Camera calibration and the search for infinity

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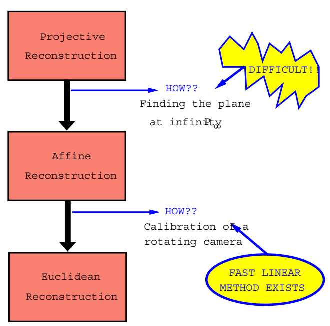
The objective

To self-calibrate a zooming camera undergoing general motion using a stratified approach.

- From an initial projective reconstruction of the scene points obtain an affine reconstruction and finally upgrade it to euclidean.



The problem

- Main difficulty: affine reconstruction, i.e. locating the plane at infinity (Π_∞) in the projective frame (an inherently non-linear problem).
- Once we have located Π_∞ the remainder of the problem is equivalent to the self-calibration of a rotating camera with varying intrinsics.
- Recently a couple of methods to achieve this goal have emerged, including a fast linear algorithm (de Agapito *et al*, CVPR '99).

Our solution: to search for Π_∞

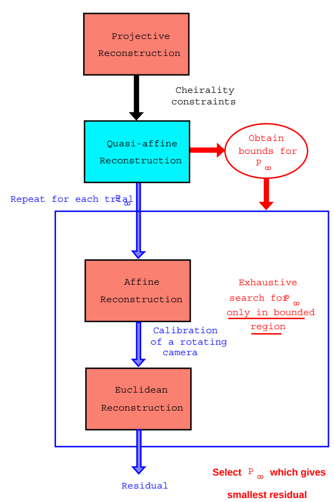
Perform an exhaustive search for the best plane at infinity:

1. Hypothesize a candidate location for the plane at infinity.
2. Obtain the corresponding affine reconstruction.
3. Upgrade to euclidean using the method of calibration of a rotating camera.
4. Use the residual from this method as the cost function to select the best Π_∞.

Key point: use *cheirality* to narrow the search for the plane at infinity

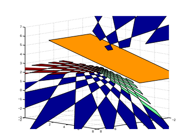
- First use cheirality to obtain an intermediate quasi-affine reconstruction (which preserves the convex hull of the set of points and camera centres).
  - By translating the scene to the origin, since Π_∞ cannot pass through the origin, the fourth coordinate of Π_∞ may be set to unity.
- Cheirality is also used to set bounds on the remaining three coordinates of Π_∞.
- Thus we may restrict the exhaustive search for Π_∞ to this bounded region.

The algorithm

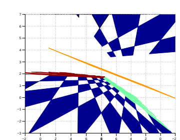


What is a quasi-affine reconstruction?

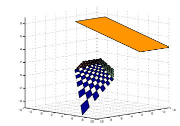
- A projective reconstruction in which Π_∞ does not split the reconstructed scene.



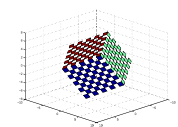
Projective reconstruction where Π_∞ intersects the scene



Projective reconstruction: side view of Π_∞



Quasi-affine reconstruction



Euclidean reconstruction

- How is one obtained? By ensuring that all points lie in front of all cameras in which they are visible by solving the *cheiral inequalities* (Hartley IJCV '98).

The cheiral inequalities

- We assume a projective reconstruction $\{\mathtt{P}_j, \mathbf{X}_i\}$ with camera centres $\mathbf{C}^{\mathtt{P}_j}$.
- We seek a non-singular transformation, $\mathtt{G}$ with fourth row $\mathbf{V}^\top$ such that each point has positive depth with respect to each camera. The necessary conditions are:

$$\begin{aligned} \mathbf{X}_i^\top \mathbf{V} &> 0 \quad \text{for all points } \mathbf{X}_i \\ \epsilon \mathbf{C}^{\mathtt{P}_j\top} \mathbf{V} &> 0 \quad \text{for all cameras } \mathtt{P}_j \end{aligned}$$

- The *signs* of $\mathbf{X}$ and $\mathbf{C}^{\mathtt{P}_j}$ matter (see the paper for details).
- The inequalities may be solved by linear programming.
- There may be two oppositely orientated solutions for $\epsilon = \pm 1$. $\mathtt{G}$ is chosen such that $\mathrm{sign}(\det \mathtt{G}) = \epsilon$.
- If $\mathbf{V}$ is $\mathbf{\Pi}_\infty$ we obtain an affine reconstruction. In this case the infinite homographies $\mathtt{H}_j$ are obtained as the left 3×3 blocks of $\mathtt{P}_j\mathtt{G}^{-1}$.

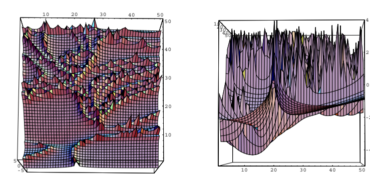
Locating the plane at infinity

- The cheiral inequalities place upper and lower bounds on the first 3 coordinates of Π_∞.
- We perform an exhaustive search in this rectangular region in parameter space with the following steps for each trial.

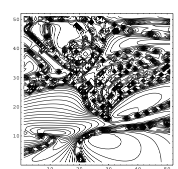
Locating the plane at infinity

1. Cheirality test : The cheiral inequalities must be satisfied.
2. If they are, obtain the infinite homographies, $\mathtt{H}_i$, for this trial.
3. Find camera intrinsics from an algorithm for self-calibration of a non-translating camera
4. IAC test : If the images of the absolute conic are not positive definite then reject this trial.
5. Otherwise, return a cost value (or vector) associated with the computed calibration.

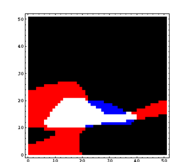
- The minimum of the search may be used to initialize a non-linear optimization in the three parameters, using the same cost function.
- The error surface is well-behaved close to the minimum, but highly indented around it. A non-linear minimization initialized at a random point would therefore be sure to fail.



Logarithmic plot of the cost function at a cross-section through the search cube.



Contour plot of the cost function.



Blue: Cheirality test satisfied
Red: IAC test satisfied
White: Both satisfied
Black: Neither satisfied

Linear algorithm for self-calibration of non-translating cameras

The matrix of intrinsic parameters is:

$$\mathtt{K} = \begin{pmatrix} \alpha_x & s & x_0 \\ & \alpha_y & y_0 \\ & & 1 \end{pmatrix}.$$

The IAC in the i-th image, $\boldsymbol{\omega}_i = \mathtt{K}_i^{-\top}\mathtt{K}_i^{-1}$, relates to $\boldsymbol{\omega}_0$ via the infinite homographies, $\mathtt{H}_i$:

$$\boldsymbol{\omega}_i = \mathtt{H}_i^{-\top} \boldsymbol{\omega}_0 \mathtt{H}_i^{-1}.$$

Under the assumption of zero skew, $s = 0$

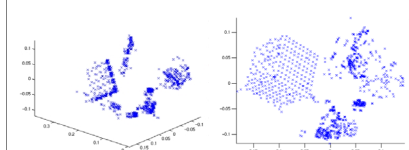
$$\boldsymbol{\omega} = \begin{pmatrix} 1/\alpha_x^2 & 0 & -x_0/\alpha_x^2 \\ 0 & 1/\alpha_y^2 & -y_0/\alpha_y^2 \\ -x_0/\alpha_x^2 & -y_0/\alpha_y^2 & 1 + x_0^2/\alpha_x^2 + y_0^2/\alpha_y^2 \end{pmatrix}.$$

We may then impose the following constraints:

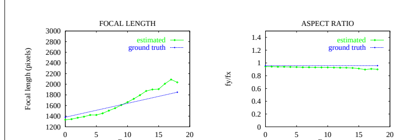
1. **Zero-skew :** If $s = \mathtt{K}_{12} = 0$, then $\boldsymbol{\omega}_{12} = 0$.
2. **Square-pixels :** If $s = 0$ and $\alpha_x = \alpha_y$, then $\boldsymbol{\omega}_{11} - \boldsymbol{\omega}_{22} = 0$.
3. **Known principal point :** If $s = 0$ and $x_0 = 0$, then $\boldsymbol{\omega}_{13} = 0$. Similarly if $y_0 = 0$ then $\boldsymbol{\omega}_{23} = 0$.

Together with the homography constraint this yields linear equations in the entries of $\boldsymbol{\omega}_0$.

Experimental results



Euclidean reconstruction from the square pixels constraint



Camera intrinsics obtained from the zero-skew constraint