From Display Calculi to Deep Nested Sequent Calculi: Formalised for Full Intuitionistic Linear Logic

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Overview

What is FILL?
Existing sequent calculi
A Display Calculus for FILL
Nested Sequent Calculus for FILL
Separation
Further Work

Categorial Semantics for FILL

$(\otimes, \mathbf{1}, \multimap)$ is a symmetric monoidal closed structure

$A \otimes B \multimap C$ iff $A \multimap (B \multimap C)$ iff $B \multimap (A \multimap C)$

$(A \otimes \mathbf{1}) \multimap A$ and $A \multimap (A \otimes \mathbf{1})$

$(\parr, \mathbf{0})$ is a symmetric monoidal structure

$(A \parr B) \multimap (B \parr A)$

$(A \parr \mathbf{0}) \multimap A$ and $A \multimap (A \parr \mathbf{0})$

interaction via either of

weak distributivity	$(A \otimes (B \parr C)) \multimap ((A \otimes B) \parr C)$
Grishin(b)	$((A \multimap B) \parr C) \multimap (A \multimap (B \parr C))$

Collapse to (classical) MLL: if we add converse of Grishin(b)

Grishin(a) $(A \multimap (B \parr C)) \multimap ((A \multimap B) \parr C)$

Proof Theory of FILL: problem and solutions

Remember: we need comma on the right to accommodate $\parr$

Problem and existing solutions:

multiple conclusions	single conclusion	existing solutions
$\dfrac{\Gamma, A \vdash B, \Delta}{\Gamma \vdash A \multimap B, \Delta}$	$\dfrac{\Gamma, A \vdash B}{\Gamma \vdash A \multimap B, \Delta}$	$\dfrac{\Gamma, A \vdash B, \Delta}{\Gamma \vdash A \multimap B, \Delta}\,(\dagger)$
unsound	no cut-elimination	cut-elimination

$\dagger$: side-conditions which ensure that A is "independent" of Δ

Hyland, de Paiva 1993: type assignments to ensure that the variable typed by A not appear free in the terms typed by Δ

Bierman 1996: $(a \parr b) \parr c \vdash a, ((b \parr c) \multimap d) \parr (e \multimap (d \parr e))$ has no cut-free derivation in the Hyland and de Paiva calculus

Display calculus for (an extension of) FILL

Structural Constant and Binary Connectives: $\Phi \quad , \quad < \quad >$

Antecedent Structure: $X_a \quad Y_a ::= A \mid \Phi \mid X_a, Y_a \mid X_a < Y_s$

Succcedent Structure: $X_s \quad Y_s ::= A \mid \Phi \mid X_s, Y_s \mid X_a > Y_s$

Sequent: $X_a \vdash Y_s$ (drop subscripts to avoid clutter)

Display Postulates: reversible structural rules

$$\frac{X_a \vdash Y_a > Z_s}{\dfrac{X_a, Y_a \vdash Z_s}{Y_a \vdash X_a > Z_s}} \qquad\qquad \frac{Z_a < Y_s \vdash X_s}{\dfrac{Z_a \vdash X_s, Y_s}{Z_a < X_s \vdash Y_s}}$$

Display Property: For every antecedent (succedent) part Z of the sequent $X \vdash Y$, there is a sequent $Z \vdash Y'$ (resp. $X' \vdash Z$) obtainable from $X \vdash Y$ using only the display postulates, thereby displaying the Z as the whole of one side

Logical rules: introduced formula is always displayed

(id) $p \vdash p$

(cut) $\dfrac{X \vdash A \qquad A \vdash Y}{X \vdash Y}$

$(\mathbf{1} \vdash)$ $\dfrac{\Phi \vdash X}{\mathbf{1} \vdash X}$

$(\vdash \mathbf{1})$ $\Phi \vdash \mathbf{1}$

$(\mathbf{0} \vdash)$ $\mathbf{0} \vdash \Phi$

$(\vdash \mathbf{0})$ $\dfrac{X \vdash \Phi}{X \vdash \mathbf{0}}$

$(\otimes \vdash)$ $\dfrac{A, B \vdash X}{A \otimes B \vdash X}$

$(\vdash \otimes)$ $\dfrac{X \vdash A \qquad Y \vdash B}{X, Y \vdash A \otimes B}$

$(\parr \vdash)$ $\dfrac{A \vdash X \qquad B \vdash Y}{A \parr B \vdash X, Y}$

$(\vdash \parr)$ $\dfrac{X \vdash A, B}{X \vdash A \parr B}$

$(\multimap \vdash)$ $\dfrac{X \vdash A \qquad B \vdash Y}{A \multimap B \vdash X > Y}$

$(\vdash \multimap)$ $\dfrac{X \vdash A > B}{X \vdash A \multimap B}$

$(-\!\!< \vdash)$ $\dfrac{A < B \vdash X}{A -\!\!< B \vdash X}$

$(\vdash -\!\!<)$ $\dfrac{X \vdash A \qquad B \vdash Y}{X < Y \vdash A -\!\!< B}$

read upwards, one rule is a "rewrite" while other "constrains"

Structural rules: no occurrences of formula meta-variables

all sub-structural properties captured in a modular way

$(\Phi \vdash)$ $\dfrac{X, \Phi \vdash Y}{X \vdash Y}$

$(\vdash \Phi)$ $\dfrac{X \vdash \Phi, Y}{X \vdash Y}$

$(\mathrm{Ass} \vdash)$ $\dfrac{W, (X, Y) \vdash Z}{(W, X), Y \vdash Z}$

$(\vdash \mathrm{Ass})$ $\dfrac{W \vdash (X, Y), Z}{W \vdash X, (Y, Z)}$

$(\mathrm{Com} \vdash)$ $\dfrac{X, Y \vdash Z}{Y, X \vdash Z}$

$(\vdash \mathrm{Com})$ $\dfrac{Z \vdash Y, X}{Z \vdash X, Y}$

$(\mathrm{Grnb} \vdash)$ $\dfrac{W, (X < Y) \vdash Z}{(W, X) < Y \vdash Z}$

$(\vdash \mathrm{Grnb})$ $\dfrac{W \vdash (X > Y), Z}{W \vdash X > (Y, Z)}$

$$((A \multimap B) \parr C) \multimap (A \multimap (B \parr C))$$

Categorial semantics for bi-intuitionistic linear logic BiILL

$(\otimes, \mathbf{1}, \multimap)$ is a symmetric monoidal closed structure

$A \otimes B \multimap C$ iff $A \multimap (B \multimap C)$ iff $B \multimap (A \multimap C)$

$(A \otimes \mathbf{1}) \multimap A$ and $A \multimap (A \otimes \mathbf{1})$

$(-\!\!<, \parr, \mathbf{0})$ is a symmetric monoidal co-closed structure

$A \multimap (B \parr C)$ iff $(A -\!\!< B) \multimap C$ iff $(A -\!\!< C) \multimap B$

$(A \parr \mathbf{0}) \multimap A$ and $A \multimap (A \parr \mathbf{0})$

interaction via either of

Grishin(b) $((A \multimap B) \parr C) \multimap (A \multimap (B \parr C))$

dualGrishin(b) $((A \otimes B) -\!\!< C) \multimap (A \otimes (B -\!\!< C))$

Collapse to (classical) MLL: if we add converse of either