

Constructive Logic and Realisability

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Abstract

We introduce the basics of constructive reasoning and realisability. We start with (standard) examples of (non-)constructive reasoning and contrast this with the BHK interpretation. We then introduce a constructive first-order calculus and relate this to classical reasoning via the Goedel-Gentzen translation. We then consider Heyting arithmetic and number realisability to prove meta-theorems such as the disjunction and existence property. We end up by considering variations of the theme, in particular modified realisability and Heyting arithmetic in higher types.

References

There are a number of good references that are both more accurate and more comprehensive than my sketchy notes.

- Thomas Streicher, Introduction to Constructive Logic and Mathematics, Lecture notes. Available from the author's home page and indeed an excellent resource that covers almost all of the material presented.
- Helmut Schwichtenberg and Stan Wainer, Proofs and Computation. Cambridge University Press, 2012. Advanced material that in particular studies different logical systems and program extraction from classical proofs. In particular, it develops a theory of computable functions that allows to discuss program extraction in the presence of general recursion.
- Anne Troelstra, Metamathematical Investigations into Intuitionistic Arithmetic, Springer 1973. A classical reference for intuitionistic mathematics.
- A curated collection of papers on realisability is maintained by Jaap van Oosten, available via his web page.