From Display Calculi to Deep Nested Sequent Calculi: Formalised for Full Intuitionistic Linear Logic

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Overview

What is FILL?
Existing sequent calculi
A Display Calculus for FILL
Nested Sequent Calculus for FILL
Separation
Further Work

Categorial Semantics for FILL

$(\otimes, \mathbf{1}, \multimap)$ is a symmetric monoidal closed structure
$A \otimes B \multimap C$ iff $A \multimap (B \multimap C)$ iff $B \multimap (A \multimap C)$
$(A \otimes \mathbf{1}) \multimap A$ and $A \multimap (A \otimes \mathbf{1})$

$(\parr, \mathbf{0})$ is a symmetric monoidal structure
$(A \parr B) \multimap (B \parr A)$
$(A \parr \mathbf{0}) \multimap A$ and $A \multimap (A \parr \mathbf{0})$

interaction via either of
weak distributivity $(A \otimes (B \parr C)) \multimap ((A \otimes B) \parr C)$
Grishin(b) $((A \multimap B) \parr C) \multimap (A \multimap (B \parr C))$

Collapse to (classical) MLL: if we add converse of Grishin(b)
Grishin(a) $(A \multimap (B \parr C)) \multimap ((A \multimap B) \parr C)$

Proof Theory of FILL: problem and solutions

Remember: we need comma on the right to accommodate $\parr$

Problem and existing solutions:

multiple conclusions	single conclusion	existing solutions
$\dfrac{\Gamma, A \vdash B, \Delta}{\Gamma \vdash A \multimap B, \Delta}$	$\dfrac{\Gamma, A \vdash B}{\Gamma \vdash A \multimap B, \Delta}$	$\dfrac{\Gamma, A \vdash B, \Delta}{\Gamma \vdash A \multimap B, \Delta}$ (†)
unsound	no cut-elimination	cut-elimination

†: side-conditions which ensure that A is "independent" of Δ

Hyland, de Paiva 1993: type assignments to ensure that the variable typed by A not appear free in the terms typed by Δ

Bierman 1996: $(a \parr b) \parr c \vdash a, ((b \parr c) \multimap d) \parr (e \multimap (d \parr e))$ has no cut-free derivation in the Hyland and de Paiva calculus