Category Theory Session 1: Questions

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March 1, 2022

1. Consider the category **Met** of metric spaces and continuous functions. Let M be a metric space, and let $D \subseteq M$ be a dense subset (viewed as a metric space with the metric on M restricted to D). Show that the inclusion $D \to M$ is epi in **Met**.

2. Hence, or otherwise, give an example of a category and a morphism that is both epi and mono, but not iso.

3. Show that all is well in the category **Top** of topological spaces and continuous functions though. That is, show that epis (monos) in **Top** are surjective (injective) mappings.

4. Let $\mathcal{C}$ be a category and $A \in \mathcal{C}$ be an object of $\mathcal{C}$. Consider the category $\mathcal{C}/A$ of morphisms with codomain A given by:

    - *objects* are pairs (X, ϕ) where $X \in \mathcal{C}$ is an object and $\phi : X \to A$ is a morphism in $\mathcal{C}$
    - *morphisms* between (X, ϕ) and (Y, ψ) are morphisms $f : X \to Y$ for which $\phi = \psi \circ f$.

    Let's think about initial and final objects in $\mathcal{C}/A$.

    - When does $\mathcal{C}/A$ have an initial object, and how can it be described?
    - When does $\mathcal{C}/A$ have a final object, and how can it be described?

5. Show that all monomorphisms in the category **Grp** of groups and group homomorphisms are injective. What structural property of groups have you used in the proof that could generalise to other settings?