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Quasi-Optimal Demodulation of Pulse-Frequency Modulated Signals

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***Abstract*—Novel demodulator structures are derived for quasi-optimal on-line demodulation of pulse-frequency modulated (PFM) signals in the presence of white Gaussian channel noise. The basic demodulator consists of a phase-locked loop with its integrators appropriately reset as each new pulse is received. This modulator may be augmented with additional integrators and gain elements to achieve quasi-optimal demodulation with delay.**

The quasi-optimal demodulation approaches optimal demodulation, in the minimum mean-square error sense, as the signal-to-noise ratio increases.

The various quasi-optimal receivers are derived by application of the extended Kalman filter theory to a state-space signal model.

INTRODUCTION

In this concise paper, the problem to be considered is the quasi-optimal demodulation of pulse-frequency modulated (PFM) signals (frequently referred to in the literature as discrete-frequency modulation or compound PAM–FM) in the presence of additive white Gaussian channel disturbances.

Existing PFM demodulators, derived using the classical frequency domain approach, consist of frequency detectors (with or without feedback) or a bank of matched filters with means for scanning the output terminals [1]. These various demodulators are only optimal in the sense that they are the result of attempts to achieve the best improvement in signal-to-noise ratio.

Optimal demodulation of PFM signals has been investigated by Wozencraft and Jacobs [2] and Van Trees [3]. The maximum *a posteriori* (MAP) estimate is derived under the assumption of a white noise uniformly distributed message source. The estimate is obtained from a two-step estimation procedure. First, an approximate estimate is determined on-line using a bank of matched filters. Next, an off-line estimate is obtained by finding the local maximum of the likelihood function.

In the discussion to follow, extended Kalman filter theory is applied to a state-space model of the PFM process to yield an approximate on-line, minimum mean-square error (MMSE) demodulator [4]–[9].

Features of this approach are now listed.

1) Real-time implementation is obtained without the introduction of a time delay.

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be regarded as decoupled. Equation (8) becomes

$$\frac{dP(t \mid t)}{dt} = -P(t \mid t)hR^{-1}h^TP(t \mid t)\frac{C^2d_f^2t^2}{2}, \quad (13)$$

the solution of which is

$$P(t \mid t) = \left[P^{-1}(0 \mid 0) + \frac{hR^{-1}h^TC^2d_f^2t^3}{6}\right]^{-1}. \quad (14)$$

The error covariance may now be obtained recursively as

$$P(t_k \mid t_{k+1}^-) = \left[P^{-1}(t_k \mid t_k^-) + \frac{hR^{-1}h^TC^2d_f^2T^3}{6}\right]^{-1},$$

$$P(t_0 \mid t_0^-) = P_0 \quad (15)$$

$$P(t_{k+1} \mid t_{k+1}^-) = \phi P(t_k \mid t_{k+1}^-)\phi^T + Q. \quad (16)$$

Steady-state performance is given by solving the two algebraic equations

$$\bar{P} = \phi\bar{\bar{P}}\phi^T + Q \quad (17)$$

$$\bar{\bar{P}} = \left[\bar{P}^{-1} + \frac{C^2d_f^2T^3hR^{-1}h^T}{6}\right]^{-1} \quad (18)$$

where we define

$$\bar{\bar{P}} \triangleq \lim_{k\to\infty} P(t_k \mid t_{k+1}^-) \quad (19)$$

and

$$\bar{P} \triangleq \lim_{k\to\infty} P(t_{k+1} \mid t_{k+1}^-). \quad (20)$$

The question of how this type of filter performs in comparison with demodulators such as in [3] or [10] where parallel processing is allowed can really only be answered by Monte Carlo simulation analysis. Most theoretical analysis of nonlinear filters involves approximations of one kind or another. One would certainly expect that the extended Kalman filter would perform well under low-noise conditions. The simulation results in Fig. 1 suggest that above 25 dB the extended Kalman filter competes strongly with alternative and more complex schemes. The curves in Fig. 1 are analogous to curves of probability of error versus CNR often used. Instead of plotting the probability of error, the probability that the demodulated signal-to-noise ratio SNR (dB) will lie below a certain level has been plotted.

SUBOPTIMAL DEMODULATION

Equations (5) and (8) could be implemented on-line without difficulty. However, to reduce the demodulator complexity, the time-varying gain given by (6) could be replaced by a suboptimal gain defined by

$$K_s(t) = P_sM_sR^{-1} \quad (21)$$

where P_s is some arbitrary constant matrix and

$$M_s = hCd_f \cos\{[w_c + d_fh^T\hat{x}(t \mid t)]t\}.$$

The equation for the suboptimal demodulator then becomes

$$\frac{d\hat{x}_s(t \mid t)}{dt} = K_s(t)[z(t) - C\sin\{[w_c + d_fh^T\hat{x}_s(t \mid t)]t\}]. \quad (22)$$

APPROXIMATE PERFORMANCE ANALYSIS OF SUBOPTIMAL DEMODULATOR

Define the suboptimal estimation error signal and its corresponding covariance by

$$e_s(t) = x(t) - \hat{x}_s(t \mid t) \quad (23)$$

$$P_s(t \mid t) = E[e_s(t)e_s^T(t)]. \quad (24)$$

Now

$$\dot{e}_s(t) = -\dot{\hat{x}}_s(t \mid t) = -K_s(t)[C\sin\{[w_c + d_fh^Tx(t)]t\} + v(t) - C\sin\{[w_c + d_fh^T\hat{x}_s(t \mid t)]t\}].$$

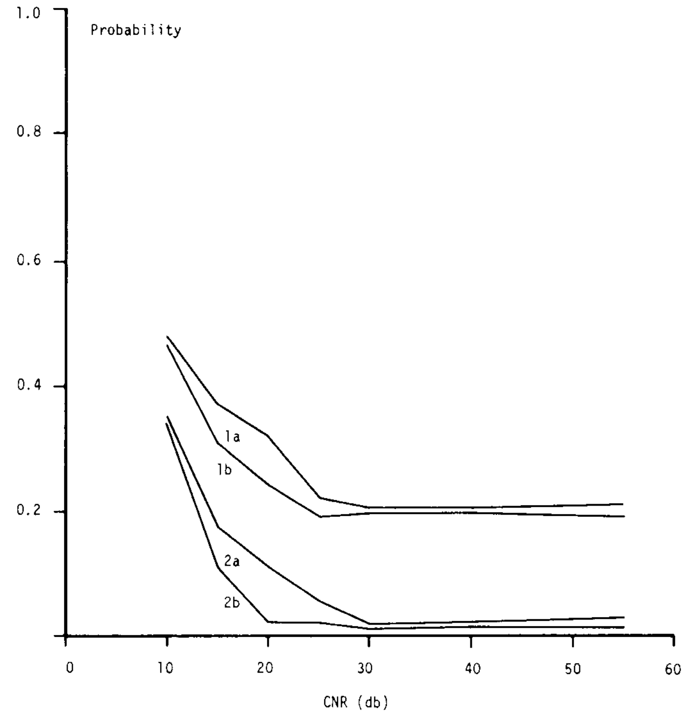

Fig. 1. Simulation results, frequency deviation $D = 3$. Curve 1. Probability of less than 5 dB improvement in SNR: a) single extended Kalman filter; b) using parallel processing. Curve 2. Probability of no improvement in SNR: a) single extended Kalman filter; b) using parallel processing.

By retaining the first two terms in a Taylor series expansion of $C\sin\{[w_c + d_fh^Tx(t)]t\}$ about $\hat{x}_s(t \mid t)$, we obtain

$$\dot{e}_s(t) = -K_s(t)M^Te_s(t) - K_s(t)v(t). \quad (25)$$

By assuming that $w_c \gg W$, we obtain the following approximate differential equation for $P_s(t \mid t)$:

$$\frac{dP_s(t \mid t)}{dt} = F(t)P_s(t \mid t) + P_s(t \mid t)F^T(t) + G(t)RG^T(t) \quad (26)$$

where

$$F(t) = -P_shR^{-1}h^T\frac{Cd_ft}{2}$$

and

$$G(t) = \frac{-Cd_f}{\sqrt{2}}P_shR^{-1}.$$

The initial condition for (8) also applies to (26). The solution of (26) is

$$P_s(t \mid t) = \Lambda(t,t_0)P_s(t_0 \mid t_0)\Lambda^T(t,t_0) + \int_{t_0}^{t}\Lambda(t,\tau)G(\tau)RG^T(\tau)\Lambda^T(t,\tau)\,d\tau$$

where $\Lambda(t,t_0)$ is the transition matrix of $F(t)$ satisfying $\dot{\Lambda}(t,t_0) = F(t)\Lambda(t,t_0)$, $\Lambda(t_0,t_0) = I$.

The problem is now to determine the choice of P_s which minimizes the steady-state suboptimal estimation error

$$\lim_{k\to\infty} \mathrm{tr}\,[P_s(t_k \mid t_{k+1}^-)].$$

The above analysis provides a rough guide only to the value of the minimizing P_s. The only reliable method is to use Monte Carlo simulation tests.

Computer simulation studies have shown that if the correct value of P_s is chosen, the same performance can be achieved as that obtained using the gain given in (6)–(8). In practical applications, this would represent a considerable saving. Figs. 2 and 3 show how the performance varies as P_s varies for two values of CNR. Note that the minimum is sharper for high CNR. In other words, under low-noise conditions, the demodulator performance is very sensitive to variations in P_s from the optimum.

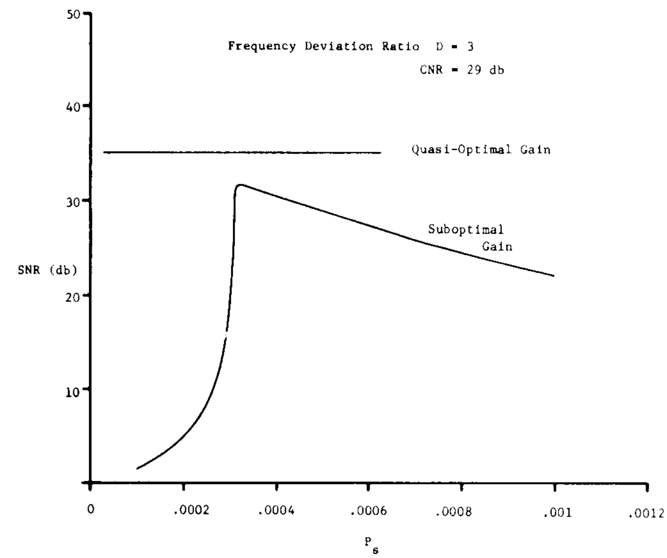

Fig. 2. Comparison of demodulator performance using different gain functions.

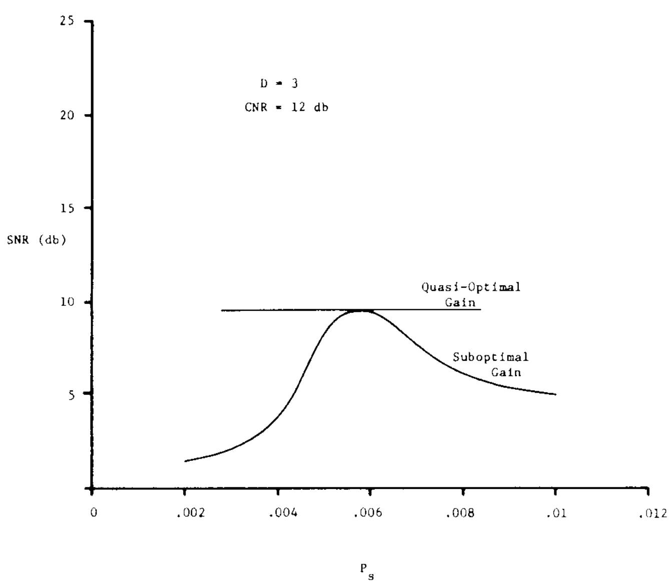

Fig. 3. Comparison of demodulator performance using different gain functions.

EXTENSIONS

A. Quasi-Optimal PFM Demodulation with Delay

As in the case of PAM demodulation with fixed delay [9], the performance of PFM demodulation can be improved by allowing a fixed amount of delay between the sending of the message and its demodulation. The derivations and performance analysis are given in [13].

B. Memory in the Channel

Most practical communications channels possess inherent dynamics. Often the channel dynamics can be modeled as a linear, finite dimensional system. In this case, extended Kalman filter theory may again be applied to achieve quasi-optimal PFM demodulation. Further discussion of this topic is contained in [10].

C. Pre-Emphasis Filters

Pre-emphasis filters could be designed for the discrete-time message source of the PFM system. System performance may be optimized using iterative techniques to search the parameter space of the pre-emphasis filter, subject to power and bandwidth constraints. Discussion of this technique using state-space analysis is given in [13]. Note that the existence of correlation in the message sequence gives rise to the possibility of improvement in performance using pre-emphasis.

D. Other Applications

The results developed here could be readily extended to other similar pulse communication systems, such as companded PAM systems and pulse-phase modulation systems.

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