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Oscillator Design Problem

***Abstract*—In the design of a class of sine-wave oscillators using resistors, capacitors, and operational amplifiers, there is either a direct tradeoff between the maximum component value and the dynamic range of signals with satisfactory sensitivity performance, or there is a direct tradeoff between the maximum component value and the sensitivity performance with satisfactory dynamic range.**

Introduction

Examination of RC oscillator circuits suggests that if a sine-wave oscillator operates at a fixed frequency ω_0 the smaller ω_0 is, the larger the lower bound on the values of R and C elements will be.

We conduct an analysis that establishes that, in theory, a 1 rad/s oscillator could be built using operational amplifiers, a small number of resistors, no larger than 1 ohm, and a small number of capacitors, no larger than 1 pF.

However, we can show that there are restrictions that impose constraints on component size reduction. The dynamic-range requirements will be increased indefinitely or the sensitivity of parameters such as frequency of oscillation to variations in component values will increase indefinitely if all component sizes are simultaneously reduced.

Dynamic Range Problem

We restrict ourselves to oscillators comprised of resistors, 2 capacitors, and operational amplifiers. One simple oscillator is shown in Fig. 1. With x_1 and x_2 denoting the voltages on the

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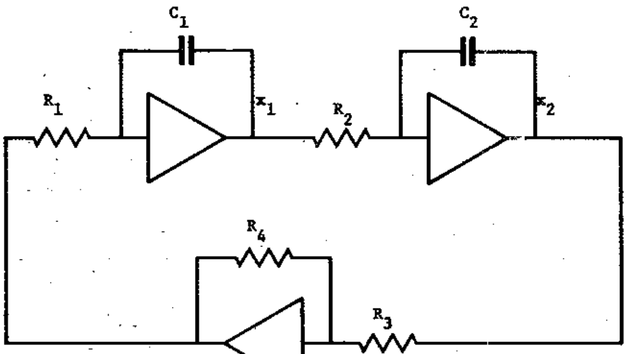

Fig. 1.

integrators, the equations describing this circuit become

$$\dot{x}_1 = -\frac{1}{R_1C_1}\left[-\frac{R_4}{R_3}\right]x_2$$

$$\dot{x}_2 = -\frac{1}{R_2C_2}\,x_1$$

or

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & \dfrac{R_4}{R_1C_1R_3}\\ -\dfrac{1}{R_2C_2} & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}. \tag{1}$$

The oscillation frequency is

$$\omega_0 = \sqrt{(R_4/R_2C_2R_1C_1R_3)}, \tag{2}$$

i.e., $j\omega_0$ is an eigenvalue of the matrix

$$\begin{bmatrix}0 & \dfrac{R_4}{R_1C_1R_3}\\ -\dfrac{1}{R_2C_2} & 0\end{bmatrix}.$$

Let us suppose that $\omega_0 = 1$ rad/s. Equation (2) alone, places no limitation on the value of R_1, R_2, etc. We require simply that

$$R_4 = R_2C_2R_1C_1R_3 \tag{3}$$

The smaller we make the component values, however, the greater will be the requirements on the dynamic range of the circuit. Suppose $x_1 = \sin t$, so that $x_2 = -(1/R_2C_2)\cos t$. As the product R_2C_2 is reduced, the amplitude of x_2 increases.

Hence the component size reduction will be limited by the dynamic range of the amplifiers.

Sensitivity Problem

In general, two integrator undriven circuits are representable by equations of the form

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}. \tag{4}$$

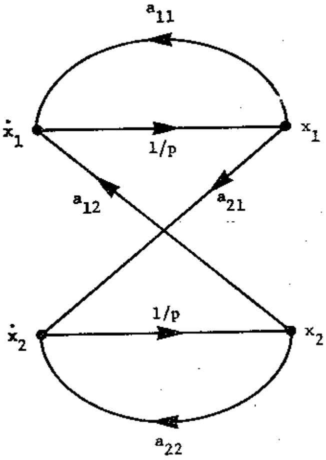

Fig. 2.

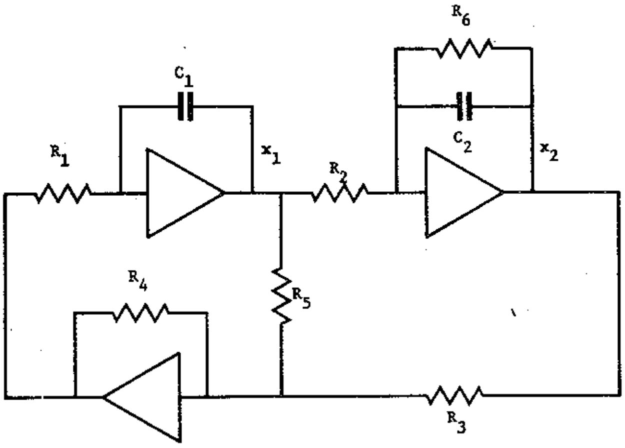

Fig. 3.

Such an equation will represent an oscillator if and only if the eigenvalues of the matrix are imaginary, for which the condition is

$$\begin{aligned} a_{11} + a_{22} &= 0\\ a_{11}a_{22} - a_{12}a_{21} &= \omega_0^2 \end{aligned} \tag{5}$$

Consider the physical circuits whose equations have the form (4). In flow-graph form, we can represent them as in Fig. 2. The circuit of Fig. 3 is described by the following equations

$$\begin{aligned} \dot{x}_1 &= \frac{R_4}{R_1C_1R_5}\,x_1 + \frac{R_4}{R_1C_1R_3}\,x_2\\ \dot{x}_2 &= -\frac{1}{R_2C_2}\,x_1 - \frac{1}{R_6C_2}\,x_2. \end{aligned} \tag{6}$$

This circuit could physically realize (4) if and only if $a_{11} > 0$, $a_{12} > 0$, $a_{21} < 0$, $a_{22} < 0$. (We could also set up other circuits and obtain expressions for the a_{ij} in terms of the values of the circuit components, which are comparable with different sign constraints on the a_{ij}.) Now consider further the circuit in

Fig. 3. For any nonnegative l and positive m, suppose that

$$a_{11} = \frac{R_4}{R_1 C_1 R_5} = l$$
$$a_{12} = \frac{R_4}{R_1 C_1 R_3} = m\sqrt{1 + l^2}. \quad (7)$$

Equations (5) then force

$$a_{21} = -\frac{1}{R_2 C_2} = -\frac{1}{m}\sqrt{1 + l^2}$$
$$a_{22} = -\frac{1}{R_6 C_2} = -l \quad (8)$$

and (4) becomes

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} l & m\sqrt{1 + l^2} \\ -\frac{1}{m}\sqrt{1 + l^2} & -l \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}. \quad (9)$$

Taking l large allows all R_i and C_i values to be small, simultaneously. This is evident from (7) and (8). (In fact, suppose m is fixed and l is very large. Then from (7), approximately a quadrupling of l is equivalent to a halving of all R and C values.)

Now consider dynamic range. If $x_1 = A \cos t$, $x_2 = m^{-1} A \cos (t + \phi)$, when $\phi = \tan^{-1} l$. So reduction in dynamic range requires keeping m close to unity, which can be consistent with ensuring small component values.

Consider the sensitivity of the oscillator performance to component changes. For example, a 1 percent change in R_6 will induce a 1 percent change in a_{22}. Aside from the fact that $a_{11} + a_{22} = 0$ will no longer be satisfied (implying physically that the circuit will cease oscillating or, more likely, will adjust the amplitude of the oscillation or the harmonic content of the waveform) consider what happens to ω_0. From (5), we see that

$$a_{11} da_{22} = 2\omega_0 d\omega_0$$

or

$$\frac{d\omega_0}{\omega_0} = \frac{a_{11} a_{22}}{2\omega_0^2} \frac{da_{22}}{a_{22}}.$$

Substituting $\omega_0 = 1$, $a_{11} = l = -a_{22}$, we see that

$$\frac{d\omega_0}{\omega_0} = -\tfrac{1}{2} l^2 \frac{da_{22}}{a_{22}},$$

which implies a percentage variation in ω_0 of $\tfrac{1}{2} l^2$.

A variation in other components may not be as critical, e.g., a 1 percent variation in C_1 induces a $\tfrac{1}{2}$ percent variation in ω_0. But the fact that the sensitivity to even one component increases according to the square of l perhaps restricts the use of very large l to achieve small component values.

Notice that the circuit considered earlier, shown in Fig. 1, corresponds to $l = 0$, and has good sensitivity peformance, but as already noted, imposes large dynamic range requirements if component size reduction is achieved.

Conclusions

Analysis of some RC operational-amplifier circuits has shown the existence of tradeoffs between component size and dynamic range requirements in the case of one oscillator circuit, and between component size and performance sensitivity in the case of another kind of circuit. One can reasonably conjecture that general reduction of component size in any oscillator circuit will generate one or both of the problems discussed. Despite the fact that unlimited reduction in component values is not possible, the fact that some reduction can be achieved will undoubtedly be advantageous in some practical situations. A comparison with standard oscillator circuits is helpful in this regard. The RC phase shift oscillator consisting of three buffered 60° phase shifts resulting from an RC pair requires $RC = \sqrt{3}$ for $\omega_0 = 1$. The Wien bridge circuit[1] has two RC pairs and $RC = 1$ for each when $\omega_0 = 1$. By contrast, we can claim here essentially that $RC = l^{-1}$ or $(1 + l)^{-1/2}$, offering a distinct improvement though at the expense of heightened sensitivity problems; alternatively reduction can be achieved at the expense of increased dynamic range.

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