Cut-free derivation in the deep nested calculus

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{a \Rightarrow a, (\cdot \Rightarrow \cdot)} \quad \dfrac{\dfrac{}{\cdot \Rightarrow (b \Rightarrow b)}}{b \Rightarrow (\cdot \Rightarrow b)} \quad \dfrac{\dfrac{}{\cdot \Rightarrow (c \Rightarrow c)}}{c \Rightarrow (\cdot \Rightarrow c)}}{a \parr b \Rightarrow a, (\cdot \Rightarrow b) \quad c \Rightarrow (\cdot \Rightarrow c)}}{(a \parr b) \parr c \Rightarrow a, (\cdot \Rightarrow b, c)}}{(a \parr b) \parr c \Rightarrow a, (\cdot \Rightarrow b \parr c) \quad \dfrac{}{\cdot \Rightarrow (d \Rightarrow d)}}}{(a \parr b) \parr c \Rightarrow a, (b \parr c \multimap d \Rightarrow d) \quad \dfrac{}{\cdot \Rightarrow (e \Rightarrow e)}}}{(a \parr b) \parr c \Rightarrow a, ((b \parr c \multimap d) \parr e \Rightarrow d, e)}}{(a \parr b) \parr c \Rightarrow a, ((b \parr c \multimap d) \parr e \Rightarrow d \parr e)}}{(a \parr b) \parr c \Rightarrow a, (b \parr c \multimap d) \parr e \multimap d \parr e}$$

No annotations, only commas as structural connective, but sequents are nested $(\cdots \Rightarrow \cdots) \cdots \Rightarrow \cdots (\cdots \Rightarrow \cdots)$

Example derivation in our display calculus

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{a \vdash a \quad b \vdash b}{a \parr b \vdash a, b} \quad c \vdash c}{(a \parr b) \parr c \vdash (a, b), c}\ (\parr\vdash)}{(a \parr b) \parr c \vdash a, (b, c)}\ \textbf{(ass)}}{(a \parr b) \parr c < a \vdash b, c}\ \textbf{(drp)}}{(a \parr b) \parr c < a \vdash b \parr c}\ (\vdash \parr) \quad d \vdash d}{b \parr c \multimap d \vdash ((a \parr b) \parr c < a) > d}\ (\multimap\vdash) \quad e \vdash e}{(b \parr c \multimap d) \parr e \vdash (((a \parr b) \parr c < a) > d), e}\ (\parr\vdash)}{(b \parr c \multimap d) \parr e \vdash ((a \parr b) \parr c < a) > (d, e)}\ (\vdash \text{Grnb})}{(b \parr c \multimap d) \parr e, ((a \parr b) \parr c < a) \vdash d, e}\ \text{(rp)}}{(b \parr c \multimap d) \parr e, ((a \parr b) \parr c < a) \vdash d \parr e}\ (\vdash \parr)}{(a \parr b) \parr c < a \vdash (b \parr c \multimap d) \parr e > (d \parr e)}\ \text{(rp)}}{(a \parr b) \parr c < a \vdash (b \parr c \multimap d) \parr e \multimap (d \parr e)}\ (\vdash\multimap)}{(a \parr b) \parr c \vdash a, (b \parr c \multimap d) \parr e \multimap (d \parr e)}\ \textbf{(drp)}$$

But we implicitly created an occurrence of $\prec$ via $<$

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No annotations, only commas as structural connective, but sequents are nested

From BiILL back to FILL

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{a \vdash a \quad b \vdash b}{a \parr b \vdash a, b} \quad c \vdash c}{(a \parr b) \parr c \vdash (a, b), c}}{(a \parr b) \parr c \vdash a, (b, c)}\ \textbf{(?)}}{(a \parr b) \parr c < a \vdash b, c}}{(a \parr b) \parr c < a \vdash b \parr c}\ (\vdash \parr) \quad d \vdash d}{b \parr c \multimap d \vdash ((a \parr b) \parr c < a) > d}\ (\multimap\vdash) \quad e \vdash e}{(b \parr c \multimap d) \parr e \vdash (((a \parr b) \parr c < a) > d), e}\ (\parr\vdash)}{(b \parr c \multimap d) \parr e \vdash ((a \parr b) \parr c < a) > (d, e)}\ (?)}{(b \parr c \multimap d) \parr e, ((a \parr b) \parr c < a) \vdash d, e}\ (?)}{(b \parr c \multimap d) \parr e, ((a \parr b) \parr c < a) \vdash d \parr e}\ (\vdash \parr)}{(a \parr b) \parr c < a \vdash (b \parr c \multimap d) \parr e > (d \parr e)}\ (?)}{(a \parr b) \parr c < a \vdash (b \parr c \multimap d) \parr e \multimap (d \parr e)}\ (\vdash\multimap)}{(a \parr b) \parr c \vdash a, (b \parr c \multimap d) \parr e \multimap (d \parr e)}\ \textbf{(?)}$$