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Automatica 42 (2006) 1959–1968
www.elsevier.com/locate/automatica

Brief paper

$\mathscr{H}_\infty$ design to generalize internal model control[☆]

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Received 10 June 2004; received in revised form 7 June 2006; accepted 12 June 2006
Available online 17 August 2006

Abstract

The internal model control (IMC) design method for stable plants has enjoyed much popularity due to its simple yet effective procedure. However, there are some shortcomings and inadequacies with the IMC design method for both stable and unstable plants which will be exposed in this paper. We propose a new $\mathscr{H}_\infty$ control design method that somehow captures the useful properties of the IMC design method but can be used for both stable or unstable plants (LTI, SISO) in a coherent framework. Moreover, some restrictive assumptions on the plant are relaxed and some of the IMC shortcomings and limitations are tackled.

Keywords: Robust control; $\mathscr{H}_\infty$ loop-shaping; Internal model control; Adaptive control

1. Introduction

Parameterization of all stabilizing controllers of a plant in terms of a parameter which belongs to the set of all stable proper transfer functions often results in a design closed-loop transfer function of interest which is affine in the parameterizing transfer function. This philosophy permeates much thinking in the $\mathscr{H}_\infty$ design. However, the idea of basing a design on this observation can probably be traced back to the late 50s (Newton, Gould, & Kaiser, 1957). The internal model control (IMC) design method uses the observation in a specialized way with the aim of introducing a simple and effective design technique for robust feedback controllers. The IMC design method for stable plants with no jω-axis zeros allows in principle the design of a controller to achieve a closed-loop form equal in magnitude to a desired transfer function (Morari & Zafiriou, 1989). The term IMC is used because, as defined in Morari and Zafiriou (1989) and Section 2, the controller can be viewed as a combination of two elements, one being a model of the plant.

In this paper, we propose a new controller design method which is based on an $\mathscr{H}_\infty$ controller approach, and which avoids limitations of the IMC design method when these are present, but retains the desirable features and much of the simplicity of the IMC design method when these shortcomings are absent. Most of the limitations are discussed in Section 3 and addressed in Section 4.

For *stable* plants, the IMC design method has received much attention over the past decade. For example, design methods for tuning PID controllers using the IMC structure are proposed in Gorez (2003) and Morari and Zafiriou (1989), a design procedure for disk drive servomechanism using the IMC structure is presented in Lee, Low, Al-Mamun, and Tan (1995), a modified IMC structure to deal with unstable processes with time delays is given in Tan, Marquez, and Chen (2003), and a linear matrix inequality (LMI) IMC-based strategy to design and tune a robust tunable controller is proposed in Boulet, Duan, and Michalska (2003). Other advantages of the IMC design method for stable plants are also exploited in the area of adaptive robust control (Anderson, 2002; Dehghani, Lanzon, & Anderson, 2004; Lee, Anderson, Mareels, & Kosut, 1995) where the IMC filter (Section 2.1) *with a single design parameter*, *which sets the bandwidth of the designed closed-loop system*, is utilized to progressively open up the closed-loop bandwidth.

☆ This paper was not presented at any IFAC meeting. This paper was recommended for publication in revised form by Associate Editor Bernard Brogliato under the direction of Editor Miroslav Krstic.
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0005-1098/$ - see front matter © 2006 Elsevier Ltd. All rights reserved.
doi:10.1016/j.automatica.2006.06.007

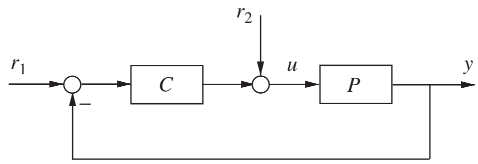

Fig. 1. Standard feedback configuration.

For *unstable* plants, however, the IMC design method requires substantial modification. Also, the abovementioned single design parameter cannot be interpreted as the closed-loop bandwidth (Campi, Lee, & Anderson, 1994). A two-degree-of-freedom approach is proposed for the IMC filter in Campi et al. (1994) which offers advantages to the standard IMC filter (Section 2.2); e.g. flatter frequency response and better stability robustness, in the sense described in Campi et al. (1994), and little overshoot in the step response. Nonetheless, such solutions are very application specific (e.g. excluding unstable plants with unstable zeros) and require additional parameter tuning to trade-off the magnitude of the overshoot and the settling time in the step response. Lee, Mareels, and Anderson (2001) proposes an alternative design method in which the use of IMC for designing the underlying control law in the application of iterative identification and control design to unstable plants is extended and a two-step control design procedure for unstable plants is used. First, an inner-loop controller is designed to stabilize the open-loop unstable plant, and then the standard IMC design method for stable plants is applied to deal with the resulting stabilized internal closed-loop transfer function.

Even for stable plants, the IMC design method may yield an unsolvable design; e.g. pole–zero cancellation may occur very close to the jω-axis, and the design method cannot handle plants with jω-axis zeros. In fact, a series of limitations and shortcomings are pointed out in Section 3. The IMC design method deals specifically with the transfer function from input r_1 to output y, T_{yr_1}, in Fig. 1 and only this quantity in terms of setting its frequency response (see Section 2.1). The IMC design method only ensures that the other transfer functions in the closed-loop mapping of Fig. 1, i.e. $[\begin{smallmatrix} r_1 \\ r_2 \end{smallmatrix}] \mapsto [\begin{smallmatrix} y \\ u \end{smallmatrix}]$, are stable but does not explicitly handle their size. However, the other three transfer functions do relate to certain input–output properties of a feedback loop as detailed in Section 3, and the IMC design method may fail to ensure that their values are acceptable.

There are some similarities and some distinct differences between our method and $\mathscr{H}_\infty$ loop-shaping. The widely accepted $\mathscr{H}_\infty$ loop-shaping ideas of McFarlane and Glover (1992) offer a design technique which is capable of addressing the problem of ensuring that the transfer functions in the feedback loop of Fig. 1 do not take large values (Papageorgiou & Glover, 1999). However, our proposed $\mathscr{H}_\infty$ design method of Section 4 sets up a model referencing design on T_{yr_1} (similar to the IMC), and offers analytical links between closed-loop performance objectives and the design of frequency cost functions (Section 4), but also ensures that the other transfer functions in Fig. 1 remain below a certain value.

Let us sketch an outline of this paper. Section 2 briefly reviews the IMC design method for stable and unstable plants, with limitations and open issues detailed in Section 3. Then in Section 4, we introduce the new systematic $\mathscr{H}_\infty$ control design method, proposed in this paper, that captures all the desirable features of IMC, but also addresses limitations/inadequacies of the existing IMC design methods. Section 4.3 outlines the proposed design procedure in a step-by-step fashion. The versatility of the proposed $\mathscr{H}_\infty$ design technique is illustrated with two examples in Section 5. We conclude this paper by emphasizing the highlights of the new design method.

2. An overview of IMC

Consider the unity feedback, LTI and SISO system in Fig. 1. The closed-loop mapping is given by

$$\begin{bmatrix} y \\ u \end{bmatrix} = \begin{bmatrix} \dfrac{PC}{1+PC} & \dfrac{P}{1+PC} \\ \dfrac{C}{1+PC} & \dfrac{1}{1+PC} \end{bmatrix} \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = H(P,C) \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} \tag{1}$$

and achieving closed-loop stability is equivalent to $H(P,C) \in \mathscr{RH}_\infty$; see Zhou, Doyle, and Glover (1996). We shall explore this relationship and the requirements of internal stability for the IMC design method for the open-loop stable/unstable plant case in the following subsections. Before that, let us briefly discuss the notations used in this manuscript.

We shall denote by $\mathbb{C}$ the field of complex numbers, $\mathbb{C}_+$ open right-half complex plane, $\Re[s]$ real part of $s \in \mathbb{C}$, $\mathscr{H}_\infty$ the space of functions bounded and analytic in $\mathbb{C}_+$, and the same function spaces with prefix $\mathscr{R}$ denoting their real-rational subspaces. Let the singular values of a matrix $A \in \mathbb{C}^{n\times m}$ be denoted by $\sigma_i(A)$ with the largest singular value $\bar{\sigma}(A)$. The transpose of A is written as A^{T}, and $G^{\sim}(s) = G^{\mathrm{T}}(-s)$ denotes the adjoint of system $G(s)$. Furthermore, let $G \in \mathbb{C}^{(p_1+p_2)\times(q_1+q_2)}$ be partitioned as $[\begin{smallmatrix} G_{11} & G_{21} \\ G_{12} & G_{22} \end{smallmatrix}]$ with $G_{11} \in \mathbb{C}^{p_1\times q_1}$ and $G_{22} \in \mathbb{C}^{p_2\times q_2}$. Then, given $Q_l \in \mathbb{C}^{q_2\times p_2}$, the lower linear fractional transformation (LFT) is defined by $\mathscr{F}_l(G, Q_l) := G_{11} + G_{12}Q_l(I - G_{22}Q_l)^{-1}G_{21}$ if the required inverse exists. The transfer functions of the plant and the controller are denoted by P and C, respectively.

2.1. Review of IMC design method for stable plants

The standard IMC design procedure was presented in Morari and Zafiriou (1989) and is outlined here for ease of reference. Following Morari and Zafiriou (1989), stable plants, which are simpler, are treated separately from unstable plants. Recall the feedback system in Fig. 1 and suppose that the plant is stable, $P \in \mathscr{RH}_\infty$; then the following choice of controller

$$C = Q(1 - PQ)^{-1} \tag{2}$$

will result in

$$H(P,C) = \begin{bmatrix} PQ & P(1-PQ) \\ Q & 1-PQ \end{bmatrix} \tag{3}$$

and internal stability requirements are satisfied if and only if $Q \in \mathcal{RH}_\infty$. Thus, stabilizing controllers are parameterized in terms of the set of all stable proper transfer functions (Zames, 1981). We can now think of C as in Eq. (2) and then focus on determining C via the proper choice of Q, which will be discussed in the sequel.[1] Note that in terms of Youla–Kucera parameterization (Kucera, 1979; Youla, Bongiorno, & Jabr, 1976) ideas, Eq. (2) is a formula for all stabilizing controllers (Doyle, Francis, & Tannenbaum, 1992, Chapter 5, Theorem 2) given stability of P.

Consider now a stable model of the plant P with no jω-axis zero.[2] One decomposes P into an inner–outer factorization of the form

$$P = P_a P_m \tag{4}$$

with P_a stable all-pass with all zeros in the right half-plane, $P_a \in \mathcal{RH}_\infty$, $P_a^\sim P_a = I$, and P_m stable minimum-phase (all zeros in $\Re[s] < 0$).

The goal of the IMC procedure broadly is to obtain a closed-loop transfer function with a desired magnitude; more precisely, one chooses an "IMC filter transfer function" $F(s)$ such that with an appropriate controller, the closed-loop transfer function is

$$T_{yr_1} = PC(1+PC)^{-1} = P_a F. \tag{5}$$

The Q-parameter defining the controller in (2) which achieves (5) is easily found to be

$$Q = P_m^{-1} F \tag{6}$$

and $|T_{yr_1}| = |F|$. Evidently $Q \in \mathcal{RH}_\infty$ provided that $F \in \mathcal{RH}_\infty$ and the relative degree of F is at least equal to the relative degree of P_m. A common choice for F is

$$F = \left(\frac{\lambda}{s+\lambda}\right)^n \tag{7}$$

for some λ which specifies the bandwidth of T_{yr_1}, and some positive integer n which should be as great as the relative degree of the plant.

The above discussions confirm the simple and efficient structure of the design method for stable plants. Nevertheless, even for stable plants the method has limitations, depends on restrictive assumptions and gives rise to certain open problems that are described in Section 3. The IMC design for unstable plants is discussed below.

2.2. Review of IMC design method for unstable plants

Open-loop unstable plants generally are more difficult to deal with than their stable peers (Doyle et al., 1992; Looze & Freudenberg, 1991) and the IMC design method is not an exception (Morari & Zafiriou, 1989). It is evident that the method of Section 2.1 will not work. Consequently, adjustment is required. Seeking a controller parameterization as in (2), one needs to develop the requirements on C that ensure internal stability. Ensuring $H(P, C) \in \mathcal{RH}_\infty$ in (3) means $Q \in \mathcal{RH}_\infty$, $PQ \in \mathcal{RH}_\infty$ and $P(1-PQ) \in \mathcal{RH}_\infty$. The last two can be interpreted as the conditions that the closed right half-plane poles of P must be cancelled both by zeros of Q and zeros of $(1-PQ)$. The correlation between these two conditions reduces the above conditions to (a) $Q \in \mathcal{RH}_\infty$, (b) $(1-PQ) = 0$ at the closed right half-plane poles of P (Morari & Zafiriou, 1989, Theorem 5.1-1). Hence, the parameterization in Eq. (2) ought to be evolved to meet these conditions on Q.

Suppose that the plant has k_p poles $p_1, p_2, \ldots, p_{k_p}$ in the open right half-plane and ℓ poles at the origin, and assume that a Q_1 exists such that $C = Q_1(1-PQ_1)^{-1}$ is a stabilizing controller. Then the parameterization of all stabilizing controllers in Eq. (2) is given by

$$Q = Q_1 + s^{2\ell} \prod_{i=1}^{k_p} \frac{(p_i^* - s)^2}{(s+p_i)^2} Q_2, \tag{8}$$

where Q_2 is any stable transfer function and p_i^* denotes the complex conjugate of p_i. It was shown in Morari and Zafiriou (1989) that such a Q_1 exists and that all Q for which $1 - PQ = 0$ at the closed right half-plane poles of P are given by Eq. (8). In fact, formula (8) is an alternative to the usual Youla parameterization.

We shall now discuss the procedure to attain a desired amplitude response for the closed-loop transfer function T_{yr_1} with the choice of Q and the notion of an IMC filter. Let $p_1, p_2, \ldots, p_k$ denote the closed right half-plane poles of P, assumed simple for convenience. We assert that any choice of $F \in \mathcal{RH}_\infty$ with relative degree of at least that of P *and additionally such that*

$$[P_a(s)F(s)]_{s=p_i} = 1 \tag{9}$$

will result, after retaining the choice of Q as in (6), in a stabilizing controller achieving (5). Evidently, we cannot expect that the choice $F = [\lambda/(s+\lambda)]^n$ will meet the requirement in (9). However, we can take

$$F(s) = (b_{k-1}s^{k-1} + \cdots + b_1 s + b_0)\left(\frac{\lambda}{s+\lambda}\right)^{n+k-1} \tag{10}$$

and choose the coefficients b_i to satisfy the requirement in (9). Thus, the closed-loop bandwidth depends on λ and on the zeros of F. Clearly, the filter parameter λ, unlike the stable case, does not directly adjust the bandwidth of the closed-loop frequency response.

Having reviewed the standard IMC design method, let us discuss some of the major limitations and/or shortcomings of the IMC design method in the sequel and pave the way for introduction of our proposed $\mathcal{H}_\infty$ method.

[1] The formula (2) helps to understand the concept of internal model. One can think of the controller as being composed itself of a feedback system with a filter Q in the forward path and a model of the true plant P in a positive feedback path. Thus, C contains an internal model of the plant.

[2] Note that the decomposition in Eq. (4) cannot be achieved if P has jω-axis zero as explained in Section 3.

3. Difficulties with the IMC approach

Let us revisit Eq. (1) and note that the reciprocal of the size of $\|H(P,C)\|_\infty$ is referred to as the generalized robust stability margin (Vinnicombe, 2000) and it corresponds to the amount of (coprime factor) uncertainty that can perturb P without destabilizing the loop (Zhou et al., 1996). Thus, we wish $\|H(P,C)\|_\infty$ to be small for a robust design in this sense.

Furthermore, the (1,1) entry of $H(P,C)$, the complementary sensitivity T, is clearly important for reference tracking design and the IMC design method deals specifically with this and only this quantity in terms of setting its frequency response (see Eq. (5)). The IMC design method only ensures stability T_{yr_2}, T_{ur_1} and T_{ur_2} in $H(P,C)$ but does not explicitly handle their values. The (1,2) entry, T_{yr_2}, must be maintained below a certain value for a sensible design as it is desirable for plant input disturbances to be attenuated at the plant output. The same holds for T_{ur_1} for the purpose of avoiding control actuator saturation and high-energy control action. In a SISO setting, the (2,2) entry, the sensitivity function S is directly related to T_{yr_1} as $S+T=1$.

With this introduction in mind, let us now record the circumstances where the IMC design method detailed in Sections 2.1 and 2.2 cannot be properly used, or its applicability is limited by restrictive assumptions, or it fails to provide a good design. The first four circumstances define situations which will cause difficulties in many design approaches, not just IMC.

(a) If the model has lightly damped stable poles within the closed-loop passband, then $P(1+PC)^{-1}$ will have large gain near the frequencies of those poles. This can result in poor design as large T_{yr_2} at some frequency means large $\bar{\sigma}[H(P,C)]$.
(b) If the model has lightly damped stable/unstable zeros within the closed-loop passband, then either the closed-loop gain will be very small near those zeros or $C(1+PC)^{-1}$ will have large gain near the frequencies of those zeros. Either poor tracking will occur or $|T_{ur_1}|$ will be large near the frequencies in question, resulting in large $\bar{\sigma}[H(P,C)]$.
(c) If the bandwidth of F is chosen to be much larger than the bandwidth of P, then $|C|$ will be very large at frequencies inside the bandwidth of F and outside the bandwidth of P, and again the control signal will be greatly amplified at these frequencies.
(d) If the roll-off rate of F is desired to be less than the roll-off rate of P (or equivalently P_m), then the IMC design method will result in an improper controller, C, since Q becomes improper.
(e) If P has zeros on the jω-axis, then the simple decomposition in (4) is not possible. If the conditions on P_m are relaxed to allow it to contain these zeros, then the standard IMC procedure breaks as Q will not be stable and again internal stability of the loop in Fig. 1 will be lost.
(f) If P is unstable, there are restrictive assumptions to be satisfied(Section 2.2) and the simple low-pass filter F in (7) is replaced with a complicated one in (10). Moreover, there are other disadvantages with this filter discussed in detail in Campi et al. (1994). Noticeably, the desired feature of the filter in (7) for an open-loop stable plant (i.e. online tuning of λ adjusts the closed-loop bandwidth) does not hold with the filter in (10); and the closed-loop step response may exhibit large overshoot due to the large peak in the filter frequency response. The search for finding new filters and/or alleviating shortcomings has resulted in much complicated procedures (see e.g. Campi et al., 1994; Lee et al., 2001).

Notice that the IMC design method described in Section 2.1 fails and cannot be used in situations (d)–(f) and the design method discussed in Section 2.2 fails and cannot be used in situations (d) and (e). The situations (a)–(d) imply that one of the entries of $H(P,C)$ in (1) will be large. In the following section, we shall introduce a new controller design method that inherits the useful desired features of the IMC design method, but explicitly addresses the above-stated problems.

4. The proposed $\mathscr{H}_\infty$ control design method

One starting point is that the IMC design method offers features that are desired to capitalize on for either stable or unstable plants (but at the same time fails to take precautions in dealing with certain plants to ensure that designs are adequate). These special attractions include having accurate control on complementary sensitivity and preserving much of the simplicity of the IMC design method. Besides, one needs to ensure that $\|H(P,C)\|_\infty$ is small for a sensible design. Put another way, accomplishment of two performance objectives is desired:

- to have the actual closed-loop transfer function $PC(1+PC)^{-1}$ close to a desired transfer function P_aF (similar to the IMC) even with P unstable or perhaps possessing a jω-axis zero; and
- to make sure that the other three transfer functions in (1) do not take large magnitudes.

Thus, the need arises for reformulation of the problem in such a way to capture these objectives. To this end, an $\mathscr{H}_\infty$ index is introduced and required to be minimized over all stabilizing controllers. Moreover, one normally has performance objectives in mind, which often require some transfer functions to be small or below certain values in some frequency regions and other transfer functions to be small or below certain values at other frequencies. The $\mathscr{H}_\infty$ index will be weighted to achieve the desired effect. At the end of the day there may be a trade-off between keeping the size of $[PC(1+PC)^{-1}-P_aF]$ small and the size of the other three transfer functions in (1) below certain values. Generally, if there is not such a trade-off, IMC design probably yields a good result.

The proposed controller design method is briefly sketched here and its procedure is presented in Section 4.3:

(i) Factor the plant transfer function P as

$$P = P_a P_m : \quad \begin{cases} P_a \in \mathscr{RH}_\infty,\ P_a^\sim P_a = I, \\ P_m \text{ has no zeros in } \mathbb{C}_+ \end{cases} \tag{11}$$

implying that P_m can now contain jω-axis zeros.

(ii) The admissible controller is given by solving

$$\gamma = \min_{C\in\underline{\mathscr{C}}} \left\| \begin{matrix} \dfrac{PC}{1+PC} - P_a F & \varepsilon_2(s)\dfrac{P}{1+PC} \\ \varepsilon_1(s)\dfrac{C}{1+PC} & \varepsilon_1(s)\varepsilon_2(s)\dfrac{1}{1+PC} \end{matrix} \right\|_\infty, \tag{12}$$

where $\underline{\mathscr{C}}$ denotes the set of all proper stabilizing controllers for P and $\varepsilon_1(s)$ and $\varepsilon_2(s)$ are SISO, stable, minimum-phase and proper weights.[3]

The selection of the weighting functions $\varepsilon_1(s)$ and $\varepsilon_2(s)$ shall be explained in detail in the following subsection.

Note that the proposed design method outlined above addresses all design difficulties, (a)–(f), marked out in Section 3, especially (e) and (f) which are peculiar to the IMC design method. This design method is applicable to stable or unstable plants, plants with or without jω-axis zeros or lightly damped poles/zeros, and the filter $F(\mathrm{j}\omega)$ can have a roll-off rate larger or smaller than that of $P(\mathrm{j}\omega)$, and a bandwidth that is larger or smaller than that of $P(\mathrm{j}\omega)$. Also, note that the (1,1) term in Eq. (12) is making $PC(1+PC)^{-1}$ close to $P_a F$ (similarly to standard IMC), the (1,2) term is limiting the size of $P(1+PC)^{-1}$, and the (2,1) term is limiting the size of $C(1+PC)^{-1}$. Furthermore, the set $\underline{\mathscr{C}}$ contains stabilizing proper controllers and hence internal stability will always be achieved and the controller achieving this will always be proper.

The index in (12) can be rewritten as

$$\gamma = \min_{C\in\underline{\mathscr{C}}} \left\| \begin{bmatrix} 1 & 0 \\ 0 & \varepsilon_1 \end{bmatrix} \begin{bmatrix} \dfrac{PC}{1+PC} - P_a F & \dfrac{P}{1+PC} \\ \dfrac{C}{1+PC} & \dfrac{1}{1+PC} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & \varepsilon_2 \end{bmatrix} \right\|_\infty$$

or alternatively

$$\gamma = \min_{C\in\underline{\mathscr{C}}} \left\| \mathcal{F}_l \left(\left[\begin{array}{cc|c} -P_a F & \varepsilon_2 P & P \\ 0 & \varepsilon_1\varepsilon_2 & \varepsilon_1 \\ \hline 1 & -\varepsilon_2 P & -P \end{array} \right], \ C \right) \right\|_\infty, \tag{13}$$

where the term in the square bracket is usually referred to as the generalized plant. One can easily verify that the assumptions of a standard $\mathscr{H}_\infty$ control problem for the generalized plant in (13) are fulfilled when $\varepsilon_1(s)$ is chosen to be proper. The reader is referred to Green and Limebeer (1995) and Zhou et al. (1996) for details on the $\mathscr{H}_\infty$ control problem and related discussions.

There exist other techniques with strong links to the size of $H(P, C)$ via a specialized weighting structure. One is the $\mathscr{H}_\infty$ loop-shaping method (McFarlane & Glover, 1992; Papageorgiou & Glover, 1999). For this an algorithm that automatically designs weights has been proposed in Hu, Bohn, and Wu (2000) and Lanzon (2001, 2005). Although there seems to be a link between our proposed $\mathscr{H}_\infty$ design method and $\mathscr{H}_\infty$ loop-shaping, the frequency cost functions $\varepsilon_1(\mathrm{j}\omega)$ and $\varepsilon_2(\mathrm{j}\omega)$ are philosophically different and capture our design objectives distinctly. This will become clearer when we discuss the procedure for designing these functions next.

4.1. Design of weighting functions $\varepsilon_1(s)$ and $\varepsilon_2(s)$

Weighting functions $\varepsilon_1(s)$ and $\varepsilon_2(s)$ were introduced as a part of the $\mathscr{H}_\infty$ index in (12) to achieve the desired effect detailed below. One has different objectives in different frequency regions based on the particular application specifications and also the characteristics of P.

(i) Let α be the desired closeness between $PC(1+PC)^{-1}$ and $P_a F$ in an $\mathscr{H}_\infty$ sense. That is to require

$$\|PC(1+PC)^{-1} - P_a F\|_\infty \leqslant \alpha.$$

(ii) Let β_p^i be the maximum tolerable gain in the appropriate frequency region for the transfer function $T_{yr_2} = P(1+PC)^{-1}$. That is to require

$$\bar{\sigma}[P(1+PC)^{-1}(\mathrm{j}\omega)] \leqslant \beta_p^i \quad \forall\omega \in [\omega_1^i, \omega_2^i].$$

(iii) Let β_c^i be the maximum tolerable gain in the appropriate frequency region for the transfer function $T_{ur_1} = C(1+PC)^{-1}$. That is to require

$$\bar{\sigma}[C(1+PC)^{-1}(\mathrm{j}\omega)] \leqslant \beta_c^i \quad \forall\omega \in [\omega_3^i, \omega_4^i].$$

Now, there are one number, α, and two sets of different numbers, namely β_p^i and β_c^i, that represent closed-loop objectives. These numbers will be used to specify $\varepsilon_1(s)$ and $\varepsilon_2(s)$ as will be discussed next. Once $\varepsilon_1(s)$ and $\varepsilon_2(s)$ are specified, one just needs to check the number γ to determine whether the design was successful in achieving the objectives or not. Towards this end, note that the index in (12) certainly guarantees that

$$\bar{\sigma}[PC(1+PC)^{-1} - P_a F] \leqslant \gamma \quad \forall\omega, \tag{14}$$

$$\bar{\sigma}[P(1+PC)^{-1}(\mathrm{j}\omega)] \leqslant \frac{\gamma}{|\varepsilon_2(\mathrm{j}\omega)|} \quad \forall\omega, \tag{15}$$

$$\bar{\sigma}[C(1+PC)^{-1}(\mathrm{j}\omega)] \leqslant \frac{\gamma}{|\varepsilon_1(\mathrm{j}\omega)|} \quad \forall\omega \tag{16}$$

are achieved. Consequently, choosing

$$|\varepsilon_1(\mathrm{j}\omega)| \geqslant \alpha/\beta_c^i \quad \forall\omega \in [\omega_3^i, \omega_4^i]$$

and

$$|\varepsilon_2(\mathrm{j}\omega)| \geqslant \alpha/\beta_p^i \quad \forall\omega \in [\omega_1^i, \omega_2^i]$$

will do the trick since $\gamma \leqslant \alpha$ will mean that the three objectives in points (i)–(iii) above are satisfied.

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[3] Note that $\min_{C\in\underline{\mathscr{C}}} \|\mathscr{F}_l(.,.)\|_\infty$ rather than $\inf_{C\in\underline{\mathscr{C}}} \|\mathscr{F}_l(.,.)\|_\infty$ is used since it is needed that both γ and $C \in \underline{\mathscr{C}}$. That is, the controller C must be proper and stabilizing and must achieve γ. In the cases considered, the minimum is attained.

4.2. Four scenarios for specifying $\varepsilon_1(\mathrm{j}\omega)$ and $\varepsilon_2(\mathrm{j}\omega)$

In this section four different scenarios are considered which specify how $\varepsilon_1(\mathrm{j}\omega)$ and $\varepsilon_2(\mathrm{j}\omega)$ ought to be chosen.

Case 1. $\varepsilon_1(\mathrm{j}\omega)=0$ and $\varepsilon_2(\mathrm{j}\omega)=0$ $\forall\omega$. The $\mathscr{H}_\infty$ index specified in Eq. (12) reduces to

$$\gamma = \min_{C\in\mathscr{C}} \|PC(1+PC)^{-1} - P_{\mathrm{a}}F\|_\infty \tag{17}$$

and hence C will be exactly the IMC controller if P is stable and has no jω-axis zeros and all other assumptions of the IMC design method outlined in Sections 2.1 and 2.2 are fulfilled (i.e. $\gamma=0$ for such a case). Note though that this $\mathscr{H}_\infty$ index can be used even if the plant, P, is unstable or has jω-axis zeros, although γ may not be equal to zero in this case. Thus, $\varepsilon_1(\mathrm{j}\omega)$ and $\varepsilon_2(\mathrm{j}\omega)$ can be set to be very small in the frequency regions where the plant characteristics and the performance objectives are such that an IMC controller can perform well at those frequencies.

Case 2. $\varepsilon_1(\mathrm{j}\omega)=0$ $\forall\omega$ and $\varepsilon_2(\mathrm{j}\omega)\neq 0$. The $\mathscr{H}_\infty$ index specified in Eq. (12) reduces to

$$\gamma = \min_{C\in\mathscr{C}} \left\| \frac{PC}{1+PC} - P_{\mathrm{a}}F \quad \varepsilon_2(s)\frac{P}{1+PC} \right\|_\infty. \tag{18}$$

It is clear that, in this situation, one is trying to make $PC(1+PC)^{-1}$ close to $P_{\mathrm{a}}F$ but simultaneously seeking to limit the size of $P(1+PC)^{-1}$. It was earlier discussed, (see Section 3), that if P had for example lightly damped poles in the closed-loop bandwidth, then an IMC controller would result in a transfer function, $P(1+PC)^{-1}$, that has large gain near the frequency of the lightly damped poles which is highly undesirable in a sensible design. Consequently, choosing $|\varepsilon_2(\mathrm{j}\omega)|\geqslant\alpha/\beta_p^i$ near the frequencies of the lightly damped poles of P, as this will then limit the size of $P(1+PC)^{-1}$ and one is free to let $|\varepsilon_2(\mathrm{j}\omega)|$ become small at frequencies far away from the lightly damped poles of P. Hence $\gamma\leqslant\alpha$ will imply

$$\bar{\sigma}[P(1+PC)^{-1}(\mathrm{j}\omega) - P_{\mathrm{a}}F(\mathrm{j}\omega)]\leqslant\alpha \quad \forall\omega$$

and

$$\bar{\sigma}[P(1+PC)^{-1}(\mathrm{j}\omega)]\leqslant\beta_p^i \quad \forall\omega : |\varepsilon_2(\mathrm{j}\omega)|\geqslant\frac{\alpha}{\beta_p^i}.$$

Therefore, the closeness of $PC(1+PC)^{-1}$ to $P_{\mathrm{a}}F$ is traded-off with limiting the size of $P(1+PC)^{-1}$ at the problematic frequencies.

Case 3. $\varepsilon_1(\mathrm{j}\omega)\neq 0$ and $\varepsilon_2(\mathrm{j}\omega)=0$ $\forall\omega$. The $\mathscr{H}_\infty$ index specified in (12) reduces to

$$\gamma = \min_{C\in\mathscr{C}} \left\| \begin{matrix} PC(1+PC)^{-1} - P_{\mathrm{a}}F \\ \varepsilon_1(s)C(1+PC)^{-1} \end{matrix} \right\|_\infty. \tag{19}$$

It is clear that, in this situation, one is trying to make $PC(1+PC)^{-1}$ close to $P_{\mathrm{a}}F$ but limit the size of $C(1+PC)^{-1}$. It was earlier noted, (see Section 3), that if P had for example lightly damped zeros in the passband, then an IMC controller would result in a transfer function, $C(1+PC)^{-1}$, that has large gain near the frequency of the lightly damped zeros. It was also explained that this is highly undesirable in a sensible design. Consequently, choosing $|\varepsilon_1(\mathrm{j}\omega)|\geqslant\alpha/\beta_c^i$ near the frequencies of the lightly damped zeros of P, as this will limit the size of $C(1+PC)^{-1}$ and one is free to let $|\varepsilon_1(\mathrm{j}\omega)|$ become small at frequencies far away from the lightly damped zeros of P. Hence $\gamma\leqslant\alpha$ will imply

$$\bar{\sigma}[P(1+PC)^{-1}(\mathrm{j}\omega) - P_{\mathrm{a}}F(\mathrm{j}\omega)]\leqslant\alpha \quad \forall\omega$$

and

$$\bar{\sigma}[C(1+PC)^{-1}(\mathrm{j}\omega)]\leqslant\beta_c^i \quad \forall\omega : |\varepsilon_1(\mathrm{j}\omega)|\geqslant\alpha/\beta_c^i.$$

Therefore, the closeness of $PC(1+PC)^{-1}$ to $P_{\mathrm{a}}F$ is traded off with limiting the size of $C(1+PC)^{-1}$ at the problematic frequencies.

Case 4. $\varepsilon_1(\mathrm{j}\omega)\neq 0$ and $\varepsilon_2(\mathrm{j}\omega)\neq 0$. Here one trades off the closeness requirement of $PC(1+PC)^{-1}$ to $P_{\mathrm{a}}F$ with limiting the size of both $P(1+PC)^{-1}$ and $C(1+PC)^{-1}$ at the appropriate frequencies. Again, $\varepsilon_1(\mathrm{j}\omega)$ and $\varepsilon_2(\mathrm{j}\omega)$ are specified separately such that $|\varepsilon_1(\mathrm{j}\omega)|\geqslant\alpha/\beta_c^i$ and $|\varepsilon_2(\mathrm{j}\omega)|\geqslant\alpha/\beta_p^i$ at the appropriate frequencies.

4.3. The proposed $\mathscr{H}_\infty$ control design procedure

The proposed $\mathscr{H}_\infty$ design method is summarized below.

Step 1: Given a model of the true plant P, perform the decomposition in (11).

Step 2: Choose an appropriate filter F according to the discussions in Section 4.4.

Step 3: Find the critical frequency regions in P based on the cases, (a)–(f), discussed in Section 3. Then set α, β_p^i and β_c^i based on the desired closed-loop objectives and specifications.

Step 4: Design the frequency weights, $\varepsilon_1(s)$ and $\varepsilon_2(s)$, according to the rules given in Sections 4.1 and 4.2, using the specified values α, β_p^i and β_c^i in Step 3, for the appropriate frequency regions.

Step 5: Solve the $\mathscr{H}_\infty$ controller design problem given in (12) and obtain γ and the admissible controller C.

Step 6: If $\gamma\leqslant\alpha$, the obtained controller C achieves the desired performance objectives[4] specified in Step 3.

In Section 5 the above-stated procedure is utilized for different plants to show the effectiveness and easy-to-use features of the proposed $\mathscr{H}_\infty$ design method. However, let us first discuss in some detail the choice of filter F in Step 2 of the procedure.

4.4. The choice of filter

For the stable plant case, the low-pass filter F can have the form of Eq. (7). However, plants with closed right half-plane poles and/or zeros impose fundamental limitations on the achievable closed-loop performance and may be difficult to handle (Freudenberg & Looze, 1985; Looze & Freudenberg, 1991).

[4] Notice that for the situation where $\gamma>\alpha$ one cannot conclude anything about achieving the performance objectives.

Suppose the plant P has closed right half-plane poles at p_i and closed right half-plane zeros at z_i, in which case $T = PC(1+PC)^{-1}$ will have to satisfy the following analytic constraints

$$T(p_i) = 1 \quad \text{and} \quad T(z_i) = 0. \tag{20}$$

These constraints were already discussed in Section 2.2, but ought to be at least roughly reflected in the choice of filter F since it is desired to have T close to $P_{\mathrm{a}}F$ in an $\mathscr{H}_\infty$ sense. Note that common engineering practice suggests that λ should be selected to be greater than the frequency of all right half-plane poles p_i and smaller than the frequency of all right half-plane zeros z_i (Doyle et al., 1992; Freudenberg & Looze, 1985; Goodwin, Graebe, & Salgado, 2000) (assuming this is possible). In fact, if one can choose λ such that $p_i \ll \lambda \ll z_i$, then $\|P_{\mathrm{a}}F\|_\infty$ will be less than unity but otherwise $\|P_{\mathrm{a}}F\|_\infty$ may be large, confirming that in such a circumstance it is an intrinsically difficult design problem. If the above-discussed condition, $p_i \ll \lambda \ll z_i$, cannot be satisfied, then one is bound to choose an alternative for F. Letting F have the form

$$F(s) = (b_{k-1}s^{k-1} + \cdots + b_1 s + b_0)\left(\frac{\lambda}{s+\lambda}\right)^{n+k-1}, \tag{21}$$

where n is at least equal to the relative degree of P, the coefficients $b_0, b_1, \ldots, b_{k-1}$ can be chosen such that $[P_{\mathrm{a}}F]_{s=p_i} = 1$ and $[P_{\mathrm{a}}F]_{s=z_i} = 0$. Notice that $P_{\mathrm{a}}F$ is automatically zero at the open right half-plane zeros of P since P_{a} contains those zeros. Thus, the second condition reduces to requiring $[P_{\mathrm{a}}F]_{s=\mathrm{j}\omega_i} = 0$, where $\mathrm{j}\omega_i$ are the $\mathrm{j}\omega$-axis zeros of P, if any.

Furthermore, in a typical design one will generally wish zero steady-state error for a unit step response. This corresponds to requiring $T(0) = 1$. Hence, in summary one shall select the filter F as in Eq. (21) with coefficients $b_0, b_1, \ldots, b_{k-1}$ chosen such that $[P_{\mathrm{a}}F]_{s=p_i,0} = 1$ and $[P_{\mathrm{a}}F]_{s=\mathrm{j}\omega_i} = 0$. For these sets of constraints to be solvable, one needs to introduce as many coefficients b_j as constraints. Hence k in Eq. (21) is chosen to be equal to the number of analytic constraints.

5. Numerical examples

Two plant models are considered to illustrate the advantages and effectiveness of the proposed design method. In all examples, the controller obtained by $\mathscr{H}_\infty$ design is order-reduced using the closed-loop controller reduction method detailed in Obinata and Anderson (2001, Section 4.3).

5.1. Example 1 (Unstable plant)

The following example illustrates the applicability of the proposed $\mathscr{H}_\infty$ method for unstable plants and the difficulty with the IMC design method discussed in situation (f) in Section 3. A simple unstable plant model is considered which has the form $P = 1/(s-1)$. Following the standard IMC design procedure of Section 2.2, and choosing $\lambda = 1$ for the filter in (10), $F = (b_1 s + b_0)[1/(s+1)]^2$, and applying the interpolation constraints $[F]_{s=1,0} = 1$, will result in $F = (3s+1)/(s+1)^2$. Thus, $Q = [P]_{\mathrm{m}}^{-1}F = (s-1)(3s+1)/(s+1)^2$ and finally the controller is $C_{\mathrm{imc}} = (3s+1)/s$ and $|T_{yr_1}| = |F|$.

Let us now use the proposed $\mathscr{H}_\infty$ algorithm and compare the results. Following the algorithm outlined in Section 4, one decomposes P into $P_{\mathrm{a}} = 1$ and $P_{\mathrm{m}} = 1/(s-1)$ and sets $\varepsilon_1(\mathrm{j}\omega) = 0\ \forall\omega$ and $\varepsilon_2(\mathrm{j}\omega) = 0\ \forall\omega$. To show the flexibility and independency of the proposed $\mathscr{H}_\infty$ algorithm in the choice of filter discussed in Section 4.4, one can choose the filter to be of the form in (7). Choosing $\tilde{\lambda} = 10$ such that $\tilde{\lambda} \gg p$, where p is the unstable pole at $s = 1$, the filter of interest is $\tilde{F} = 10/(s+10)$ so that $[\tilde{F}]_{s=1,0} \approx 1$ (although not exactly equal to unity). Let us seek to: (a) have the closeness between $PC/(1+PC)$ and $P_{\mathrm{a}}\tilde{F}$ in an $\mathscr{H}_\infty$ sense below 0.1 ($\alpha = 0.1$); (b) keep T_{ur} below $\beta_c = 10$ ($\approx 20\,\mathrm{dB}$). Here it is assumed that the actuators can pump up a maximum gain of 10. Based on the rules stated in Section 4.1, the frequency cost $\varepsilon_1(\mathrm{j}\omega)$ is designed and $\varepsilon_2(\mathrm{j}\omega) = 0\ \forall\omega$. The frequency cost function $\varepsilon_1(\mathrm{j}\omega)$ is chosen to gradually reach the maximum gain of almost $-40\,\mathrm{dB}$ ($\alpha/\beta_c = \frac{0.1}{10}$) as the plant model loses its bandwidth to the controller. Solving the $\mathscr{H}_\infty$ index in Eq. (12) using the MATLAB command HINFLMI with the generalized plant constructed as in (13) with the desired filter $\tilde{F}$ will result in

$$\tilde{C}^\infty = \frac{1.099 \times 10^6 (s+18.34)(s^2+6s+9)}{(s+1.15\times 10^5)(s+17.14)(s^2+5.94s+8.85)}$$

and $\tilde{\gamma} = 0.10$. This, supported by the discussion in Section 4.1, means that the desired closed-loop objectives (α=0.1, β_c=10($\approx$ 20 dB)) have been achieved.

The closed-loop controller reduction method detailed in Obinata and Anderson (2001, Section 4.3, pp. 137–140) is employed, and it is assumed that the Hankel singular values of the graph symbol (Vinnicombe, 2000) of the controller are decreasingly ordered ($\sigma_1 > \sigma_2 > \sigma_3 > \cdots > \sigma_7$). One performs balanced realization of the normalized coprime representation and the result is truncated to retain all Hankel singular values greater than $0.01\sigma_1$. The resulting controller after truncation is

$$C^\infty = \frac{1.099\times 10^6 (s+18.41)(s+4.061)}{(s+1.16\times 10^5)(s+17.22)(s+3.992)}.$$

One may choose to use the filter in (7) based on the discussion in Section 4.4. Let us now choose the filter $F = (3s+1)/(s+1)^2$ as the desired filter transfer function and hence obtaining $C_\infty = (3s+1)/s((s/2.6\times 10^5)+1)$ and $\gamma = 8.68 \times 10^{-5}$. This together with (14) show that the controller C_∞ achieves a closed-loop frequency response very close to that of the filter F. It is also obvious that C_{imc} and C_∞ have the same structure.

5.2. Example 2 (Lightly damped poles of plant in the closed-loop passband)

This example illustrates the difficulty with the IMC approach discussed in situation (a) in Section 3. Consider a plant model with the transfer function $P = 1/(s^2 + 0.005s + 1)$ with lightly damped poles at $s = -0.0025 \pm \mathrm{j}$. Following the standard IMC design procedure of Section 2.1, let us choose $\lambda = 10\,\mathrm{rad/s}$ for

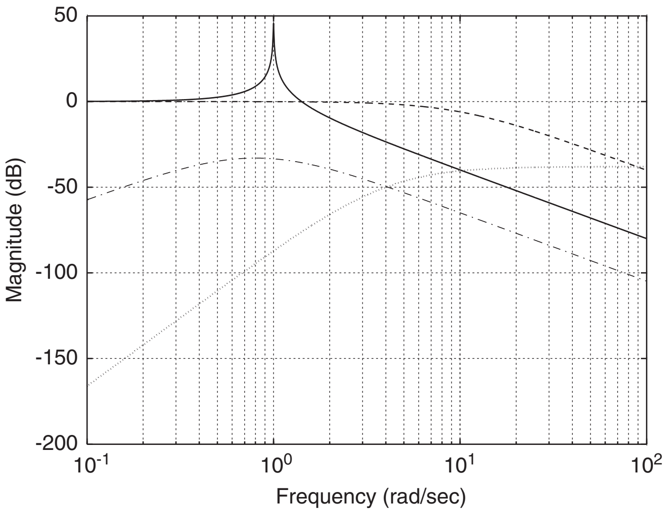

Fig. 2. Magnitude responses of P (solid), F (dashed), ε_1 (dotted), and ε_2 (dash-dotted) of Section 5.2.

the filter[5] in (7), $F=[10/(s+10)]^2$, hence $Q=P_{\mathrm{m}}^{-1}F=100(s^2+0.005s+1)/(s+10)^2$ and finally the controller $C_{\mathrm{imc}}=100s^2+0.5s+100/s(s+20)$ and $|T_{yr_1}|=|F|$.

One can easily verify that $T_{yr_2}=P/(1+PC_{\mathrm{imc}})$, has the maximum gain of $40(\simeq 32\,\mathrm{dB})$ near the frequency of the lightly damped poles (1 rad/s). This is undesirable for a sensible design as any plant input disturbances will be largely amplified in the proximity of 1 rad/s.

Following the proposed design method of Section 4 and after decomposing P into $P_{\mathrm{a}}=1$ and $P_{\mathrm{m}}=1/(s^2+0.005s+1)$, let us seek to: (a) have the closeness between $PC/(1+PC)$ and $P_{\mathrm{a}}F$ in an $\mathscr{H}_\infty$ sense below 0.15 ($\alpha=0.15$); (b) keep T_{yr_2} below $\beta_p=2(\approx 6\,\mathrm{dB})$ which is by a factor of 20 less than the gain of $P/(1+PC)$ if C_2 was used; (c) keep T_{ur} below $\beta_c=55(\approx 35\,\mathrm{dB})$. Here it is assumed that the actuators can pump up a maximum gain of 35 dB. Based on the aforementioned objectives, the frequency cost functions $\varepsilon_1(\mathrm{j}\omega)$ and $\varepsilon_2(\mathrm{j}\omega)$ are designed (Fig. 2). Notice that $\varepsilon_1(\mathrm{j}\omega)$ has to be designed since a closed-loop bandwidth is chosen which is larger than that of P.

Solving the $\mathscr{H}_\infty$ index in (12) using the MATLAB command HINFSYN with the generalized plant constructed as in (13) with given $F=[10/(s+10)]^2$ as the desired filter results in

$$\tilde{C}^{\infty}(s)=300821\frac{\prod_{i=1}^{7}(s-z_i)}{\prod_{i=1}^{8}(s-p_i)}$$

with its zeros and poles given in Table 1.

The norm $\gamma=0.149<\alpha$ which means that the desired closed-loop objectives specified above have been achieved (Section 4.1).

[5] Note that the open-loop bandwidth is around 1.5 rad/s and hence choosing a closed-loop bandwidth to exceed this, $\lambda=10\,\mathrm{rad/s}$ is with the intention to combine these two situations, (a) and (c) in Section 3, to illustrate the circumstance discussed in Case 4 in Section 4.1 where it is required to design both $\varepsilon_1(\mathrm{j}\omega)$ and $\varepsilon_2(\mathrm{j}\omega)$.

Table 1
Poles and zeros of $\tilde{C}^{\infty}$

z_i	p_i
-37.7171	-5847.104
$-0.0601\pm\mathrm{j}1.0069$	-0.408
$-0.8657\pm\mathrm{j}0.0674$	$-20.2348\pm\mathrm{j}6.5197$
$-0.7343\pm\mathrm{j}0.0623$	$-0.8385\pm\mathrm{j}0.0390$
	$-0.7615\pm\mathrm{j}0.0380$

Employing the closed-loop controller reduction according to the method discussed in Example 1, the resulting controller after truncation is

$$C^{\infty}=\frac{300821(s+37.7)(s^2+0.12s+1)}{(s+5847)(s+0.4)(s^2+40.47s+452)}.$$

The above detailed examples showed key attributes of the proposed design procedure and its easy-to-use features. The IMC design method limitations and open problems, discussed in Section 3, were put in contrast with the proposed $\mathscr{H}_\infty$ design method in practice and the adeptness of the proposed method was asserted. It has been shown that the proposed method involves one simple procedure for both stable and unstable plants while the IMC design method has two different methods with many conditions and restrictive assumptions to be satisfied (Sections 2.1 and 2.2). Furthermore, there are situations (see Section 3) for which it is possible to design an IMC controller, but the designed controller leads to poor performance, unlike in the proposed $\mathscr{H}_\infty$ criterion.

The proposed $\mathscr{H}_\infty$ design method provides assurance and reliability that the controller obtained meets the pre-specified performance specifications and objectives by simply checking the flag γ.

6. Conclusions

In this paper, we have introduced a new controller design method for SISO plants which inherits the desirable features of the IMC but extends its applicability. Sections 2.1 and 2.2 reaffirm that there exist distinct procedures for stable and unstable plants; each with its own limitations and restrictive assumptions. The proposed method uses an $\mathscr{H}_\infty$ control design method, in which the shortcomings and deficiencies of the previously used IMC design methods (Morari & Zafiriou, 1989) are addressed. Moreover, this $\mathscr{H}_\infty$ design method can be used for stable or unstable plants or plants with jω-axis zeros. In addition, it handles well plants with lightly damped poles and zeros and situations where the bandwidth of F is orders of magnitude greater than that of P. This algorithm also gives us one number, γ, that easily flags whether the desired performance specifications have been achieved. These advantageous features were illustrated in numerical examples of Section 5. An extension of this research to deal with MIMO systems is proposed in Dehghani, Lanzon, and Anderson (2005).

Acknowledgements

This work was supported by an ARC Discovery-Projects Grant (DP0342683) and National ICT Australia. National ICT Australia is funded by the Australian Government's Department of Communications, Information Technology, and the Arts and the Australian Research Council through Backing Australia's Ability and the ICT Research Centre of Excellence programs.

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