Efficient exact inference in Ising graphical models applied to Go

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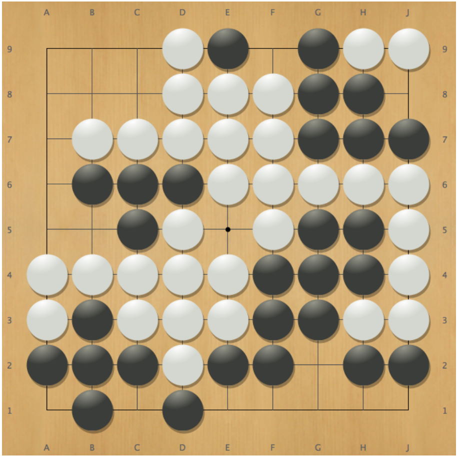
What is Go?



- Two players alternate in placing stones on the intersections of a grid
- Neighbouring stones of the same colour form a contiguous *block*
- A block can be *captured* if all its empty neighbours (*liberties*) are occupied by opponent stones

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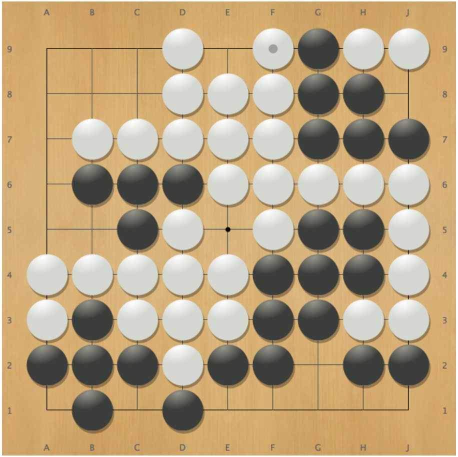

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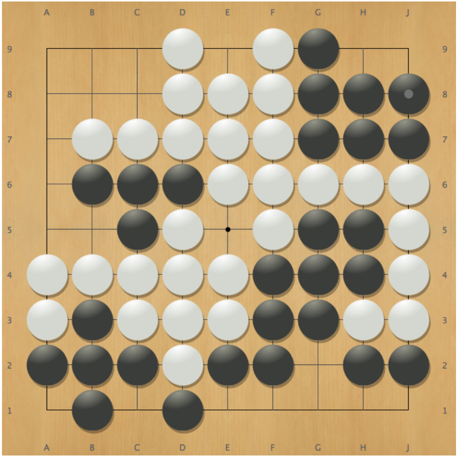

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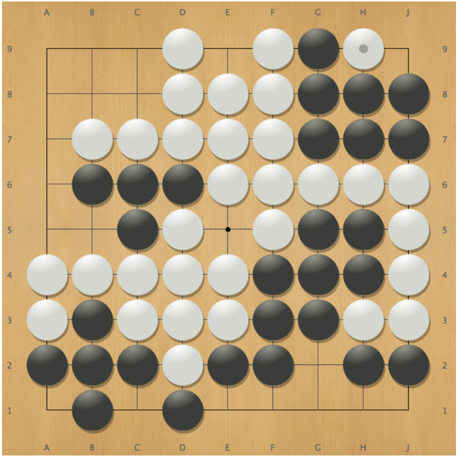

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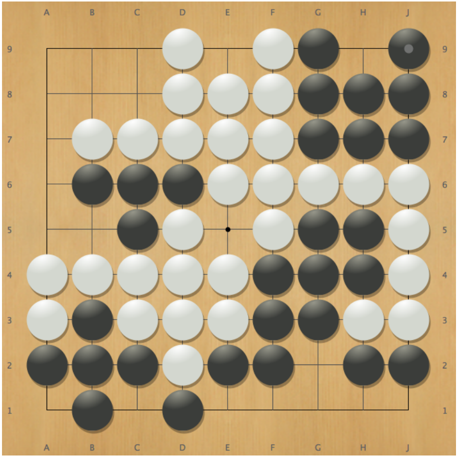

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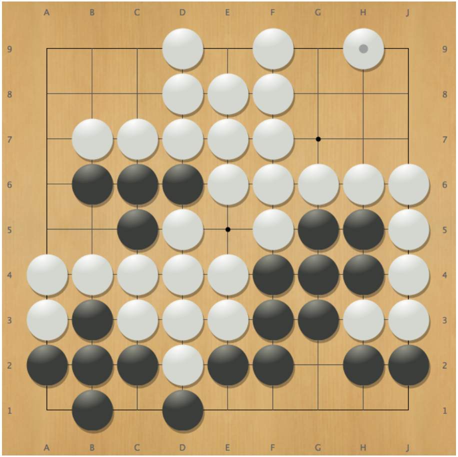

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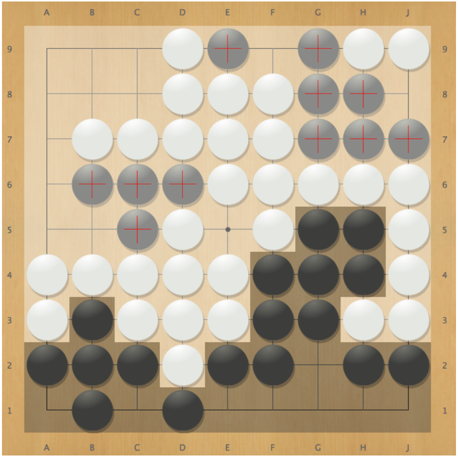


- The game terminates once players agree on the life status of blocks
- The blocks and their surrounding area count towards *territory*
- Territory is used to determine the winner of the game

Why is Go challenging for computers?

- Huge branching factor
  - $\simeq 200$ on 19×19 board, $\simeq 40$ in Chess
  - $\simeq 10^{172}$ legal board positions, $\simeq 10^{50}$ in Chess
  - Standard alpha-beta min-max is too inefficient
- Position evaluation is difficult
  - Hard to judge strength of blocks statically
  - Stones have both local and long-range interactions

Heuristic-based programs

- Rely on hand-tuned patterns and results from local searches
- **Advantages:** Strong locally, especially if pattern is known
- **Disadvantages:**
  - Weak at global play
  - Weak at judging unseen situations
  - Board evaluation is slow

Learning-based programs

- Learn an evaluation function using self-play or expert games
- **Advantages:** Loads of expert games available
- **Disadvantages:** Relatively weak playing strength
- *Gut feeling:*
  - State-space is too large
  - Hard to define features
  - Evaluation function is highly non-smooth

Sampling-based programs

- Bandit-based tree search (UCT)
  - Each tree node (board position) is a multi-arm bandit
  - Sample child positions, maximize total reward
  - Store all node statistics in a tree data structure
  - If a node is not in the tree then use an evaluation function
- Evaluation function
  - Playout position randomly until no moves remain. Final position is trivial to score
  - Enhanced through the use of patterns and other heuristics

Sampling-based programs

- **Advantages:**
  - Evaluation function is fast and accurate for many samples
  - Increase in samples gives increase in playing strength
  - Assymetric tree growth - more time spent on difficult positions
  - Best performance. Reached 3-dan (professional) level on 9×9
- **Disadvantages:**
  - Weak performance early in the game
  - Still weak on larger boards
- **Conclusion:**
  - Framework has good potential
  - But need to improve both search and evaluation

Learning in Go

- Go is played on a grid graph G, so it is natural to model it with a graphical model such as CRF
- Major problem is inference:
  - Approximate: Loopy Belief Propagation
  - Exact: Junction-Tree, Graph Cuts

Junction Tree Algorithm

- Exact method for computing partition function, marginals and MAP (*maximum a posteriori*) state
- Graph is **a tree**: complexity polynomial in graph size
- Graph is **not a tree**:
  - Convert the graph into a tree of cliques
  - Complexity exponential in the treewidth = size of the maximal clique
  - For $N \times N$ grid the treewidth is N

Graph Cuts

- Exact method for computing MAP state of a binary-labeled problem
- Treat MAP computation as finding the min-cut of a particular graph (with positive edge weights)
- **Theorem:** finding the graph's min-cut is equivalent to finding its max-flow
- Can use Ford-Fulkerson. Complexity is polynomial in graph size

Other methods?

- **Question:** Is there a method that can compute partition function and marginals like Junction Tree, but in polynomial time like Graph Cuts?
- **Answer:** Yes!

Other methods?

- **Question:** Is there a method that can compute partition function and marginals like Junction Tree, but in polynomial time like Graph Cuts?
- **Answer:** Yes!

Ising problem

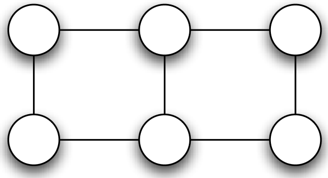
- Graph $G = (V, E)$, binary variables (spins): $y_i \in \{+, -\}$
- Spins only interact in pairs. One energy for agreement: $\psi_{--} = \psi_{++}$, another for disagreement: $\psi_{-+} = \psi_{+-} = 0$
- Model distribution:

$$\mathbb{P}(y) = \frac{1}{Z(\psi)} \exp(\sum_{ij \in E} [y_i = y_j] \psi_{ij}) \text{ , where}$$

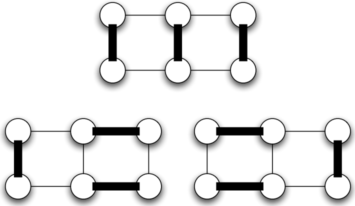
$$Z(\psi) = \sum_{y} \exp(\sum_{ij \in E} [y_i = y_j] \psi_{ij}) \text{ is the partition function}$$

Dimer problem

- How many perfect matchings does a graph have?

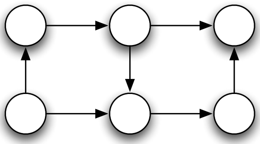


- **Perfect Matching**: A set of non-overlaping edges (dimers) that cover all vertices



Counting Matchings

- Every planar graph has a **Pfaffian orientation**: each face (except possibly outer) has an odd number of edges oriented clockwise



- Define a skew-symmetric matrix K such that:

$$K_{ij} = \begin{cases} 1 & \text{if } i \rightarrow j \\ -1 & \text{if } i \leftarrow j \\ 0 & \text{otherwise} \end{cases}$$

Kasteleyn Theorem

$$K = \begin{array}{|c|c|c|c|c|c|} \hline 0 & 1 & 0 & 0 & 0 & -1 \\ \hline -1 & 0 & 1 & 0 & 1 & 0 \\ \hline 0 & -1 & 0 & -1 & 0 & 0 \\ \hline 0 & 0 & 1 & 0 & -1 & 0 \\ \hline 0 & -1 & 0 & 1 & 0 & -1 \\ \hline 1 & 0 & 0 & 0 & 1 & 0 \\ \hline \end{array}$$

Kasteleyn Theorem:
Number of perfect matchings is $\mathrm{Pf}(K) = \sqrt{|K|}$

The connection

- Let $G_{\triangle}$ be G plane triangulated: each face becomes a triangle
- Let G^* be the dual of graph $G_{\triangle}$: each face in $G_{\triangle}$ is a vertex in G^*
- Let G^*_e be the expanded version of G^*: each vertex is replaced with 3 vertices in triangle
- **Connection**: There is a 1:1 correspondence between perfect matchings in G^*_e and agreement edge sets in G

Our method: overview

- No need to compute the dual G^* and expanded dual G_e^*
- Show how to compute the marginals and hence perform parameter estimation
- Show how to compute the MAP state
- All computations are **polynomial** in graph size

Our method: overview

- Model distribution:

$$\mathbb{P}(y) = \frac{1}{Z(\psi)} \exp(\sum_{ij \in E} [y_i = y_j] \psi_{ij})$$

$$\mathbb{P}(y|x;\theta) = \frac{1}{Z(x;\theta)} \exp(\sum_{ij \in E} [y_i = y_j] < \phi_{ij}(x), \theta >)$$

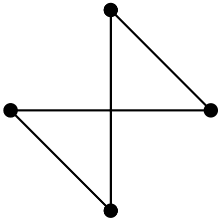
- Restrictions:
  - Graph is planar: can be drawn without crossing edges
  - Binary labels
  - No node potentials (*no external field*)
  - Edge potentials: one for agreement, one for disagreement

Comparison to Graph Cuts

	Graph Cuts	Ising Model
Need planarity?	No	For polynomial runtime
2-label problem	Exact and polynomial runtime	
N-label problem	approx. with $\alpha-$expansion	Not yet
Node potentials?	Yes	Only *outerplanar* graph
Energy restriction	Submodularity: $E_{0,0} + E_{1,1} \leq E_{0,1} + E_{1,0}$ non-submodular: partial sol.	$E_0 = E_1 = 0$ $E_{0,1} = E_{1,0}\ (= 0)$ $E_{0,0} = E_{1,1}$
Combined restriction	$E_{0,0} = E_{1,1} \leq 0,\ E_0 = E_1 = 0$: trivial solution	
Partition function?	No	Yes
Marginals?	No	Yes
Parameter Estimation	Max-Margin	Max-Likelihood

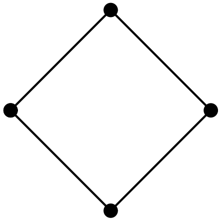
Algorithm

- Original graph $G = (V, E)$



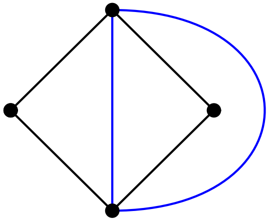
Algorithm: Step 1

- Obtain a planar embedding
- Using Boyer-Myrvold algorithm the complexity is $O(n)$, where $n = |E|$



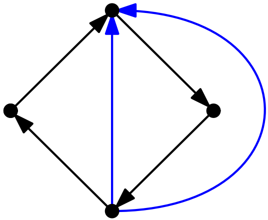
Algorithm: Step 2

- Add edges to plane triangulate the graph
- Using simple ear-clipping the complexity is $O(n)$



Algorithm: Step 3

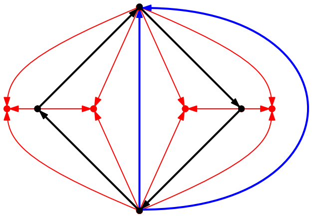
- Orient the edges such that each vertex has odd in-degree
- Equivalent to having a Pfaffian orientation in the dual graph
- Complexity is $O(n)$

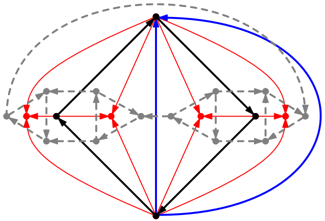


Algorithm: Step 4 (intuition)

- Add nodes to each face
- Orient edges towards those nodes
- Equivalent to expansion in the dual graph

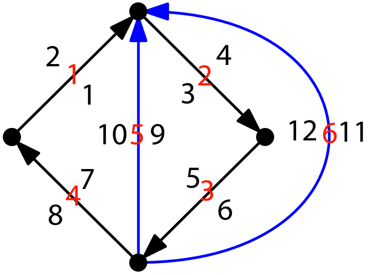
- Construct a skew-symmetric $2|E| \times 2|E|$ matrix K (for dual edges):
  - $K_{ij} = \pm e^{\psi_{ij}}$ if ij crosses **original**
  - $K_{ij} = \pm 1$ if ij crosses added
- Complexity is $O(n)$





Algorithm: Step 4 (implementation)

- Number each edge
- **Number** the sides of each edge k
  - LHS $= 2k$
  - RHS $= 2k - 1$



Pseudo Code

For each vertex v:

- For each edge k incident on v (clockwise):
  - if k points away from v:
    - $K_{2k,\mathrm{prev}} = 1$ $(2 \to 8)$
  - else
    - $K_{2k-1,\mathrm{prev}} = 1$ $(7 \to 1)$
    - $K_{2k-1,2k} = e^{\psi_k}$ $(7 \to 8)$

Return $K - K^\top$

Algorithm: Parameter estimation

- Compute partition function: $Z(\psi) = 2\sqrt{|K|}$
- Compute gradients:

$$\frac{\partial \ln Z(\psi)}{\partial \theta_k} = \frac{2}{Z(\psi)} \frac{\partial \sqrt{|K|}}{\partial \psi_k} = \ldots = -[K^{-1} \odot K]_{2k-1,2k}$$

- Computing inverse and determinant takes at most $O(n^3)$ time

Algorithm: MAP state

- Maximum *a posteriori* state (MAP):

$$y^* = \operatorname*{argmax}_y \mathbb{P}(y|x;\theta^*) \text{ , where}$$

$$\theta^* = \operatorname*{argmin}_\theta \mathcal{L}(\theta) \text{ , } \mathcal{L}(\theta) = \frac{||\theta||^2}{2\sigma^2} - \sum_{k=1}^{m} \ln \mathbb{P}(y|x;\theta)$$

- Max-weight perfect matching on G_e^* gives the max-weight agreement edge set. Use blossom-shrinking (Edmonds 1965)
- This takes $O(n^2 \log(n))$ time

Algorithm: Numerical problems

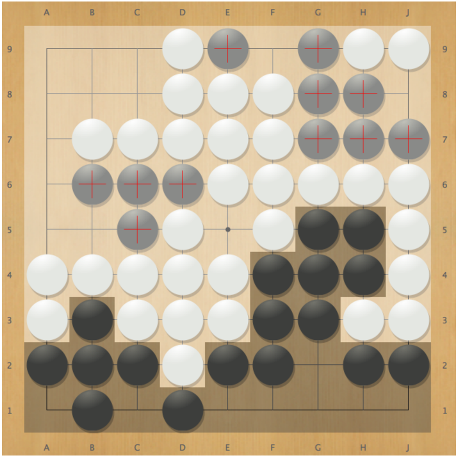
- Computation of $|K|$ and K^{-1} are prone to numerical problems
- Method 1: for skew-symmetric matrices K and constant q:

$$|K| = \frac{|qK|}{q^n}$$

- Method 2: use LU decomposition of K:

$$K^{-1} = U^{-1}L^{-1}\ ,\ \ln|K| = \sum_{i=1}^{n} \ln U_{i,i}$$

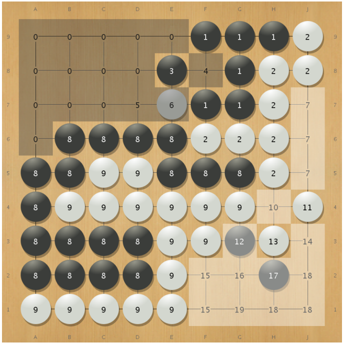
Territory prediction in Go

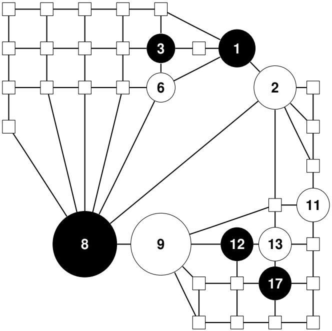


- The blocks and their surrounding area count towards *territory*
- **Territory prediction:** Given a board position predict the owner of each intersection
- Challenging problem for ML!

Graph abstraction: common fate graph

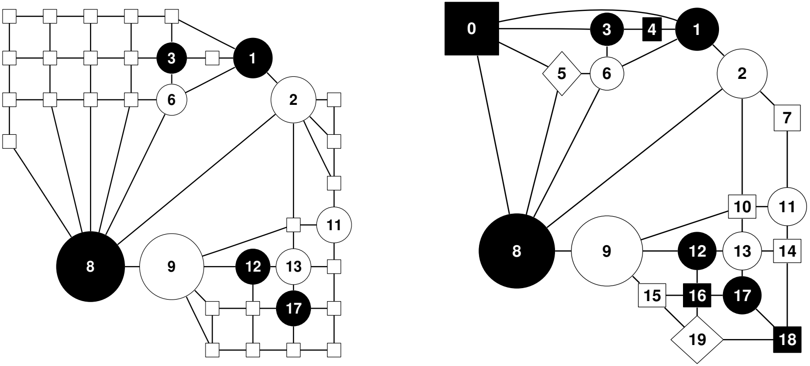
- Grid graph G does not capture the fact that **stones in a block always live or die as a unit**
- *Common fate graph* G_{cfg} (Graepel et al., 2001) merges all stones in a block into a single node





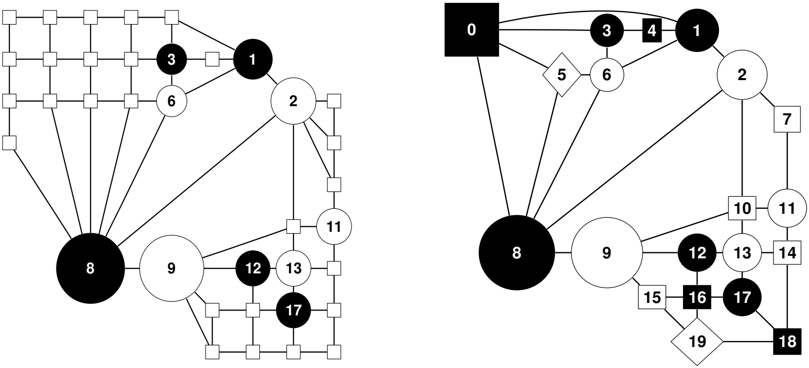
Graph abstraction: block graph

- Use Manhattan distance to classify empty regions into 3 types: *black surround* (■), *neutral*(◇) and *white surround*(□)
- Collapse empty regions to form the *block graph* G_b



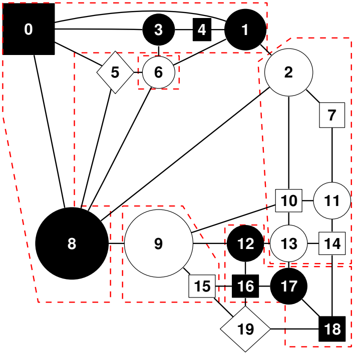
Graph abstraction: block graph

- Surrounds encode the possibility for obtaining territory
- G_b is more concise than G_{cfg}, but preserves the kind of information required for predicting territory



Graph abstraction: group graph

- *Group*: set of blocks of the same colour that share at least one surround
- Construct the *group graph* G_g by collapsing groups of G_b

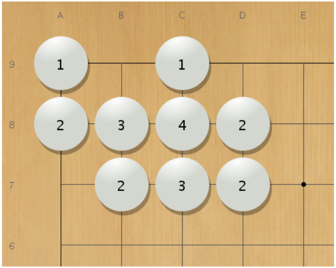


Feature engineering: nodes

- Given node $v \in G_b$, compute feature vector F:

$$F_k = \text{num. intersections in v with k neighbours in v}$$

- Provides a powerful summary of the region's shape



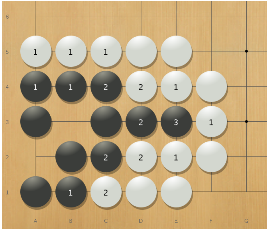
$$F = \{2, 4, 2, 1\}$$

Feature engineering: edges

- Given nodes $v^1, v^2 \in G_b$, compute their corresponding features F^1 and F^2:

$$F_k^1 = \text{num. intersections in } v^1 \text{ with k neighbours in } v^2$$
$$F_k^2 = \text{num. intersections in } v^2 \text{ with k neighbours in } v^1$$

- Provide information of node's liberties and boundary shape



$$F^1 = \{3, 3, 1\}, F^2 = \{6, 3, 0\}$$

Datasets

- 9×9 games: Van der Werf et al. collection, 1000 training and 906 testing
- 19×19 games: scored by our cooperative scorer, 1000 for training and testing
- Oversize games: 22 games manually scored. Sizes range from 21×21 to 38×38. Only for testing

Task

- Given an endgame position determine the label (BLACK or WHITE) of each intersection
- We train our CRF on the block graph G_b, using BFGS as the optimizer
- Prediction determined using MAP state of the group graph G_g

Controls

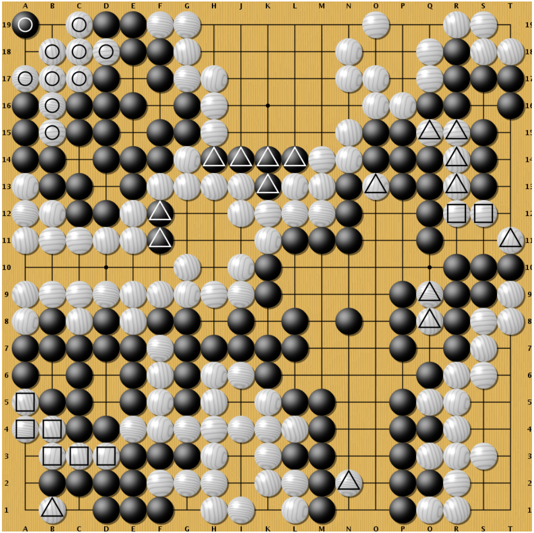
- **Naive:** assume all stones are alive
- **GnuGo 3.6:** open source program. Uses Go-specific knowledge and local searches
- **NN:** neural net classifier (van der Werf et al. 2005). Uses 63 Go-specific features of various board abstractions
- **MRF:** a simple MRF on G with just 6 parameters (Stern et al. 2004). Inference via 50 iterations of LBP. Prediction via marginal expectations at each intersection

Prediction Accuracy

Size	Algorithm	Error (%) Block	Stone	Game	Winner
9×9	Naive	17.57	8.80	75.70	30.79
	MRF	8.19	5.97	38.41	13.80
	CRF approx	2.73	2.46	9.93	2.65
	CRF exact	2.57	2.32	9.05	2.32
	GnuGo*	-	**0.05**	1.32	-
	NN*	$\leq$ **1.00**	0.19	**1.10**	**0.50**
19×19	Naive	16.52	6.96	98.30	32.60
	MRF	4.91	3.80	63.90	20.50
	CRF approx	5.25	4.93	49.00	11.80
	CRF exact	3.93	3.81	43.40	9.30
	GnuGo	-	**0.11**	**5.10**	-
greater than 19×19	Naive	19.64	10.25	100.00	31.81
	MRF	7.80	6.83	100.00	22.73
	CRF approx	7.51	6.84	**81.82**	**9.09**
	CRF exact	**4.52**	**5.02**	**81.82**	**9.09**

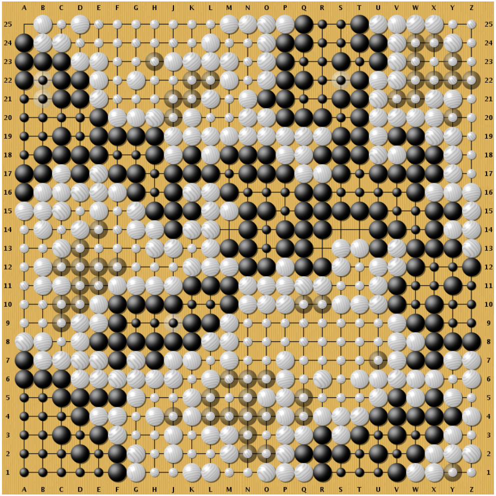
* Was used to label data

Errors made by different methods



- ○ misclassified by Naive, MRF and CRF
- □ by Naive and MRF
- △ by Naive only
- Gnugo made no errors
- MRF inconsistent due to use of marginals

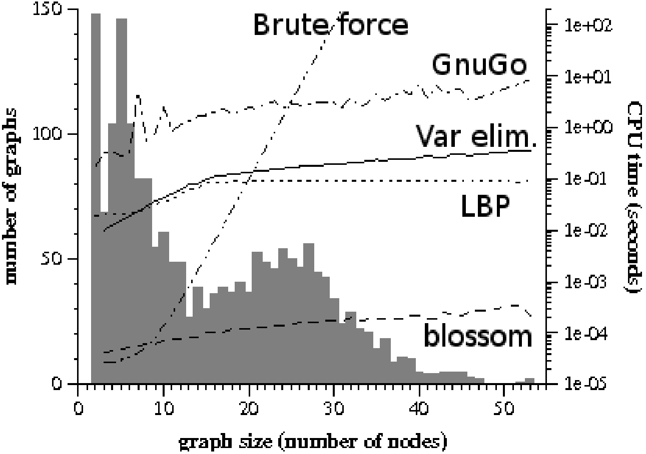
CRF's perfect prediction for an oversize game



Prediction speed: methods

- **GnuGo 3.6:** scoring in *aftermath* mode
- **LBP:** 50 iterations of LBP for marginal expectations (Stern et al. 2004)
- **Brute force:** variable elimination with arbitrary elimination ordering
- **Variable elimination:** using min-fill heuristic (Kjaerulff 1990)
- **Our method:** blossom-shrinking (Edmonds 1965)

Prediction Speed



This work

- Not much luck with conference papers
- Going to publish this as a journal paper in JMLR
- Want to try territory prediction for middle-game positions and move prediction
- Want to apply this method to images and compare directly to Graph Cuts

Bandit-based tree search

- Assume each node n_i has a reward distribution X_i
- UCT samples node

$$n_i^* = \underset{n_i}{\operatorname{argmax}}(\mathbb{E}(X_i) + c * \text{Var}(X_i))$$

- Instead assume $X_i = B(\alpha_i, \beta_i)$. Now sample node

$$n_i^* = \underset{n_i}{\operatorname{argmax}}(x_i \sim X_i)$$

- Performance not as good as UCT's
- Now want to try Gittins indices

Bandit-based tree search

- UCT falsely assumes that arms (siblings) are independant
- Instead sample from dependant arms (Pandey et al., 2007)
  - Cluster arms (eg. based on group graph)
  - Step 1: Select a cluster to sample
  - Step 2: Select an arm within that cluster to sample
  - Update statistics of **all** arms in that cluster
- Can expect huge speed-up

Other Go work

- Cooperative Scorer
- Fast influence function
- Go playout on GPU
  - GPUs are designed for floating point and matrix operations
  - Nvidia Tesla has up to 128 parallel cores, 512 Gflops
  - Developed a random Go player that only uses matrix operations
  - Huge potential if works on the GPU!

Questions?