Belnap's Eight Conditions a lá Kracht

(C1) Each formula variable occurring in some premise of a rule ρ is a subformula of some formula in the conclusion of ρ.

(C2) *Congruent parameters* is a relation between parameters of the identical structure variable occurring in the premise and conclusion

(C3) Each parameter is congruent to at most one structure variable in the conclusion. Equivalently, no two structure variables in the conclusion are congruent to each other.

(C4) Congruent parameters are either all antecedent or all succedent parts of their respective sequent.

(C5) A formula in the conclusion of a rule ρ is either the entire antecedent or the entire succedent. Such a formula is called a **principal formula** of ρ.

(C6/7) Each rule is closed under simultaneous substitution of arbitrary structures for congruent parameters.

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(C8) If there are rules ρ and σ with respective conclusions $X \vdash A$ and $A \vdash Y$ with formula A principal in both inferences (in the sense of C5) and if *cut* is applied to yield $X \vdash Y$, then either $X \vdash Y$ is identical to either $X \vdash A$ or $A \vdash Y$; or it is possible to pass from the premises of ρ and σ to $X \vdash Y$ by means of inferences falling under *cut* where the cut-formula always is a proper subformula of A.

$$\dfrac{\dfrac{X \vdash C > D}{X \vdash C \multimap D} \quad \dfrac{U \vdash C \qquad D \vdash Z}{C \multimap D \vdash U > Z}}{X \vdash U > Z}\ \text{cut}$$

$$\dfrac{U \vdash C \qquad \dfrac{\dfrac{\dfrac{X \vdash C > D}{X, C \vdash D} \qquad D \vdash Z}{X, C \vdash Z}}{C \vdash X > Z}\ \text{cut}}{\dfrac{\dfrac{U \vdash X > Z}{X, U \vdash Z}}{X \vdash U > Z}}\ \text{cut}$$