

Category Theory Session 6: Questions

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1. Show that **Cat**, the category of all small categories, has coproducts.
2. Given an example of two categories \mathbb{C} and \mathbb{D} , two functors $F, G : \mathbb{C} \rightarrow \mathbb{D}$ and an indexed family $(\eta_X : FX \rightarrow GX)_{X \in \text{obj}(\mathbb{C})}$ that is *not* a natural transformation.
3. Let (X, \leq) be a poset. An *upper set* in (X, \leq) is a subset $U \subseteq X$ that is upwards closed, i.e. $x \in U$ and $x \leq y$ implies $y \in U$. The *Alexandroff topology* on a poset (X, \leq) consists of all upper sets in (X, \leq) .
 - Show that this defines a functor $\text{Pos} \rightarrow \text{Top}$ between the categories of posets (and monotone maps) and topological spaces (and continuous maps).
 - Is this functor full? faithful? Justify your answer.
 - How Would this change if we would take the open sets to be the downwards closed sets?