Supplementary Material – *Multi-View 3D Reconstruction from Uncalibrated Radially-Symmetric Cameras*

Appendix

In this supplementary material, we present implementation details of our alternating direction continuation method for complete measurements, as well as incomplete measurements. The supplementary video showing 3D reconstruction results is also included as the supplementary material.

1. Efficient implementation through ALM

In the paper, we present an Augmented Lagrangian Multiplier (ALM) based efficient implementation (similar implementations have been applied to various problems, such as recovery of corrupted low-rank matrices [2], unsupervised visual learning [3] and projective factorization [1]). Algorithm 1 illustrates the efficient implementation under complete measurements case.

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Initialization: Given Φ^(0), set W^(0) = Φ^(0) ⊙ M,
  compute U^(0), V^(0) through low rank projection, select
  ε, μ̄ and η_μ, fix the sequence of μ_l as
  μ_{l+1} = max{μ_l η_μ, μ̄}, l = 1, ··· , L − 1.
for μ = μ_1, μ_2, ··· , μ_L do
  while Not converged do
    Update U as U^(k+1) = (Φ ⊙ M)V(V^T V + μI)^{-1};
    Update V as V^(k+1) = (Φ ⊙ M)^T U(U^T U + μI)^{-1};
    Update W as W^(k+1) = U^(k+1) V^(k+1)^T;
    Update the projective depth element-wisely as
      φ_ij^(k+1) = (w_ij^T m_ij)/(m_ij^T m_ij);
    Normalize the projective depth matrix Φ^(k+1) to
      satisfy the column-sum and row-sum constraints
      Φ1_n = n1_m and Φ^T 1_m − m1_n.
    Compute stopping criteria
      ε^(k+1) = ‖W^(k+1) − W^(k)‖_F / max{1, ‖W^k‖_F}.
    if ε^(k+1) < ε then
      Inner iteration converged;
    end if
  end while
end for
```

Algorithm 1: Radial camera factorization via alternating direction continuation.

2. Missing data handling with MALM

Under incomplete measurements case, through the introduction of mask matrix Ω, the imaging process for radially-symmetric cameras with missing data can be compactly expressed $\Phi \odot \mathtt{M} = \Omega \odot \mathtt{W}$. In this section, we extend the implementation for complete measurements case to handling missing data. Under incomplete measurements case, Hadamard factorization formulation for radially-symmetric cameras becomes:

$$\begin{aligned}
&\min_{\mathtt{U},\mathtt{V},\Phi} \frac{1}{2}\|\Omega \odot (\mathtt{U}\mathtt{V}^T) - \Phi \odot \mathtt{M}\|_F^2 + \frac{\mu}{2}(\|\mathtt{U}\|_F^2 + \|\mathtt{V}\|_F^2) \\
&\text{s.t. } \Phi^T \mathtt{1}_m = m\mathtt{1}_n, \\
&\Phi \mathtt{1}_n = n\mathtt{1}_m, \\
&\phi_{ij} > 0,
\end{aligned}$$

The corresponding Augmented Lagrangian Multiplier formulation is stated as:

$$\begin{aligned}
\mathcal{L}(\mathtt{U},\mathtt{V},\Phi,\Gamma,\Upsilon) = &\frac{1}{2}\|\Omega \odot (\mathtt{U}\mathtt{V}^T) - \Phi \odot \mathtt{M})\|_F^2 \\
&+ \frac{\mu}{2}(\|\mathtt{U}\|_F^2 + \|\mathtt{V}\|_F^2) \\
&+ \langle \Gamma, \Phi^T \mathtt{1}_m - m\mathtt{1}_n \rangle + \langle \Upsilon, \Phi \mathtt{1}_n - n\mathtt{1}_m \rangle \\
&+ \frac{\beta}{2}(\|\Phi^T \mathtt{1}_m - m\mathtt{1}_n\|_F^2 + \|\Phi \mathtt{1}_n - n\mathtt{1}_m\|_F^2),
\end{aligned} \tag{1}$$

Obviously, cost function Eq.-(1) is not jointly convex over $\mathtt{U}, \mathtt{V}, \Phi$, therefore we propose to minimize it with respect to $\mathtt{U}, \mathtt{V}$ and Φ one at a time while fixing the others. The partial derivation of $\mathcal{L}(\Phi, \mathtt{U}, \mathtt{V})$ with respect to U_{ij} is computed as:

$$\frac{\partial \mathcal{L}(\Phi,\mathtt{U},\mathtt{V})}{\partial U_{ij}} = \sum_{n=1}^{N} (\Omega_{in} \sum_{k=1}^{4} U_{ik} V_{nk} - \phi_{in} M_{in}) \Omega_{in} V_{nj} + \mu U_{ij} \tag{2}$$

We can obtain a linear equation on the variable U_{ij} by setting $\frac{\partial \mathcal{L}(\Phi,\mathtt{U},\mathtt{V})}{\partial U_{ij}} = 0$,

$$\sum_{n=1}^{N} \sum_{k=1}^{4} \Omega_{in}^2 V_{nj} V_{nk} U_{ik} + \mu U_{ij} = \sum_{n=1}^{N} \phi_{in} \Omega_{in} M_{in} V_{nj}. \tag{3}$$

Similarly, we can obtain a linear equation of V_{ji} as:

$$\sum_{m=1}^{M}\sum_{k=1}^{4}\Omega_{mj}^{2}U_{mj}U_{mk}V_{jk}+\mu V_{ji}=\sum_{m=1}^{M}\phi_{mj}\Omega_{mj}M_{mj}U_{mi}. \tag{4}$$

From these linear equations, we can update U_{ij} and V_{ij} in sequel. ϕ_{ij} can be updated in a similar way as under complete measurements case except that we have to normalize the projective depth matrix with missing data. Complete algorithm is presented in Algorithm 2.

Initialization: Given $\Phi^{(0)}$, set $\mathtt{W}^{(0)} = \Phi^{(0)} \odot \mathtt{M}$, compute $\mathtt{U}^{(0)}, \mathtt{V}^{(0)}$ through low rank projection, select ϵ, $\bar{\mu}$ and η_μ, fix the sequence of μ_l as $\mu_{l+1} = \max\{\mu_l \eta_\mu, \bar{\mu}\}, l = 1, 2, \ldots, L-1$.
for $\mu = \mu_1, \mu_2, \cdots, \mu_L$ **do**
  while Not converged **do**
    Update $\mathtt{U}^{(k+1)}$ by solving Eq.-(3);
    Update $\mathtt{V}^{(k+1)}$ by solving Eq.-(4);
    Update $\mathtt{W}$ as $\mathtt{W}^{(k+1)} = \mathtt{U}^{(k+1)}{\mathtt{V}^{(k+1)}}^T$;
    On the visible position, update ϕ_{ij} as $\phi_{ij}^{(k+1)} = (\mathbf{w}_{ij}^T \mathbf{m}_{ij})/(\mathbf{m}_{ij}^T \mathbf{m}_{ij})$;
    Normalize $\Phi^{(k+1)}$ to satisfy the column-sum and row-sum constraints.
    Compute stopping criteria $\varepsilon^{(k+1)} = \frac{\|\mathtt{W}^{(k+1)} - \mathtt{W}^{(k)}\|_F}{\max\{1, \|\mathtt{W}^k\|_F\}}$.
    if $\varepsilon^{(k+1)} < \epsilon$ **then**
      Inner iteration converged;
    end if
  end while
end for

Algorithm 2: Radial camera factorization via alternating direction continuation under incomplete measurements case.

References

[1] Y. Dai, H. Li, and M. He. Projective multiview structure and motion from element-wise factorization. *PAMI*, 35(9):2238–2251, 2013.

[2] Z. Lin, M. Chen, and Y. Ma. The Augmented Lagrange Multiplier Method for Exact Recovery of Corrupted Low-Rank Matrices. *ArXiv e-prints*, Sept. 2010.

[3] R. Liu, Z. Lin, F. De la Torre, and Z. Su. Fixed-rank representation for unsupervised visual learning. In *CVPR*, pages 598–605, 2012.