Problem Sheet 1
Spatial Descriptions and Transformations

Q 1. Calculating rotations

i) Question 2.2, Chapter 2, Craig.

ii) Question 2.3, Chapter 2, Craig.

Q 2. Structure of a rotation matrix

Let ${}^A_BR$ be a rotation matrix between two frames. Show that the eigenvalues of the matrix are $\left(1, e^{j\theta}, e^{-j\theta}\right)$. What are the properties of the eigenvectors associated with the eigenvalues. What is the physical meaning of the eigenvector associated with the eigenvalue 1. What is the physical meaning of the parameter θ. (Questions 2.5 and 2.25, Chapter 2, Craig).

Q 3. Angle axis representation of a rotation matrix

Question 2.6, Chapter 2, Craig. (Hint: Use a known rotation to transform the arbitrary axis of rotation k into a form for which the rotation matrix is easy to compute.)

Q 4. Commutativity of rotation matrices

Question 2.11, Chapter 2, Craig.

Q 5. Computing Frames of reference

i) Question 2.32, Chapter 2, Craig.

ii) Question 2.33, Chapter 2, Craig.

iii) Question 2.34, Chapter 2, Craig. (Work it out directly and then verify your answer using the results from parts i) and ii).

Q 6. What is a good screw

Consider a rigid body motion ${}^A_BT$, where, starting from initial coincidence, $\{B\}$ is obtained by simultaneously rotating around an axis ω, located at point ${}^AP$, and translating along the axis ω. The motion is parameterized by the direction ω, the location ${}^AP$ of ω, the angle of rotation θ and the distance translated $V = h\theta$, for $h \in \mathbb{R}$ a scalar. Motion of this nature is known as screw motion due to the analogy of rotation around and translation along a single axis that occurs when fixing a screw. The scalar h is termed the 'pitch' of the motion and measures the ratio of linear displacement to rotation analogous to the pitch of a screw thread. (cf. also Question 2.14, Chapter 2, Craig. An interested student can also look at the more advanced text Murray and Sastry, 'A mathematical introduction to robotic manipulation', pp.45-.)

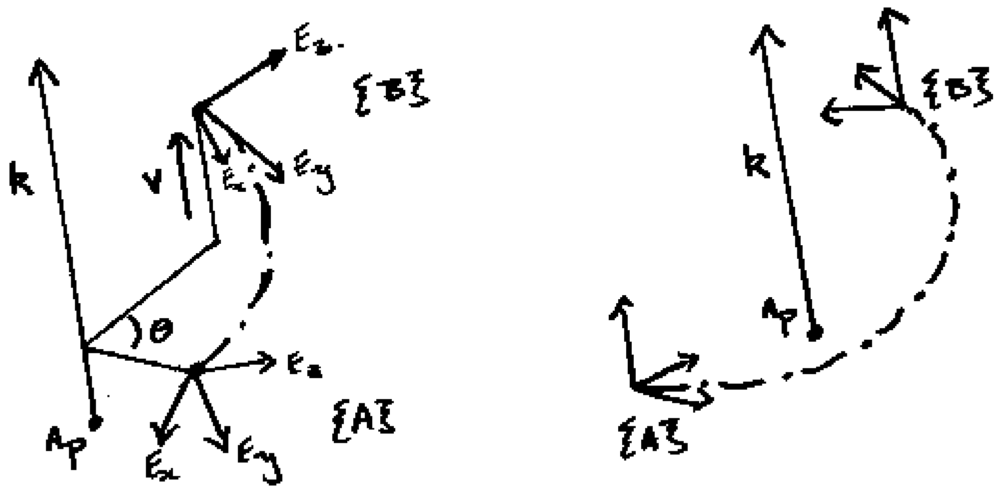

Figure 1: Two examples of screw motions.

i) Develop a general formula for ${}^B_AT$ in terms of θ and V. (Hint: Use an initial rotation of the frame of reference to simplify the screw motion.)

ii) Is it possible to obtain any (fixed) rigid body motion via a screw motion. Discuss.

Screw motion itself is not assessable in this course, however, working through this example is a nice exercise in calculating frames of reference and screw motion is fundamental in advanced robotics such as grasping and manipulation.

Q 7. Euler Angles

Question 2.19, Chapter 2, Craig. Also you can check your answers against appendix B, pg.442-444 of Craig.

i) Calculate the rotation matrix associated with the Z-Y-Z Euler angles.

ii) Calculate the rotation matrix associated with fixed axis rotations around the Z-Y-Z axis.

Q 8. Rigid body motion in 2D

Consider an object moving in 2 dimensions.

i) Write down a description of a frame of reference for two dimensional space from first principals.

ii) Calculate a rigid-body mapping ${}^{A}_{B}T$ to transform between two frames of reference

$$ {}^{A}_{B}T : \{B\} \rightarrow \{A\} $$

in two dimensions from first principals.

iii) Write down homogeneous coordinates for rigid body transformations in two dimensions.

iv) Show how rigid-body motion can be calculated for movement in two dimensions.

v) Show how the motion of rigid bodies in the two dimensional case can be embedded in the representation of motion of rigid bodies in three dimensions. Or alternatively, how to extract two dimensional rigid body motion from the representation of three dimensional rigid body motion.