tilde{U}_0 \Delta G)^{-1} && (3.5)\\ &= \tilde{M}_0 (\Delta G - \Delta G_0) M_0 [I - \tilde{U}_0 (\Delta G - \Delta G_0) M_0]^{-1} && (3.6)\end{aligned}$$

or in terms of the closed-loop system operators as

$$S = \begin{bmatrix} -\tilde{N}_0 & \tilde{M}_0 \end{bmatrix} \left[\begin{bmatrix} I & -K_0 \\ -\Delta G & I \end{bmatrix}^{-1} - \begin{bmatrix} I & -K_0 \\ -\Delta G_0 & I \end{bmatrix}^{-1} \right] \begin{bmatrix} M_0 \\ N_0 \end{bmatrix} \quad (3.7)$$

Moreover, $\{N(S), M(S)\}$ are coprime and obey a Bezout identity

$$\tilde{V}_0 M(S) - \tilde{U}_0 N(S) = I \quad (3.8)$$

Proof Now simple manipulations allow (3.5) to be reorganized under the well-posedness assumption as

$$\begin{bmatrix} I \\ S \end{bmatrix} = \begin{bmatrix} \tilde{V}_0 & -\tilde{U}_0 \\ -\tilde{N}_0 & \tilde{M}_0 \end{bmatrix} \begin{bmatrix} I \\ \Delta G \end{bmatrix} (I - K\Delta G)^{-1} \tilde{V}_0^{-1}$$

and via the Bezout identity, as

$$\begin{bmatrix} M(S) \\ N(S) \end{bmatrix} = \begin{bmatrix} M_0 & U_0 \\ N_0 & V_0 \end{bmatrix} \begin{bmatrix} I \\ S \end{bmatrix} = \begin{bmatrix} I \\ \Delta G \end{bmatrix} (I - K\Delta G)^{-1} \tilde{V}_0^{-1} \quad (3.9)$$

Thus under (3.5) then $M^{-1}(S)$ exists and,(3.1) holds as follows

$$N(S)M^{-1}(S) = \Delta G (I - K\Delta G)^{-1} \tilde{V}_0^{-1} \left[(I - K\Delta G)^{-1} \tilde{V}_0^{-1} \right]^{-1} = \Delta G$$

To prove the equivalence of (3.1) and (3.2), simple manipulations give

$$\begin{aligned} \Delta G &= \Delta G_0 + (N_0 + V_0 S)(I + M_0^{-1} U_0 S)^{-1} M_0^{-1} - N_0 M_0^{-1} \\ &= \Delta G_0 + (V_0 - N_0 M_0^{-1} U_0) S (I + M_0^{-1} U_0 S)^{-1} M_0^{-1} \\ &= \Delta G_0 + (V_0 - \tilde{M}_0^{-1} \tilde{N}_0 U_0) S (I + M_0^{-1} U_0 S)^{-1} M_0^{-1} \\ &= \Delta G_0 + \tilde{M}_0^{-1} (\tilde{M}_0 V_0 - \tilde{N}_0 U_0) S (I + M_0^{-1} U_0 S)^{-1} M_0^{-1} \\ &= \Delta G_0 + \tilde{M}_0^{-1} S (I + M_0^{-1} U_0 S)^{-1} M_0^{-1} \end{aligned}$$

so that under (2.8), then (3.2) holds. Likewise (3.5) is equivalent to (3.6) as follows

$$\begin{aligned} S &= \tilde{M}_0 (\Delta G - \Delta G_0)(\tilde{V}_0 - \tilde{U}_0 \Delta G)^{-1} \\ &= \tilde{M}_0 (\Delta G - \Delta G_0) M_0 (\tilde{V}_0 M_0 - \tilde{U}_0 \Delta G M_0)^{-1} \\ &= \tilde{M}_0 (\Delta G - \Delta G_0) M_0 (I + \tilde{U}_0 N_0 M_0^{-1} M_0 - \tilde{U}_0 \Delta G M_0)^{-1} \\ &= \tilde{M}_0 (\Delta G - \Delta G_0) M_0 [I - \tilde{U}_0 (\Delta G - \Delta G_0) M_0]^{-1} \end{aligned}$$

To see that the operator of (3.1) is equivalent to that depicted in Figure (3.1a), observe from Figure (3.1a) that $l = M_0^{-1}(e_1 - U_0 S l)$, or equivalently, $l = (M_0 + U_0 S)^{-1} e$. Also, $(e_2 - w_2) = (N_0 + V_0 S) l = (N_0 + V_0 S)(M_0 + U_0 S)^{-1} e_1$ which is equivalent to (3.1).

Now suppose there is some other $(S + \Delta S)$ which also satisfies (3.1), then

$$\begin{bmatrix} I \\ \Delta G \end{bmatrix} = \begin{bmatrix} M_0 & U_0 \\ N_0 & V_0 \end{bmatrix} \begin{bmatrix} I \\ S \end{bmatrix} (M_0 + U_0 S)^{-1} = \begin{bmatrix} M_0 & U_0 \\ N_0 & V_0 \end{bmatrix} \begin{bmatrix} I \\ S + \Delta S \end{bmatrix} (M_0 + U_0 S + U_0 \Delta S)^{-1}$$

for some ΔS. Then, using (2.8),

$$\begin{bmatrix} \tilde{V}_0 & -\tilde{U}_0 \\ -\tilde{N}_0 & \tilde{M}_0 \end{bmatrix} \begin{bmatrix} I \\ \Delta G \end{bmatrix} = \begin{bmatrix} I \\ S \end{bmatrix} (M_0 + U_0 S)^{-1} = \begin{bmatrix} I \\ S + \Delta S \end{bmatrix} (M_0 + U_0 S + U_0 \Delta S)^{-1} \tag{3.10}$$

Premultiplication by $[I \quad 0]$ gives $M_0 + U_0 S = M_0 + U_0 S + U_0 \Delta S$, and premultiplication by $[0 \quad I]$ gives then in turn that $\Delta S = 0$.
To verify (3.7), first observe that

$$\begin{bmatrix} I & -K_0 \\ -\Delta G & I \end{bmatrix} = \begin{bmatrix} M_0 & -U_0 \\ -N_0 & V_0 \end{bmatrix} \begin{bmatrix} I & 0 \\ -S & I \end{bmatrix} \begin{bmatrix} M_0 + U_0 S & 0 \\ 0 & V_0 \end{bmatrix}^{-1} \tag{3.11}$$

Thus

$$\begin{bmatrix} I & -K_0 \\ -\Delta G & I \end{bmatrix}^{-1} - \begin{bmatrix} I & -K_0 \\ -\Delta G_0 & I \end{bmatrix}^{-1} = \begin{bmatrix} M_0 & U_0 \\ N_0 & V_0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ S & 0 \end{bmatrix} \begin{bmatrix} M_0 & -U_0 \\ -N_0 & V_0 \end{bmatrix}^{-1}$$

and applying the double Bezout (2.8)gives

$$\begin{bmatrix} \tilde{V}_0 & -\tilde{U}_0 \\ -\tilde{N}_0 & \tilde{M}_0 \end{bmatrix} \left[\begin{bmatrix} I & -K_0 \\ -\Delta G & I \end{bmatrix}^{-1} - \begin{bmatrix} I & -K_0 \\ -\Delta G_0 & I \end{bmatrix}^{-1} \right] \times \begin{bmatrix} M_0 & -U_0 \\ -N_0 & V_0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ S & 0 \end{bmatrix}$$

or equivalently (3.3) holds, and (3.7). (This result is generalized in Theorem 3.2)
Simple manipulations from Figure 3.1(b) give the transfer function of the G block to be $J_{21} S (1 - J_{11} S)^{-1} J_{12} + J_{22}$, and substitution of (3.4) gives ΔG by (3.2).
To establish coprimeness of $N(S), M(S)$ observe that under the double bezout (2.8)

$$\tilde{V}_0 M(S) - \tilde{U}_0 N(S) = \tilde{V}_0 M - \tilde{U}_0 N + (\tilde{V}_0 U_0 - \tilde{U}_0 V_0) S = I$$

which is unimodular, Thus from [7] Lemma 2.1, $N(S) M(S)^{-1}$ is a right co-prime factorization.
□

Remarks

1. When ΔG is linear, the above results specialize to known results in [5], although the details of the theorem proof appears quite different so as to avoid using superposition when nonlinear operators $\Delta G, S$ are involved.

2. The fact that $\tilde{M}_0, \tilde{N}_0, M_0, N_0, \tilde{U}_0, \tilde{V}_0, U_0, V_0$ are linear has allowed derivations to take place without differential boundedness or other such assumptions as in a full nonlinear theory as developed in [6], [7] using left coprime factorizations.

3. Dual left coprime factorization results , apart from those in [6], [7] involving differential boundedness, are elusive at this time. Certainly dualizing certain of the above proof steps requires superposition and thus linearity of $\Delta G, S$.

4. Dual results apply for fractional mappings of $K = K(Q)$, as in (3.12),(3.13) along with duals of the other results. Thus $K(Q)$ can be expressed as a linear controller K_0 augmented with a non-linear Q.Also, by duality, Figure (3.1a) depicts a block diagram arrangement for

$$K \triangleq K(Q) = U(Q) V^{-1}(Q) = (U_0 + M_0 Q)(V_0 + N_0 Q)^{-1} \tag{3.12}$$

where

$$Q = (-\tilde{U}_0 + \tilde{V}_0 K)(\tilde{M}_0 - \tilde{N}_0 K)^{-1} \tag{3.13}$$

Stabilization Results

We define a system $\{G, K\}$ to be *internally stable* iff for all bounded inputs, the outputs are bounded.

Theorem 3.2 *Consider the well-posed feedback system $\{\Delta G, K\}$ under the conditions of Theorem 3.1, with ΔG and K parametrised by S, Q as in (3.1), (3.12) and as depicted in Figures 3.1(a) and (b). Then $\{\Delta G(S), K(Q)\}$ is well posed and stable if and only if the feedback system $\{Q, S\}$ depicted in Figure 3.1(d) is well posed and internally stable. Moreover, referring to Figure 3.1(c), the $J_k/\Delta G$ block with input/output operator T satisfies*

$$T = S \tag{3.14}$$

Proof Observe that from (3.1),(3.12)

$$\begin{bmatrix} I & -K(Q) \\ -\Delta G(S) & I \end{bmatrix} = \begin{bmatrix} M_0 & -U_0 \\ -N_0 & V_0 \end{bmatrix} \begin{bmatrix} I & -Q \\ -S & I \end{bmatrix} \begin{bmatrix} M_0 + U_0 S & 0 \\ 0 & V_0 + N_0 Q \end{bmatrix}^{-1} \tag{3.15}$$

Clearly, under the double Bezout identity (2.8), or equivalently under $\{\Delta G_0, K_0\}$ well posed and internally stable,
$\begin{bmatrix} I & -K(Q) \\ -\Delta G(S) & I \end{bmatrix}^{-1}$ exists $\Longleftrightarrow$ $\begin{bmatrix} I & -Q \\ -S & I \end{bmatrix}^{-1}$ exists.
Equivalently, $\{G(S), K(Q)\}$ is well posed if and only if $\{Q, S\}$ is well posed. Thus under well posedness assumptions, taking inverses in, and exploiting (3.15) then simple manipulations yield

$$\begin{bmatrix} I & -K(Q) \\ -\Delta G(S) & I \end{bmatrix}^{-1} = \begin{bmatrix} I & -K_0 \\ -\Delta G_0 & I \end{bmatrix}^{-1} + \begin{bmatrix} U_0 & M_0 \\ V_0 & N_0 \end{bmatrix} \begin{bmatrix} S & 0 \\ 0 & Q \end{bmatrix} \begin{bmatrix} I & -Q \\ -S & I \end{bmatrix}^{-1} \begin{bmatrix} \tilde{V}_0 & \tilde{U}_0 \\ \tilde{N}_0 & \tilde{M}_0 \end{bmatrix} \tag{3.16}$$

Now internal stability of $\{\Delta G_0, K_0\}$, $\{S, Q\}$, and stability of $N_0, \tilde{N}_0$ etc leads to internal stability of the right hand side and thus of $\{\Delta G(S), K(Q)\}$ as claimed. Moreover from (3.16),(2.8)

$$\begin{bmatrix} S & 0 \\ 0 & Q \end{bmatrix} \begin{bmatrix} I & -Q \\ -S & I \end{bmatrix}^{-1} = \begin{bmatrix} -\tilde{N}_0 & \tilde{M}_0 \\ \tilde{V}_0 & -\tilde{U}_0 \end{bmatrix} \times \left[\begin{bmatrix} I & -K(Q) \\ -\Delta G(S) & I \end{bmatrix}^{-1} - \begin{bmatrix} I & -K_0 \\ -\Delta G_0 & I \end{bmatrix}^{-1} \right] \begin{bmatrix} M_0 & -U_0 \\ -N_0 & V_0 \end{bmatrix}$$

Thus well posedness and internal stability of $\{\Delta G(S), K(Q)\}$ and $\{\Delta G_0, K_0$ gives well posedness and internal stability of $\{Q, S\}$ to complete the first part of the proof.
Now with J_K defined as in (2.10) then the operator T in Figure 3.1(c) can be represented as

$$\begin{aligned} T &= V_0^{-1} \Delta G (I - \tilde{V}_0^{-1} \tilde{U}_0 \Delta G)^{-1} \tilde{V}_0^{-1} - \tilde{N}_0 \tilde{V}_0^{-1} \qquad (3.17) \\ &= V_0^{-1} \Delta G (\tilde{V}_0 - \tilde{U}_0 \Delta G)^{-1} - \tilde{N}_0 \tilde{V}_0^{-1} \\ &= [V_0^{-1} \Delta G - \tilde{N}_0 + \tilde{N}_0 \tilde{V}_0^{-1} \tilde{U}_0 \Delta G](\tilde{V}_0 - \tilde{U}_0 \Delta G)^{-1} \\ &= \tilde{M}_0 [\tilde{M}_0^{-1} (V_0^{-1} + \tilde{N}_0 \tilde{V}_0^{-1} \tilde{U}_0) \Delta G - \Delta G_0](\tilde{V}_0 - \tilde{U}_0 \Delta G)^{-1} \\ &= \tilde{M}_0 [\Delta G - \Delta G_0](\tilde{V}_0 - \tilde{U}_0 \Delta G)^{-1} \\ &= S \end{aligned}$$

□

Remarks

1. Note that this proof does not use superposition associated with operators S, Q, but does in regard to M_0, N_0 etc. The results following Theorem 3.1 also apply for Theorem 3.2.Thus the proof approach differs (of necessity) from the proof approach given in [5] for the linear S, Q case based on work with the left factorizations , since when working with left factorizations, superposition is used associated with the operators Q, S.More general versions of this approach where G_0, K_0 are nonlinear will be explored in subsequent work.

2. If $|S| < \epsilon$ then by the small gain theorem for closed feedback loops, if $|Q| < 1/\epsilon$ then Q stabilizes the loop. From this, and Theorem 3.2 with $(\Delta G - \Delta G_0)$ suitably small in norm, then there exists some Q which will guarantee stability.

3. In the case where $\Delta G = \Delta G_0$ then trivially $S = 0$, and any Q selection based on identification of S will be trivially $Q = 0$. This contrasts the awkwardness of one alternative design approach which would seek to identify the closed-loop system as a basis for a controller augmentation design.

4. Observations on examples in the linear ΔG case have shown that if K_0 is robust for G, then S can be approximated by a low order system [3], so making any Q selection more straightforward than might be otherwise expected.

5. In [8] stability results are studied for nested linear systems based on the Q/S parametrization approach. The authors demonstrate how an $(n+1)$ loop control diagram can be specialised to an equivalent n–loop diagram, and shows that internal stability of an $(n+1)$ control loop is equivalent to that of the controller in the last loop stabilizing the n-th frequency-shaped plant-model error. It is clear that our results could also likewise extend, at least in the case when all approximations but the last were linear.

Averaging Convergence Analysis The adaptive scheme has the property that when $\Delta G = \Delta G_0$, then Q_k converges to zero, so that when K_0 is the nominal optimal regulator, then the adaptive regulator $K(Q_k)$ converges to $K_0 = K(0)$, the optimal one. Such details are studied in [1]. More general results are given in [9] for the case of linear ΔG, based on an averaging analysis. One result concerns the case for when $\{\Delta G, K_0\}$ is a stabilizing pair, as well as $\{\Delta G_0, K_0\}$. There is guaranteed performance enhancement when $\{\Delta G, K_0\}$ is not stabilizing, but is small in that $\{\Delta G, K(Q)\}$ is stabilizing for some Q with $\|Q\| < \epsilon$ with ϵ known, then with Q_K projected into the domain $\{\|Q\| < \epsilon\}$, there is guaranteed performance enhancement. For the more general case where ΔG is nonlinear, then new results are needed. One approach is the averaging analysis as used in [9] but for nonlinear systems as in [10], but clearly any results obtained will be problem specific and beyond the scope of the present paper. A first step in such an analysis is to derive appropriate stability results. Stability results for the proposed scheme in the nonlinear ΔG, but linear $K_0, \Delta G_0$ case are studied in the next section. These are more developed than those for the nonlinear $K_0, \Delta G_0, \Delta G$ studied in references [6],[7]. Convergence results for a learning-Q approach for linear systems as in Remark 8 in Section 2, would follow similar lines to the adaptive-Q approach, at least when $\Delta G_0, J_k$ and Q are functions only of x^*. But in the more general case when the operators are functions of x, or δx, a stabilization theory coping with nonlinear $\Delta G_0, J_K$ is developed in the companion paper [4].

4 Simulations

In this section, we demonstrate the efficacy of our approach through simulation studies. Consider an optimal control problem based on the van der Pol equation

$$\dot{x}_1 = (1 - x_2^2)x_1 - x_2 + u, \quad \dot{x}_2 = x_1, \quad y = x_1 \tag{4.1}$$

with $x_1(0) = 0$, $x_2(0) = 1$ and the performance index defined by

$$I = \frac{1}{2}\int_0^5 (x_1^2 + x_2^2 + u^2)dt \tag{4.2}$$

A second-order algorithm [11], using 400 integration steps, was adopted for the numerical solution of the open-loop optimal control signal u^*. An arbitrary initial nominal control $u \equiv 0, t \in [0, 5]$, was chosen. The value of the performance index was reduced to the optimal one in 4 iterations in updating $u(\cdot)$ over the range $[0, 5]$.

Four situations have been studied in simulations. For each case we add a stochastic or a deterministic disturbance which disturbs the optimal input signal. Also, in some of the simulations we apply a plant with unmodelled dynamics. The objective is to regulate perturbations from the optimal by means of the index $\Delta I = \int_0^5 (\delta x_1^2 + \delta x_2^2 + \delta u^2)dt$ which is expressed in terms of perturbations $\delta x, \delta u$. For each of the disturbances added, and for the unmodelled dynamics case, we compare five controller strategies, and demonstrate the robustness and performance properties of the adaptive-Q methodology.

Case 1: Open-loop design.
Here we adopt the optimal control signal u^* as an input signal of the nonlinear system with added disturbance. Fig 4.1 shows that the open-loop design is quite sensitive to such disturbances in that x_1, x_2 differ significantly from x_1^*, x_2^*.

Case 2: LQG's design.
In order to construct feedback controllers, we adopt the standard LQG theory based on the linearized plant model of (4.1) about the optimal trajectories and the performance index (4.3). Of course, the input signals $u^* + \delta u$ are no longer 'optimal' for the nominal plant. The LQG controller's design yields better performance than the open-loop case in that the errors $x_1 - x_1^*, x_2 - x_2^*$ are mildly smaller than in the previous figure for the open-loop case. See Table 4.1. It is well known, however, that the LQG controller, although optimal for the nominal plant model under the assumed noise environment, may lose performance and perhaps its stability even for small variations from the nominal plant model.

Case 3: LQG/LTR design
In order to enhance the robustness properties of LQG controllers, we adopt well known loop transfer recovery (LTR) techniques [12]. Thus the system noise covariance Q_f in a state estimator design, is parametrized by a scaler $q > 0$, and a loop recovery property is achieved as q becomes large. In our scheme the state estimator 'design system and measurement noise covariances', $Q_f(q)$ and R_f, are given by $Q_f(q) = I + q^2 \begin{bmatrix} 1 \\ 0 \end{bmatrix} [1\ 0]$, $\quad R_f = I$ with $q = 50$. There is a more dramatic reduction of errors $x_1 - x_1^*, x_2 - x_2^*$ over that for the LQG design of the previous case as indicated in Table 4.1. Of course, the Case 3 is identical to the Case 2 when $q = 0$. Also, simulations not reported here show that the LQG/LTR design performs virtually identically to an LQ design where states δx are assumed available for feedback.

Case 4: Adaptive Q design
The adaptive Q, two-degree-of-freedom, controller design for optimal control problem is studied, with the LQG or LQG/LTR controller K_0 and the adaptive $Q = [Q_1,\ Q_2]$ using least square techniques. Third-order FIR models are chosen for the forward $Q_1(z)$ and the backward $Q_2(z)$. Simulations, summarized in Table 4.1 show that adaptive-Q controller design strengthens the robustness/performance properties of both the LQG and LQG/LTR design without the need for any high gains in the controller. The intention in this first design example has not been to demonstrate that an adaptive-Q approach works dramatically better than all others, although one example is shown where such is the case. Rather, we have sought to stress that the adaptive-Q method is perhaps best used only after a careful robust fixed controller design, and then only to achieve fine turning. Actually, for the design study here, the robust LQG/LTR design performed better than the LQG adaptive-Q design. The values of ΔI for all five cases are summarized in Table 4.1 for a deterministic disturbance $d = .2$ (disturbance #1), and then two stochastic disturbances, with, in the first instance d uniformly distributed between .1 and .3 (#2), and in the second d uniformly distributed between 0 and 1 (#3).

Table 4.1 - ΔI for Trajectory 1. I.C. = [0 1], for disturbances #1, #2, #3

Open loop	LQG	LQG/LTR	LQG/Ad-Q	LQG/LTR/Ad-Q
3.0712	0.7486	0.2295	0.3478	0.1600
3.0556	0.7435	0.2278	0.3449	0.1592
6.2587	3.2557	1.3483	1.9925	1.0010

To demonstrate the robustness of the adaptive-Q control strategy, the simulations were repeated with unmodelled dynamics in the actual plant. The state equations of the actual plant in this case are $\dot{x}_1 = (1 - x_2^2)x_1 - x_2 + x_3 + u$, $\dot{x}_2 = x_1$, $\ddot{x}_3 = -\dot{x}_3 - 4x_3 + u$, $y = x_1$ with initial state vector [0 1 0].

Table 4.2 - ΔI for Trajectory 1. I.C. = [0 1 0], for disturbance #1 and unmodelled dynamics.

Open loop	LQG	LQG/LTR	LQG/Ad-Q	LQG/LTR/Ad-Q
5.9077	1.9438	0.6623	0.984	0.4251

Similar results are obtained when the simulations are repeated for other initial conditions and hence optimal trajectories.

Remarks

1. In our simulation for the adaptive Q controller, two passes are needed for "warming up" of the controller.

2. The prefilters $P_{12}M_0$ used in our study are as follows.

$$\dot{x}_{ps} = (A + BF)x_{ps} + Bu^*, \quad \xi_1 = \begin{pmatrix} F \\ I \end{pmatrix} x_{ps} + \begin{pmatrix} I \\ 0 \end{pmatrix} u^*$$

with input u^* and output ξ_1. Likewise for the prefilters driven by δr and s.

3. Our simulations not reported here show significant improvements when scaling adjustments are made to r and e. Also, other simulations not reported here show that there is insignificant benefit with increasing the dimensions $p = 3, m = 3, n = 0$ in Q, although the cost of reducing p or m is significant.

5 Conclusions

A method to combine off-line (open-loop) optimal control approaches with robust feedback control and on-line (closed-loop) adaptive control techniques is presented, with emphasis on nonlinear cases. Stability properties for the nonlinear case are discussed. Simulation results show that our proposed method can enhance robustness/performance properties, in the presence of unmodelled dynamics, and deterministic or stochastic disturbances. The method can be generalized to a learning-Q approach where the Q feedback operator is a function of the optimal state trajectory x^*, or of x itself, and this will be discussed in a companion paper [4].

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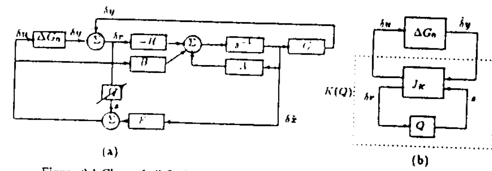

Figure 2.1 Class of all Stabilizing Controllers for ΔG_n.

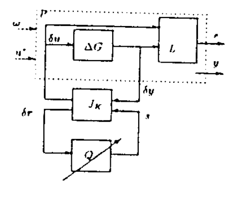

Figure 2.2 - Adaptive Q for Disturbance Response Minimization

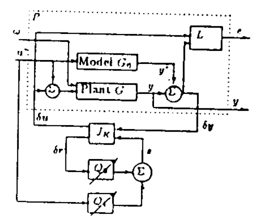

Figure 2.3 - Two degree-of-freedom adaptive-Q scheme

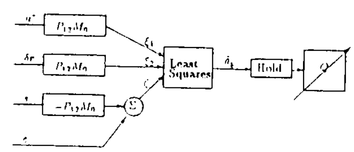

Figure 2.4 The Least Squares Adaptive-Q Arrangement

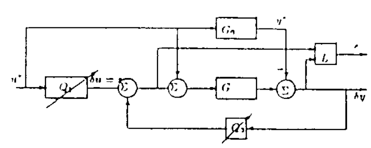

Figure 2.5 Model Reference Adaptive Control Special Case.

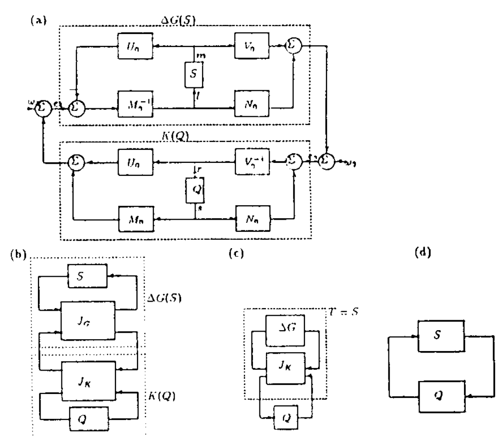

Figure 3.1- The feedback systems $\{\Delta G(S), K(Q)\}$, $\{Q, S\}$.