Category Theory Session 2: Questions

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1. Construct a coproduct diagram of two abelian groups in the category Ab of abelian groups (and group homomorphsims). Where did you need that the groups were abelian?

2. Construct the coproduct of two free groups in the category Grp of (not necessarily abelian) groups and group homomorphisms.

3. Given an example of a category that does not have products.

4. A functor $F : \mathcal{C} \to \mathcal{D}$ *preserves products* if the image under F (left)

$$FA \xleftarrow{F\pi_1} F(A \times B) \xrightarrow{F\pi_2} FB \qquad\qquad A \xleftarrow{\pi_1} A \times B \xrightarrow{\pi_2} B$$

   of each product diagram (right) in $\mathcal{C}$ is again a product diagram. Give an example of a functor F that preserves products, and an example of a functor F that doesn't.

5. Given a category $\mathcal{C}$ with products, the product functor $\times : \mathcal{C} \times \mathcal{C} \to \mathcal{C}$ maps each pair (C, D) of objects to a chosen product $C \times D$ and each pair (f, g) of morphisms to the unique morphism that makes

$$\begin{array}{ccccc} C' & \xleftarrow{\pi_1'} & C' \times D' & \xrightarrow{\pi_2'} & D' \\ {\scriptstyle f}\downarrow & & \downarrow{\scriptstyle f \times g} & & \downarrow{\scriptstyle g} \\ C & \xleftarrow[\pi_1]{} & C \times D & \xrightarrow[\pi_2]{} & D \end{array}$$

   commute. Show that $\times$ preserves morphism composition, i.e. $(f \circ f') \times (g \circ g') = (f \times g) \circ (f' \times g')$ for all $f : B \to C$, $f' : A \to B$, $g : Y \to Z$ and $g' : X \to Y$.

6. Show that an equaliser of a parallel pair of arrows is always monic.