European Journal of Control (2002)8:181–182

European
Journal of
Control

Guest Editorial

This special issue of the *European Journal of Control* is dedicated to V.M. Popov.

The years preceding 1960 witnessed the seeds of a true paradigm shift in the mathematical approach to control. It was the birth of what would in later years be called *modern* control theory, or mathematical systems theory. The influence in this scientific revolution of the work of Soviet scientists such as Pontryagin, Yacubovich, and US scientists such as Kalman, Bellman, among others, is well-known and has been documented in many subsequent publications. Less well-known, perhaps, to the younger generation of researchers, may be the deep impact of the marvelous work of the Romanian electrical engineer V.M. Popov.

The best-known scientific result to which his name became attached is the *Popov stability criterion*. This result pertains to dynamical systems that contain one memoryless nonlinearity. It is customary to model such systems as feedback systems with a linear time-invariant dynamical system with transfer function G in the forward loop and a nonlinear element f in the feedback loop (see Fig. 1).

Popov obtained elegant sufficient conditions for the global asymptotic stability of this system. These are

1. There exists $\alpha \in \mathbb{R}$ such that

$$\text{Real}(1+\alpha s)G(s) > -\frac{1}{k} \quad \text{for } s \in \mathbb{C} \text{ with Real}(s) \geq 0, \tag{1}$$

2. $f(0) = 0$ and $0 \leq \frac{f(\sigma)}{\sigma} \leq k$ for all $0 \neq \sigma \in \mathbb{R}$.

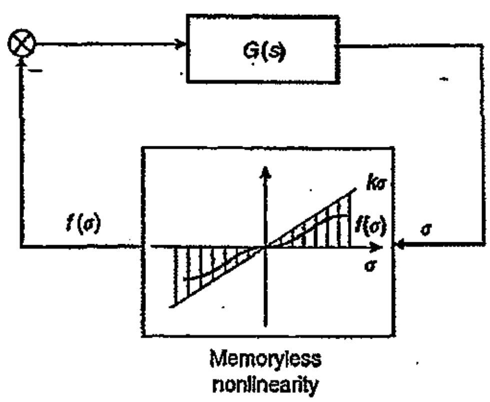

Fig. 1. A nonlinear feedback system.

This elegant criterion and its nice graphical interpretation became instantly central results and *role models* in the field. Its importance does not arise only, or mainly, from its direct applicability. Rather, the Popov criterion was the catalyst for a wealth of theoretical ideas, seminal concepts, and practical design methods that became central in the development of the field.

Popov's stability criterion explained, and greatly simplified, the nature of Lur'e's *resolving equations* for the stability of feedback systems. These equations were discovered already in 1944, and condition (1) gave an extremely simple interpretation of these resolving equation.

In a series of papers, Popov obtained proofs that are both based on integral equations and on the use of Lyapunov functions.

The proof based on integral equations marked the path that lead to the *small gain theorem*, and the positive and conic operator criteria for the stability of feedback systems. These are the ideas that lie at the basis of the input/output stability theory and, indirectly, robust $\mathscr{H}_\infty$-control.

The Lyapunov-based proof emanates from the equivalence of the positive realness of a transfer function and the existence of a positive definite symmetric matrix that satisfies certain matrix inequalities. This result became known as the Kalman–Yacubovich–Popov (KYP) lemma. It is one of the classics in the field. It became central in essentially all aspects of control, ranging from robust control, to covariance generation, to electrical circuit theory.

In addition, condition 1 is a first example of a stability result that involves a multiplier. *Multipliers* allow to exploit "excess passivity" to obtain sharper stability criteria, and to bring a feedback system into an equivalent form in which the passivity structure becomes evident. Looking for multipliers for structured linear and nonlinear feedback systems is the key to many stability results that were discovered in the sixties.

Positive realness of a transfer function

$$\mathrm{Real}G(s) \geq 0 \quad \text{for } s \in \mathbb{C} \text{ with } \mathrm{Real}(s) \geq 0, \tag{2}$$

was, at the time of Popov's writing, a condition whose importance (while very well appreciated in electrical circuit theory) was essentially absent and unknown in control. Popov's work introduced this property, both as a mathematical condition, and as a general concept, which he called *hyperstability*. Hyperstability and its generalization, *dissipativeness* are among the basic ideas that underlie systems and control, with unexpected relevance to areas as systems identification and adaptive control.

Popov's work also had a deep impact on linear system theory. He was the first one to discuss canonical forms, the equivalence of state space descriptions with polynomial matrix descriptions, the interaction of systems with quadratic forms on the state space, etc. Finally, it was in his work that the pole placement problem for multivariable systems was first discussed and solved (for linear systems over the complex field).

The breadth and depth of Popov's work is of historical importance, and lives on as a fiber touching all aspects of the field of control.

The Editors
B.D.O. Anderson
P. Kokotovic
I.D. Landau
J. Willems