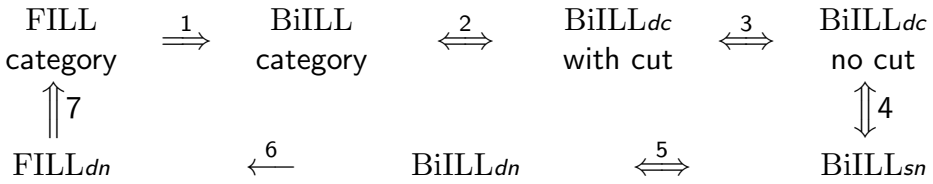
From BiILL back to FILL



Nested FILL-sequent: nested sequent that has no nesting of sequents on the left of $\Rightarrow$ and no occurrences of $-\!\!<$

Why? entire BiILL$_{dn}$-derivation of a nested FILL-sequent contains only nested FILL-sequents (look at the rules!)

FILL$_{dn}$: remove $-\!\!<^d_l$, $-\!\!<^d_r$, pl_2 and pr_1 from BiILL$_{dn}$

Separation Thm: nested FILL-sequents are derivable in FILL$_{dn}$ iff they are derivable in BiILL$_{dn}$.

Thm: every rule of FILL$_{dn}$ preserves FILL-validity downwards

Cor: FILL$_{dn}$ is sound and complete for FILL-validity

Cor: BiILL is a conservative extension of FILL

Formalisation

use of Isabelle: work verified in Isabelle theorem prover

value of formal verification: an earlier proof was found to be flawed (after some months' work)

time taken: formal proof took about 1/2 year

most difficult: showing that shallow nested rules admissible in deep nested calculus — *many* cases, since (eg) $X[\mathcal{S} \Rightarrow \mathcal{T}]$ ($\mathcal{S}$ and $\mathcal{T}$ multisets!) can match given sequent Z in *many* ways

programmed tactics: many programming of tactics and combinations of them — SML progamming interface invaluable

Formalisation: multisets in nested sequents

Display Calculus structure in Isabelle: involves (sub-)structures (recursively), with binary operators, and formulae

nested sequents in Isabelle ??: would involve *multisets* of nested sequents

Isabelle couldn't do this: (lists — yes, multisets — no) so we just used a ',' operator, and defined an equivalence relation (so, eg $A \Rightarrow (B, B' \Rightarrow C) \equiv A \Rightarrow (B', B \Rightarrow C)$)

consequential change: definition of merge, $X_1[\,] \bullet X_2[\,]$, becomes much simpler

many lemmas: we needed many lemmas about using this $\equiv$: how much easier if we could use multisets directly ??

Isabelle developments: possibility to use multisets recently introduced into Isabelle

this work is in Isabelle 2005: too much incompatible change in Isabelle developments for me to change all my proofs

Cut-free derivation in our display calculus

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{a \vdash a \qquad b \vdash b}{a \parr b \vdash a, b} \qquad c \vdash c}{(a \parr b) \parr c \vdash a, b, c}}{(a \parr b) \parr c < a \vdash b, c}}{(a \parr b) \parr c < a \vdash b \parr c} \qquad d \vdash d}{b \parr c \multimap d \vdash ((a \parr b) \parr c < a) > d} \qquad e \vdash e}{(b \parr c \multimap d) \parr e \vdash (((a \parr b) \parr c < a) > d), e}}{(b \parr c \multimap d) \parr e \vdash ((a \parr b) \parr c < a) > d, e}}{(b \parr c \multimap d) \parr e, ((a \parr b) \parr c < a) \vdash d, e}}{(b \parr c \multimap d) \parr e, ((a \parr b) \parr c < a) \vdash d \parr e}}{(a \parr b) \parr c < a \vdash (b \parr c \multimap d) \parr e > d \parr e}}{(a \parr b) \parr c < a \vdash (b \parr c \multimap d) \parr e \multimap d \parr e}}{(a \parr b) \parr c \vdash a, (b \parr c \multimap d) \parr e \multimap d \parr e}$$

No annotations, but many extra structural connectives