THE AUSTRALIAN NATIONAL UNIVERSITY

Second Semester 2003

Practice Examination

ENGN4627

Robotics

Study Period: 15 minutes.

Writing Period: 3 hours

Permitted Materials: Calculators, 2 double sided A4 pages of handwritten notes, frames of reference, good luck charms.

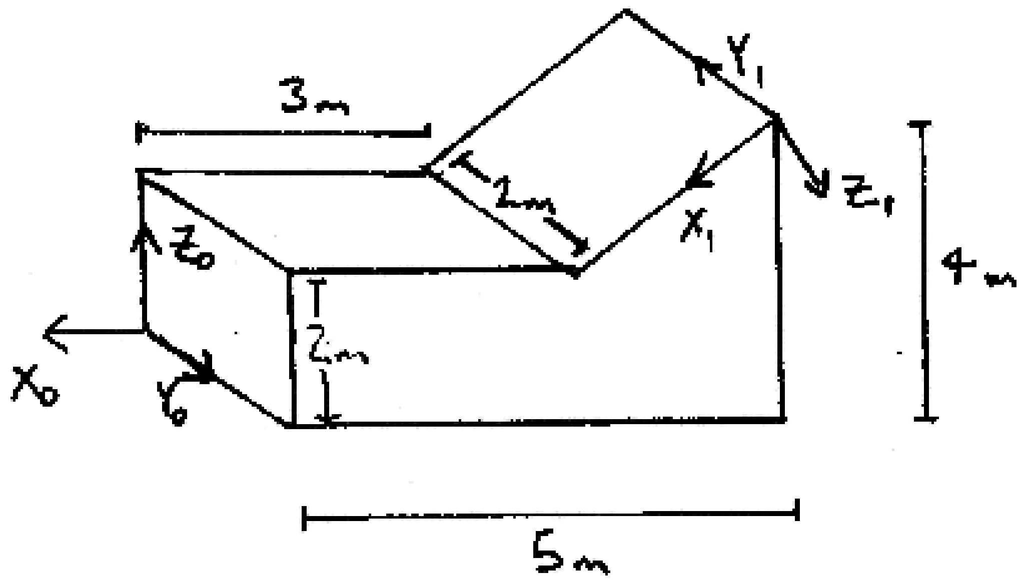
Answer ALL questions – Total of 58 marks

Answer all questions or as many parts of questions as you are able. The clarity and precision of your explanations and answers substantially affect the marks you are awarded for each answer. If you know an answer you arrive at doesn't make complete sense, indicate why you believe this, and what that may tell you about correcting your work if you had more time. If you are unable to fully complete a question, you should outline in as precise a fashion as possible (time-allowing) what steps need to be taken to complete the question.

The final exam paper is not to be removed from the exam room.

Question 1. 6 Marks (10%) [15 minutes]

Consider the following diagram:



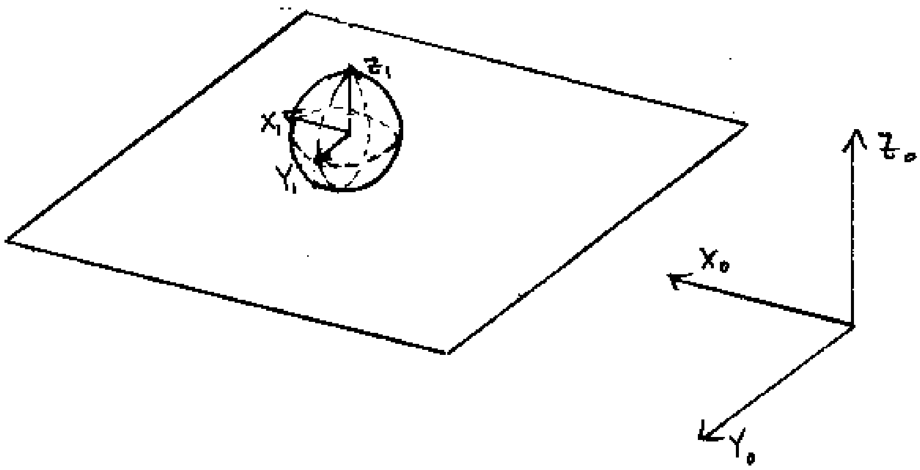
(a) 2 Mark: Compute the linear translation between frame {0} and frame {1}.

(b) 2 Mark: Compute the rotation between frame {0} and frame {1}.

(c) 2 Mark: Write down the homogeneous transformation ${}^{0}_{1}T$ (in matrix form) relating frame {0} and {1}.

Question 2. 6 Marks (10%) [20 Minutes]

Consider a glass ball with a right-hand-frame of reference attached at its centre point, rolling on a flat plane as shown in the figure



The position of the ball is irrelevant, only its attitude is of interest. The ball can be rolled forward in the X_0 direction or sideways in the Y_0 direction.

(a) 1 Mark: Rolling the ball forward in the position X_0 direction causes a change in the attitude of the ball. Compute the rotation matrix ${}^0_1R$ associated with α degrees of rotation due to linear movement in the X_0 direction.

(b) 1 Mark: Rolling the ball sideways in the position Y_0 direction causes a change in the attitude of the ball. Compute the rotation matrix ${}^0_1R$ associated with β degrees of rotation due to linear movement in the Y_0 direction.

(c) 2 Mark: Consider a composite movement associated with

i) Firstly α degrees of rotation due to linear motion in the X_0 direction.

ii) Followed by β degrees of rotation due to linear motion in Y_0 direction.

Compute the rotation matrix ${}^0_1R$ for this movement.

(d) 1 Mark: If a third rotation, once again in the X_0 direction, is undertaken and denoted by γ degrees. Do the angles α, β and γ provide fixed-angle representation or an Euler angle representation of attitude. Explain your answer in a sentence.

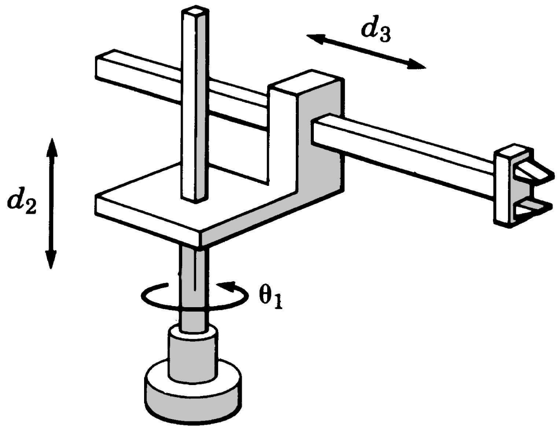
(e) 1 Mark: Is it possible to roll the ball only in the $\{X_0, Y_0\}$ plane and achieve an attitude

$$ {}^0_1R = \begin{pmatrix} c_\theta & -s_\theta & 0 \\ s_\theta & c_\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} $$

Write a sentence to justify your answer.

Question 3. 5 Marks (10%) [20 Minutes]

Consider the RPP manipulator shown below



The Denavit-Hartenberg parameters for this robot are

Link	α_{i-1}	a_{i-1}	θ_i	d_i
1	0	0	θ_1	0
2	0	0	0	d_2
3	+90°	0.1m	0	d_3

(a) 2 Mark: Write down a set of mathematical constraints that define the manipulator subspace for the manipulator shown.

(b) 3 Marks: A goal position

$$^0P_G = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

is specified is specified in the base frame. Compute the inverse kinematics of the manipulator. That is compute $\{\theta_1, d_2, d_3\}$ as functions of (x, y, z).

Question 4. 10 Marks (20%) [30 Minutes]

A spray painting robotic manipulator is constructed as a four link anthropomorphic geometry mounted on a base that runs on rails. The complete manipulator is classed as a PRRRR. Its Denavit-Hartenberg parameters are

Link	α_{i-1}	a_{i-1}	θ_i	d_i
1	0	0	0	d_1
2	-90	0	θ_2	1
3	+90	0	θ_3	0.2
4	0	0.5	θ_4	0
5	-90	0	θ_5	0.5

(a) 3 Mark: Sketch a valid geometry for the first two links of the robot. That is sketch the world frame {0}, the first link frame {1} and the second link frame {2} in the correct geometry. Show frame orientations and offsets clearly.

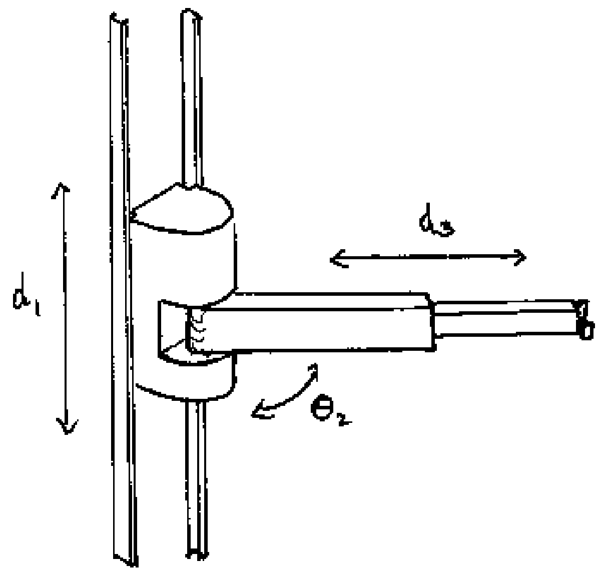
(b) 3 Mark: Sketch a valid geometry for links 2, 3, and 4 of the manipulator. That is imagine frame {2} as a world frame and sketch the relative positions of link frame {3} and {4}. Show frame orientations and offsets clearly.

(c) 2 Mark: On a separate diagram sketch the geometry of the full robotic manipulator showing frame attachments.

(d) 2 Mark: Based on the parameters given calculate the link transformation ${}^1_2T$ between the first and second links of the manipulator.

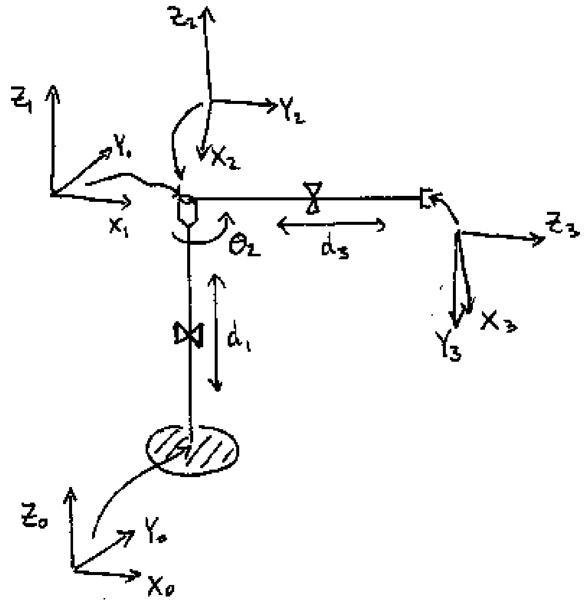
Question 5. 7 Marks (10%) [25 Minutes]

Consider the PRP robot shown in the figure



The home position for the robot (with $d_1 = 0$, $\theta_2 = 0$, $d_3 = 1m$) has the base of the robot on the ground, the arm pointing straight out and the end effector one metre from the vertical axis. (Note that joint limits prevent $d_3 = 0$.) The revolute joint θ_2 is measured positive in a counter-clockwise direction from home.

The frame assignments are shown in the following schematic



The link transformations and forward kinematics for the robot are

$$
{}^0_1T = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad
{}^1_2T = \begin{pmatrix} c_2 & -s_2 & 0 & 0 \\ s_2 & c_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad
{}^2_3T = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},
$$

$$
{}^0_2T = \begin{pmatrix} c_2 & -s_2 & 0 & 0 \\ s_2 & c_2 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad
{}^1_3T = \begin{pmatrix} c_2 & 0 & -s_2 & -d_3s_2 \\ s_2 & 0 & c_2 & d_3c_2 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},
$$

$$
{}^0_3T = \begin{pmatrix} c_2 & 0 & -s_2 & -d_3s_2 \\ s_2 & 0 & c_2 & d_3c_2 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{pmatrix}.
$$

The inertial velocity of the base frame is zero

$$
{}^0\omega_0 = 0, \quad {}^0v_1 = 0.
$$

(a) 2 Mark:

i) What are the inertial velocities of the first link ${}^0v_1$, ${}^0\omega_1$ in terms of the link variables $(\dot{d}_1, \dot{\theta}_2, \dot{d}_3)$.

ii) Compute the inertial velocities, ${}^1\omega_1$ and ${}^1v_1$, of link 1 expressed in frame $\{1\}$.

(b) 2 Mark: Based on your solution from part a).

i) Compute the inertial velocities of the second link ${}^1v_2$, ${}^1\omega_2$ expressed in frame {1} in terms of the link variables $(\dot{d}_1, \dot{\theta}_2, \dot{d}_3)$.

ii) Compute the inertial velocities, ${}^2\omega_2$ and ${}^2v_2$, of link 2 expressed in frame {2}.

(c) 2 Mark: Based on your solution from part b).

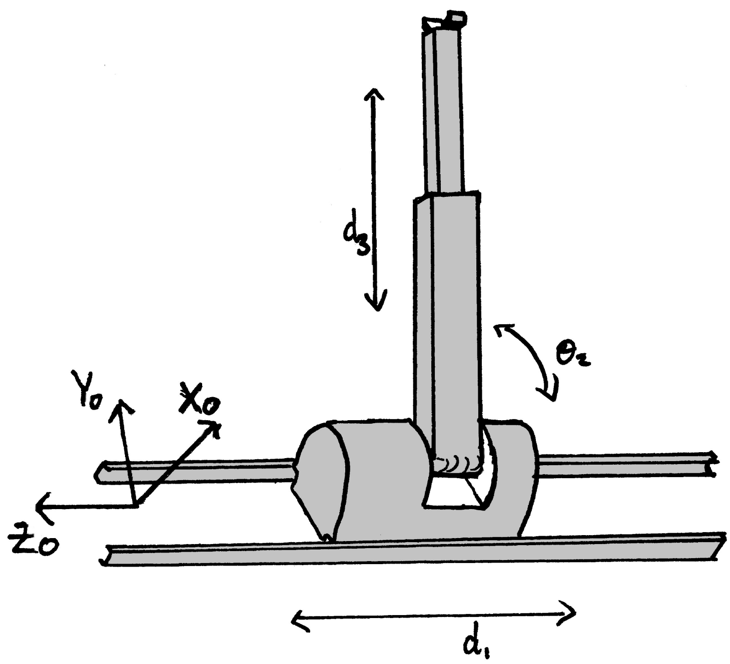
i) Compute the inertial velocities of the third link ${}^2v_3$, ${}^2\omega_3$ expressed in frame {2} in terms of the link variables $(\dot{d}_1, \dot{\theta}_2, \dot{d}_3)$.

ii) Compute the inertial velocities, ${}^3\omega_3$ and ${}^3v_3$, of link 3 expressed in frame {3}.

(d) 1 Marks: Compute the velocity Jacobian ${}^3J_v$ of the robot in the end effector frame.

Question 6. 12 Marks (20%) [35 minutes]

The the PRP robot discussed in Question 5. has been built by an unnamed Australian university to store radioactive samples in an unused tunnel under the Engineering building.



The robot is mounted sideways on the tunnel wall (as shown in the figure) and inserts samples into the slots cut into the opposing wall. The base frame is oriented with the X_0 axis horizontal to the floor of the tunnel, the Y_0 axis is in the vertical positive direction and Z_0 lies along the axis of the tunnel. The base cart of the robot weighs 20kg. The inner and outer arm links are both 1m in length and weigh 5kg each mass is uniformly distributed over the length of each arm. The gripper weighs 2kg.

The robot end effector is gripping a sample weighing 1kg. Due to gravity the object exerts a force of

$$^{0}F = \begin{pmatrix} 0 \\ -10 \\ 0 \end{pmatrix} N$$

and a torque of

$$^{0}N = \begin{pmatrix} 0 \\ 0 \\ -\cos(\theta_2) \end{pmatrix} Nm$$

on the end effector of the robot.

The task of the manipulator is to insert the object sideways into a slot in the wall. The joint configuration at the goal frame is

$$d_1 = 10m, \quad \theta_2 = +45^\circ, \quad d_3 = 1.7m$$

The force required to insert the object is

$$^{0}F_G = \begin{pmatrix} 5 \\ 0 \\ 0 \end{pmatrix} N$$

(a) 1 Mark: Compute the force Jacobian $^{0}J_f$ for the manipulator.

(b) 3 Mark: Compute the force and torque required by each joint to support the robot in the desired configuration against the forces of gravity.

(c) 2 Marks: Compute the joint torques and forces that are required to hold the sample in the desired location prior to inserting it in its slot.

(d) 2 Marks: Compute the total force that must be applied by the robot to insert the sample in its holding spot.

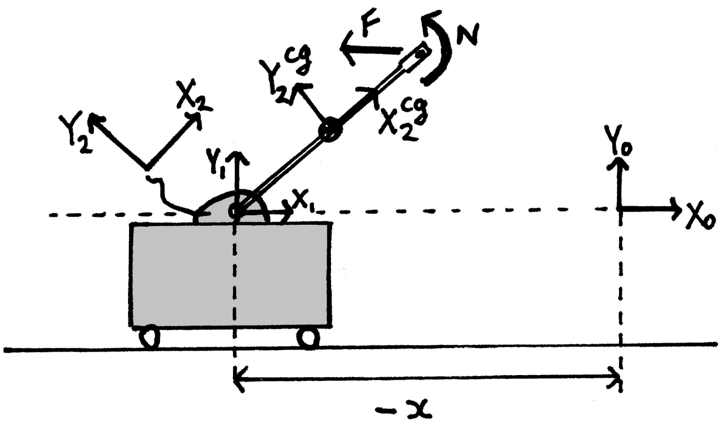
Storage slots are located at a distance of 1.2m from the robot tracks between $\theta_2 = -45^\circ$ to $+45^\circ$. The robot must always supply a force $^{0}F_G = (5, 0, 0)$ to insert a sample in the storage slot.

(e) 2 Marks: What orientation would require the largest joint torques and force to insert the object. Justify your answer with a short sentence.

(f) 2 Marks: What would be the smallest force and torque specifications that could be specified for the actuators for links 2 and 3 that would still achieve the required task.

Question 7. 12 Marks (20%) [35 minutes]

Consider the idealized diagram below of a robotic device design as an active walking aid. The design incorporates a small robotic cart with an actuated inverted pendulum that will act as a walking stick for the client



In the modelling of the dynamics an external force and torque (F, N) is generated at the handle of the walking stick by the client.

The mass of the cart is denoted M. The mass of the arm is m and its length is l. Assume that the arm mass is uniformly distributed along its length and its centre of mass is located mid-way along its length. The inertia of the arm is $I = ml^2/12$ around it centre of mass. The 'joint' variables are the cart displacement x and the stick angle θ. The stick angle is measured from the stick in horizontal position pointing towards the positive X_0 direction. Thus, the stick in the vertical position corresponds to a stick angle of $+90°$. The link transformations and forward kinematics for the frames of reference shown are

$$
{}^0_1T = \begin{pmatrix} 1 & 0 & 0 & x \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad {}^1_2T = \begin{pmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad {}^0_2T = \begin{pmatrix} c_1 & -s_1 & 0 & x \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},
$$

The velocities of the frames of the reference shown are

$$
{}^0v_0 = 0, \quad {}^0\omega_0 = 0, \quad {}^0v_1 = \begin{pmatrix} \dot{x} \\ 0 \\ 0 \end{pmatrix}, \quad {}^0\omega_1 = 0, \quad {}^0v_2 = \begin{pmatrix} \dot{x} \\ 0 \\ 0 \end{pmatrix}, \quad {}^0\omega_2 = \begin{pmatrix} 0 \\ 0 \\ \dot{\theta} \end{pmatrix}
$$

where all velocities have been expressed in the base frame of reference.

The accelerations of the frames of the reference shown are

$$
{}^0\dot{v}_0 = 0, \quad {}^0\dot{\omega}_0 = 0, \quad {}^0\dot{v}_1 = \begin{pmatrix} \ddot{x} \\ 0 \\ 0 \end{pmatrix}, \quad {}^0\dot{\omega}_1 = 0, \quad {}^0\dot{v}_2 = \begin{pmatrix} \ddot{x} \\ 0 \\ 0 \end{pmatrix}, \quad {}^0\dot{\omega}_2 = \begin{pmatrix} 0 \\ 0 \\ \ddot{\theta} \end{pmatrix}
$$

where all accelerations have been expressed in the base frame of reference.

(a) 2 Marks: Compute the linear and rotational acceleration of the frame of reference attached to the centre of mass of the the stick.

(b) 2 Marks: Compute the forces and torques generated on the stick link due to gravity.

(c) 4 Marks: Apply the backward iteration to the device to compute all Euclidean forces and torques applied to the cart and the stick. Use the centre of gravity frame of reference for the stick and frame $\{1\}$ for the cart. (You will need to apply the iteration twice, once for each link and compute the linear force and torque for each link.)

(e) 2 Marks: Compute the linear force f_1 applied to the cart and torque τ_1 applied to the stick in terms of the system dynamics and externally applied forces and torques (F, N).

(f) 2 Marks: Compute dynamic equations for the evolution of the link displacements x and θ based on the above analysis.