Category Theory Session 9: Questions

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June 14, 2022

1. Let $\mathbb{C}$ be cartesian closed. Show that the functor $S(A) = A \times S^A$ extends to a monad – the *side effet monad*. Why is it called that name?
2. Let $\mathcal{P}(X)$ be the powerset of a set X. Then $\mathcal{P}$ extends to a monad on Set where the unit is singleton, and the multiplication is union. Show that the category of $\mathcal{P}$-algebras is isomorphic to the category of join-semilattices.

   Hint. Given a $\mathcal{P}$-algebra $\alpha : \mathcal{P}(A) \to A$, one can define an ordering by $a \leq b$ iff $\alpha(\{a, b\}) = b$.
3. Consider the category Meas of measurable spaces, and define a functor $M : \mathsf{Meas} \to \mathsf{Meas}$ where $\mathsf{Meas}(S)$ is the set of all probability measures on S, endowed with the smallest σ-algebra that makes the evaluation maps $\mathsf{ev}_U : M(X) \to [0, 1]$, $\mathsf{ev}_U(\mu) = \mu(U)$ measurable for all measurable $U \subseteq X$.
   - Can you define a unit and a co-unit that turn M into a monad?
   - Can you Algebras for the *functor* M in terms of a different mathematical structure?