PROOF OF THE MANLEY–ROWE RELATIONS FROM QUANTUM CONSIDERATIONS*

Prof. Brown's proof of the Manley–Rowe relations as given in his original letter[1] and later remarks[2] cause some misgivings.

It seems an integral part of the argument that A_{mn} be an integer, or at least rational. The quantity A_{mn}, being an *average* rate of arrival of quanta per unit time, may be regarded as the limit of a number of integral or rational quantities, i.e. the number of arrivals observed in a given time interval, divided by the length of that interval, where the limit arises from letting the length of the time interval become arbitrarily large. However, the limit of a sequence of rational numbers need by no means be rational. For example, if 3 quanta were observed in the first second, 31 in the first ten seconds, 314 in the first hundred seconds, 3142 in the first thousand seconds, and so on, it is evidently possible that $A_{mn} = \pi$.

It seems that the assumption in the usual Manley–Rowe derivations that certain frequencies must be incommensurable is not well motivated by physical considerations, and a proof that made clear the physical interpretation of the incommensurability (or otherwise) of the various frequencies involved would be most welcome.

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References

1 BROWN, J.: 'Proof of the Manley–Rowe relations from quantum considerations', *Electronics Letters*, 1965, **1**, p. 23
2 VALKO, I. P., and BROWN, J.: 'Proof of the Manley–Rowe relations from quantum considerations', *ibid.*, 1965, **1**, p. 129

Mr. Anderson's first point is covered by the modification made to the original proof in reply to Prof. Valko. Since the modification does not appear to have been clearly stated, it is repeated here. In the original proof, A_{mn} was defined as the number of quanta per second associated with the input power P_{mn} supplied at frequency $mf_1 + nf_2$. There is no particular reason why the period of one second should be regarded as fundamental, and the revised proof may be expressed in terms of A_{mn}' quanta in a period of T seconds. The average power over this period is then

$$P_{mn} = \frac{hA_{mn}'(mf_1 + nf_2)}{T}$$

The validity of the proof rests on the assumption that A_{mn}' is an integer, and does not require that A_{mn}'/T be an integer. If T is allowed to tend to infinity,

$$P_{mn} = h(mf_1 + nf_2)\,\mathrm{Lt}\,(A_{mn}'/T)$$

and the proof is valid for arbitrarily small powers.

Dr. Rowe has pointed out in a private communication that the quanta will have a Poisson distribution of arrival times. This, of course, implies that there is noise associated with each single frequency. However, since the modified proof is now expressed in terms of average powers evaluated over an indefinitely long time interval, the results remain valid for the frequencies considered.

Mr. Anderson's second point regarding the incommensurability of the frequencies does appear physically plausible. The simplest case involves two independent oscillators of frequencies f_1 and f_2. It is well known that the density of irrational numbers is infinitely greater than the density of rational numbers, and hence it is virtually certain that the ratio f_2/f_1 is irrational, provided that the two oscillators are independent.

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