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Brief Paper

Recursive identification algorithms for continuous-time nonlinear plants operating in closed loop☆

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Abstract

A family of algorithms for the identification of continuous-time nonlinear plants operating in closed loop is presented. An adjustable closed-loop output error-type predictor parameterized in terms of the existing controller and the estimated plant model is used. The algorithms are derived from stability considerations in the absence of noise and assuming that the plant model is in the model set. Some convergence results based on passivity concepts are presented. Subsequently, the algorithms are analyzed in the presence of noise and when the plant model is not in the model set.

Keywords: Closed-loop identification; Nonlinear systems; Adaptive systems, Output error; Passivity

1. Introduction

Motivated by practical needs, in the context of linear models, recursive and batch algorithms for plant model identification in closed loop have been proposed, analyzed and evaluated experimentally (de Bruyne, Anderson, Linard, & Gevers, 1999; Linard, Anderson, & de Bruyne, 1999; Landau & Karimi, 1997; Van Donkelaar & Van den Hof, 1996; Gevers, 1993; Landau, Lozano, & M'Saad, 1997). One of the successful ways to develop algorithms for identification in closed-loop is to consider "closed-loop output error" schemes (Landau & Karimi, 1997; Van Donkelaar & Van den Hof, 1996).

The problem of identification of nonlinear models in closed loop is definitely of practical importance for the same reasons as for linear models. In addition, identifying nonlinear models in continuous time makes possible the direct estimation of physical parameters which have a clear significance for the end user.

The problem of closed-loop identification of nonlinear time-varying systems in the presence of a linear or a nonlinear controller has been discussed in Dasgupta and Anderson (1996), Linard et al. (1999), and de Bruyne et al. (1999). The convergence of the algorithms is not discussed.

In the present paper we focus on the recursive identification of *nonlinear* plants operating in closed loop with a *nonlinear* controller using a closed-loop output error identification scheme. An important aspect is that we are addressing the problem of identifying nonlinear plants whose outputs cannot be expressed linearly in terms of the unknown parameters.

The paper is organized as follows. The problem setting is given in Section 2. In Section 3, the derivation of the algorithms is done in continuous time and in a deterministic noise-free environment assuming that the plant is in the model set for a particular value of the unknown parameter vector. A stability analysis in this context is provided. The case when the plant model is perturbed by noise and possibly not in the model set as well as the case when the high-order terms in the Taylor series expansions cannot be neglected is discussed in Section 4. The concept of strong strict passivity and related properties are used extensively in this paper and outlined in Appendix A.

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2. The basic equations and problem setting

The objective is to estimate the parameters of a single-input–single-output (SISO) nonlinear time-invariant system described by

$$S:\quad y = P_0(u, v), \tag{2.1}$$

where P_0 is an unknown causal nonlinear operator, u is the control input signal, y is the achieved output signal and v is the disturbance signal allowed to enter the system nonlinearly. It is not assumed that the output y can be expressed linearly in terms of some parameter vector θ_0. For ease of notation the time argument will be omitted when there are no ambiguities.

The plant is operated in closed loop with a known nonlinear controller, i.e.

$$C:\quad u = -C(y, r), \tag{2.2}$$

where r is an external reference which is assumed to be quasi-stationary and uncorrelated with v. The controller C is a causal nonlinear operator of both r and y.

It is required that the closed-loop system is bounded input–bounded output (BIBO) stable. In the sequel, we often make use of linearizations of some nonlinear operators around their operating trajectories. We therefore require that the plant, the model (to be defined subsequently), the controller and all closed-loop operators are smooth functions of the reference signal, the input signal, the output signal and the disturbance signal. This means that if the closed-loop operator is linearized around any (stable) trajectory, the resulting linear (time-varying) system is BIBO stable. See Desoer and Vidyasagar (1975) for more details.

We consider the following adjustable model for the closed-loop system defined by (2.1) and (2.2) (closed-loop output error predictor):

$$y(\theta) = P(\theta, u(\theta)), \tag{2.3}$$

$$u(\theta) = -C(y(\theta), r), \tag{2.4}$$

where $P(\theta, u)$ defines the adjustable plant model, $y(\theta)$ is the output of the closed-loop predictor and $u(\theta)$ is the plant model input.

The closed-loop output error is defined as

$$\varepsilon_{CL} = y - y(\theta). \tag{2.5}$$

The following assumptions will be made till further notice:

(i) $\exists \theta_0$ such that $P(\theta_0, u) = P_0(u, 0)$ for all $u \in \mathscr{L}_{2e}$ and $v \equiv 0$
(subsequently in the case $v \equiv 0$ the argument v will be deleted).

(ii) *Notation*: The operator $\partial P_u(\theta, u)$ is the linearization of $P(\theta, u)$ in response to a perturbation in u along the input trajectory u. The operator $\partial C_y(r, y)$ is the linearization of $C(y, r)$ in response to a perturbation in y along the trajectories produced r and y.
It is assumed that $\partial P_u(\theta, u)$ and $\partial C_y(r, y)$ exist for all allowable u, y and r. They are linear time-varying operators along the trajectories of the closed-loop system.

(iii) *Notation*: The partial derivative of $P(\theta, u)$ with respect to θ_j is denoted by $P'_{\theta_j}(\theta, u)$ for $j = 1, \dots, d$ where d is the dimension of the parameter vector θ.
The operator $P'_{\theta_j}(\theta, u)$ and its time derivatives exist and are norm-bounded $\forall j$ along the trajectories of the closed-loop predictor which requires $\dot{r}$ to be bounded. This assumption is not particularly restrictive as P and $P(\theta)$ are assumed to be smooth operators.

(iv) Let us define the operator

$$P_{CL}(\theta) = [I + \partial P_u(\theta, u(\theta)) \partial C_y(r, y(\theta))]. \tag{2.6}$$

It is assumed that $P_{CL} = P_{CL}(\theta_0)$ and its inverse P_{CL}^{-1} exist along every trajectory of the closed-loop system encountered during the identification process. Both operators are BIBO linear time-varying operators.

(v) The reference r and the stochastic disturbance v are independent.

Assumption (i) means that at least for $\theta = \theta_0$ and in the absence of noise, the plant is in the model set. (The case when this is not true will be discussed separately in Section 4.)

The generic parameter adaptation algorithm (PAA) which will be used throughout the paper is the continuous-time version of the general PAA used in Landau et al. (1997):

$$\dot{\theta}(t) = F(t)\phi(t)\varepsilon_{CL}(t), \tag{2.7}$$

$$\dot{F}^{-1}(t) = -[1 - \lambda_1(t)]F^{-1}(t) + \lambda_2(t)\phi(t)\phi^{T}(t),$$

$$0 < \lambda_1(t) \le 1,\quad 0 \le \lambda_2(t) < 2,$$

$$F(0) > 0,\quad F^{-1}(t) > \alpha F^{-1}(0),\quad 0 < \alpha < \infty, \tag{2.8}$$

where $\theta(t)$ is the estimated parameter vector, $\varepsilon_{CL}(t)$ is the closed-loop output error, $\phi(t)$ is the observation vector, $F(t)$ is the adaptation gain matrix, $\lambda_1(t)$ is a time-varying forgetting factor and $\lambda_2(t)$ allows one to weight the rate of decrease of the adaptation gain. The two functions $\lambda_1(t)$ and $\lambda_2(t)$ allow one to have different laws of evolution of the adaptation gain. See Landau et al. (1997) for details. We will consider subsequently that assumptions (i)–(iv) are valid and furthermore, for some analysis, that:

(vi) $v \equiv 0$.

(vii) The higher-order terms in the Taylor series involving expansions in powers of $(u - u(\theta))$, $(y - y(\theta))$ and $(\theta_0 - \theta)$ along the trajectories of the system can be neglected.

3. Nonlinear closed-loop output error algorithms

In this section, we present the derivations of the algorithm and we provide a stability analysis in a deterministic environment assuming that the system can be modeled exactly and that one can neglect terms of power higher than one in certain Taylor series expansions.

One has the following result (the NL-CLOE algorithm):

Theorem 3.1. *Under assumptions* (i)–(iv), (vi) *and* (vii) *one has for*

$$\phi(t) = [P'(\theta, u(\theta))]^{\mathrm{T}} = [P'_{\theta_1}(\theta, u(\theta)) \quad \cdots \quad P'_{\theta_d}(\theta, u(\theta))]^{\mathrm{T}} \tag{3.1}$$

that

$$\lim_{t\to\infty} \varepsilon_{\mathrm{CL}}(t) = 0 \tag{3.2}$$

if the linear time-varying operator

$$H = P_{\mathrm{CL}}^{-1} - \frac{\lambda(t)}{2} I; \qquad \lambda(t) > \lambda_2(t), \quad \forall t \tag{3.3}$$

is strongly strictly passive.[1]

If furthermore P_{CL}^{-1} has a finite-dimensional description as in (A.9) *and* (A.10) *one has also*

$$\lim_{t\to\infty} \phi^{\mathrm{T}}(t)(\theta(t) - \theta_0) = 0. \tag{3.4}$$

Remark 1. (1) For the particular case when one can write

$$y(\theta) = P(\theta, u(\theta)) = \phi^{\mathrm{T}}(t)\theta,$$

where $\phi(t)$ is a vector of linear or nonlinear functions of $y(\theta)$ and $u(\theta)$ one has

$$[P'(\theta, u(\theta))]^{\mathrm{T}} = \phi(t).$$

(2) Condition (3.3) assures that the closed-loop output error goes asymptotically to zero, and that the estimated parameter vector θ, converges to a set defined as

$$\mathscr{D}_{\mathrm{c}} = \{\theta\colon \phi^{\mathrm{T}}(t)(\theta - \theta_0) = 0\}. \tag{3.5}$$

If

$$\phi^{\mathrm{T}}(t)(\theta - \theta_0) = 0 \tag{3.6}$$

has a unique solution $\theta = \theta_0$, the parameter vector will converge toward this value. In fact, this condition is a "persistence of excitation" condition for the nonlinear case.

Proof. The proof will be done in several steps. *Step* I: Establishing the expression $\varepsilon_{\mathrm{CL}} = \mathbf{f}(\theta_0 - \theta(t))$.

One has the following lemma:

Lemma 3.1. *Under assumptions* (i)–(iv), (vi) *and* (vii) *the closed-loop output error is given by*

$$\varepsilon_{\mathrm{CL}} = P_{\mathrm{CL}}^{-1} P'(\theta, u(\theta))[\theta_0 - \theta(t)]. \tag{3.7}$$

Proof. From (2.1) with $v \equiv 0$ one gets

$$y = P(\theta_0, u) = P(\theta_0, u(\theta)) + [P(\theta_0, u) - P(\theta_0, u(\theta))] \tag{3.8}$$

and using a series expansion around u while neglecting higher-order terms in $(u - u(\theta))$ one gets

$$P(\theta_0, u) - P(\theta_0, u(\theta)) = - \partial P_u(\theta_0, u),[C(y, r) - C(y(\theta), r)]. \tag{3.9}$$

On the other hand, $[C(y,r) - C(y(\theta), r)]$ can be expressed as

$$[C(y,r) - C(y(\theta), r)] = \partial C_y(r, y)(y - y(\theta)) \tag{3.10}$$

(neglecting higher-order terms in $(y - y(\theta))$) and therefore

$$P(\theta_0, u) - P(\theta_0, u(\theta)) = - \partial P_u(\theta_0, u)\partial C_y(r, y)(y - y(\theta)). \tag{3.11}$$

Using the definition of $\varepsilon_{\mathrm{CL}}$ given in (2.5), (3.11) can be re-written as

$$y = P(\theta_0, u(\theta)) - \partial P_u(\theta_0, u)\partial C_y(r, y)\varepsilon_{\mathrm{CL}}. \tag{3.12}$$

Subtract now (2.3) from (3.12) and use (2.5). One gets

$$\varepsilon_{\mathrm{CL}} = P(\theta_0, u(\theta)) - P(\theta, u(\theta)) - \partial P_u(\theta_0, u)\partial C_y(r, y)\varepsilon_{\mathrm{CL}}. \tag{3.13}$$

Using a series expansion around θ, one has

$$P(\theta_0, u(\theta)) - P(\theta, u(\theta)) = P'(\theta, u(\theta))(\theta_0 - \theta), \tag{3.14}$$

neglecting higher-order terms in $(\theta_0 - \theta)$. Here $P'(\theta, u(\theta))$ has to be read as $P'(\theta, u)|_{u=u(\theta)}$. Therefore (3.13) becomes

$$\varepsilon_{\mathrm{CL}} = P'(\theta, u(\theta))(\theta_0 - \theta) - \partial P_u(\theta_0, u)\partial C_y(r, y)\varepsilon_{\mathrm{CL}} \tag{3.15}$$

from which one obtains

$$[I + \partial P_u(\theta_0, u)\partial C_y(r, y)]\varepsilon_{\mathrm{CL}} = P'(\theta, u(\theta))(\theta_0 - \theta) \tag{3.16}$$

and (3.7) results using the definition of P_{CL} given in (2.6).

Step II (*Stability proof*): With $\phi(t)$ given by (3.1), (3.7) together with the P.A.A. given by (2.7) and (2.8) define an equivalent feedback system characterized by the following equations:

$$\varepsilon_{\mathrm{CL}} = y_1 = P_{\mathrm{CL}}^{-1}(-P'(\theta, u(\theta))\tilde{\theta}(t)) = P_{\mathrm{CL}}^{-1} u_1 = -P_{\mathrm{CL}}^{-1} y_2, \tag{3.17}$$

$$\dot{\tilde{\theta}}(t) = F(t)[P'(\theta, u(\theta))]^{\mathrm{T}}\varepsilon_{\mathrm{CL}} = F(t)[P'(\theta, u(\theta))]^{\mathrm{T}} u_2, \tag{3.18}$$

$$y_2 = P'(\theta, u(\theta))\tilde{\theta}(t), \tag{3.19}$$

[1] It is assumed here that H has the form (A.2) and (A.3). See Definition A.1 in the appendix for a definition of strong strict passivity.

where

$$\tilde{\theta}(t) = \theta(t) - \theta_0 \tag{3.20}$$

and u_j, y_j, $j = 1,2$ define the inputs and outputs of the equivalent feedforward and feedback blocks, respectively.

In the general case with $F(t)$ time varying, the feedback path is not provably passive and we have to use an extension of the passivity theorem, given in the appendix (Theorem A.1) as well as the definitions of the systems belonging to the class $L(\Lambda)$ (excess of passivity) and $N(\Gamma)$ (lack of passivity) (see Definitions A.2 and A.3).

Consider Eqs. (3.18) and (3.19) together with (3.20). Eqs. (3.18) and (3.19) correspond to a state-space representation considered in Lemma A.2 with

$$A = 0, \quad B = F(t)\phi(t), \quad C = \phi^{\mathrm{T}}(t), \quad D = 0,$$

$$x = \tilde{\theta}, \quad u = \varepsilon_{\mathrm{CL}} = u_2, \quad y = y_2 = P'(\theta, u(\theta))\tilde{\theta}(t). \tag{3.21}$$

Taking $P(t) = F^{-1}(t)$ in Lemma A.1 one gets using (2.8), (A.11)

$$[1 - \lambda_1(t)]F^{-1}(t) - \lambda_2(t)\phi(t)\phi^{\mathrm{T}}(t) = Q(t) \tag{3.22}$$

also,

$$S(t) = 0 \quad \text{and} \quad R(t) = 0. \tag{3.23}$$

Notice that the positive semidefiniteness of (A.14) is not being claimed. Now using Lemma A.2 one has

$$\begin{aligned} \int_{t_0}^{t} y_2^{\mathrm{T}} u_2 \,\mathrm{d}\tau &= \int_{t_0}^{t} P'(\theta, u(\theta))\tilde{\theta}\varepsilon_{\mathrm{CL}} \,\mathrm{d}\tau \\ &\geq \frac{1}{2}\tilde{\theta}^{\mathrm{T}}(t)F^{-1}(t)\tilde{\theta}(t) - \frac{1}{2}\tilde{\theta}^{\mathrm{T}}(t_0)F^{-1}(t_0)\tilde{\theta}(t_0) \\ &\quad - \frac{1}{2}\int_{t_0}^{t} \lambda_2(\tau)\|y_2(\tau)\|^2 \,\mathrm{d}\tau. \end{aligned} \tag{3.24}$$

Therefore, it follows from Definition A.3 that the equivalent feedback block belongs to the class $N(\Gamma)$ with $\Gamma = \lambda_2(t)$ (i.e. it falls short of being provably passive).

By hypothesis, P_{CL}^{-1} belongs to class $\Lambda(\lambda(t))$ with $\lambda(t) > \lambda_2(t)$. It now follows by a straightforward application of Theorem A.1 that $u_1 \in \mathscr{L}_2$, $x_1 \in \mathscr{L}_\infty$, $\tilde{\theta} \in \mathscr{L}_\infty$ and $\lim_{t\to\infty} x_1(t) = 0$. By hypothesis (see Assumption (iii)), $\phi(t)$ (given by (3.1)) and all its time derivative are bounded; this implies that $u_1 = -y_2 = -\phi(t)^{\mathrm{T}}\tilde{\theta} \in \mathscr{L}_\infty$. The boundedness of $u_2 = y_1$ follows from the boundedness of x_1 and u_1 and Eq. (A.10). It is now straightforward to see that $\dot{u}_1 \in \mathscr{L}_\infty$. Indeed,

$$\dot{u}_1 = -[\phi(t)^{\mathrm{T}}\dot{\tilde{\theta}} + \dot{\phi}(t)^{\mathrm{T}}\tilde{\theta}]$$

and both term on the right-hand side of the equality sign are individually in $\mathscr{L}_\infty$. By Barbalat's lemma (see (Narendra & Annaswamy, 1989, Corollary 2.9, p. 86), $u_1 \in \mathscr{L}_2$, $u_1 \in \mathscr{L}_\infty$ and $\dot{u}_1 \in \mathscr{L}_\infty$ imply that $\lim_{t\to\infty} u_1(t) = 0$. □

3.1. *Relaxation of the strong strictly passive condition*

Algorithm F_1NL-CLOE
Neglecting the swapping correction terms which anyway become negligible when one uses decreasing adaptation gains ($\lambda_2(t) > 0$, $\lim_{t\to\infty} \lambda_1(t) = 1$), (3.7) can be also written as

$$\varepsilon_{\mathrm{CL}} = P_{\mathrm{CL}}^{-1}\bar{P}_{\mathrm{CL}}(\bar{P}_{\mathrm{CL}}^{-1}P'(\theta, u(\theta))[\theta_0 - \theta(t)]), \tag{3.25}$$

where the time-varying operator $\bar{P}_{\mathrm{CL}}$ is defined by

$$\bar{P}_{\mathrm{CL}} = [I + \partial P_u(\theta, u(\theta))\partial C_y(r, y(\theta))]_{\text{for } \theta = \theta(t_0) = \text{const.}} \tag{3.26}$$

In this case, following the same procedure as for the NL-CLOE algorithm one has to choose

$$\phi(t) = \bar{P}_{\mathrm{CL}}^{-1}P'(\theta, u(\theta)). \tag{3.27}$$

In this case one filters $P'(\theta, u(\theta))$ through a linear time-varying closed-loop system which depends upon an initial estimate $\theta(t_0)$.

The corresponding strongly strictly passive condition will become

$$H = P_{\mathrm{CL}}^{-1}\bar{P}_{\mathrm{CL}} - \frac{\lambda(t)}{2}I; \qquad \lambda(t) > \lambda_2(t), \quad \forall t > t_0 \tag{3.28}$$

should be strongly strictly passive. Clearly in the vicinity of θ_0, this condition is much more likely to be satisfied, than condition (3.3) for NL-CLOE.

Algorithm AFNL-CLOE
The strongly strict passivity condition can also be relaxed by replacing the filter $\bar{P}_{\mathrm{CL}}^{-1}$ given in (3.26) by an estimate of $P_{\mathrm{CL}}(\theta)$ based on the current parameter estimate $\theta(t)$, i.e. using (2.6). This, of course, requires that at each instant $P_{\mathrm{CL}}^{-1}(\theta)$ derived by (2.6) is stable. If this is not the case, then as in the identification of linear models one uses the last stable estimated filter $P_{\mathrm{CL}}^{-1}(\theta)$.

Remark 2. We refer to de Bruyne, Anderson, and Landau (2000) for a theory based on kernel representations which allows the recursive closed-loop identification of unstable nonlinear plants.

4. Robustness analysis

It is important to analyze the robustness of the identification schemes when the plant is not in the model set, when the output is affected by a disturbance that is allowed to enter the system nonlinearly and when the higher terms in the Taylor series expansion around the nominal trajectory cannot be neglected.

The plant will be described by

$$y = P_0(u, v) + \Delta P(u, v), \tag{4.1}$$

where $P_0(u,v)$ is the "reduced" order plant, $v(t)$ is a zero mean bounded disturbance, and $\Delta P(u,v)$ is a BIBO operator that is due to the unmodeled part of the system. Note that the BIBO assumption might be unnecessarily restrictive.

The estimated model is assumed to be represented by

$$y(\theta) = P(\theta, u) \tag{4.2}$$

with the property that $P_0(u,0) = P(\theta_0, u)$.

The true input u and the estimated input $u(\theta)$ are generated by (2.2) and (2.4), respectively.

To start with, we show that the effect of the noise and the unmodeled dynamics upon the closed-loop system can be considered to be additive. Denote by

$$y = P(\theta_0, u) = P_0(u, 0), \tag{4.3}$$

$$u = -C(y, r), \tag{4.4}$$

the values of the input and output obtained for the reduced order plant in the absence of noise.

Denote by

$$\bar{y} = P_0(\bar{u}, v) + \Delta P(\bar{u}, v), \tag{4.5}$$

$$\bar{u} = -C(\bar{y}, r), \tag{4.6}$$

the values of the plant input and output, i.e. in the presence of noise and with the unmodeled dynamics.

Define

$$\bar{y} = y + y_p, \tag{4.7}$$

$$\bar{u} = u + u_p, \tag{4.8}$$

where y_p and u_p are the perturbations coming from the noise v and the unmodeled plant dynamics.

From (4.5) and (4.6) one gets taking into account (4.3) and (4.4):

$$y_p = [\partial P_u(\theta_0, u) + \partial \Delta P_u(u, 0)]u_p + [\partial P_{0_v}(u, 0) + \partial \Delta P_v(u, 0)]v + \Delta P(u, 0), \tag{4.9}$$

$$u_p = -\partial C_y(r, y) y_p. \tag{4.10}$$

Here, $\partial P_{0_v}(u,0)$ denotes the linearization of P_0 in response to a perturbation in v around the trajectory u and $v = 0$. Note that terms of order higher than one in the Taylor series expansion have been neglected; these are taken care of subsequently. Also, $\partial \Delta P_u(u,0)$ and $\partial \Delta P_v(u,0)$ denote the linearization of ΔP, respectively, in response to a perturbation in u and v around the trajectory u and $v = 0$. Combining (4.9) and (4.10) one gets

$$y_p = \tilde{P}_{CL}^{-1}[(\partial P_{0_v}(u, 0) + \partial \Delta P_v(u, 0))v + \Delta P(u, 0)], \tag{4.11}$$

where $\tilde{P}_{CL}^{-1} = [I + (\partial P_u(\theta_0, u, 0) + \partial \Delta P_u(u, 0))\partial C_y(r, y)]^{-1}$ is assumed to be a BIBO (asymptotically) stable I/O operator leading to a bounded y_p.

On the other hand, the neglected terms in the developments leading to (3.7) for the closed-loop output error and (4.11) for the perturbation term have also to be taken into account. Therefore, the equation of the closed-loop output error will take the form

$$\varepsilon_{CL} = P_{CL}^{-1}\phi(t,\theta)^T[\theta_0 - \theta(t)] + w(t), \tag{4.12}$$

where w reflects the perturbation due to the unmodeled part of the plant and the possible bounded output disturbances, i.e. y_p, and the effect of the high-order terms in all Taylor series expansions.

Remark 3.

- It can be shown following the same lines as in Landau and Karimi (1997) and using (4.12) that all the signals remain bounded under the passivity conditions (3.3) provided that $r(t)$ is bounded.
- It follows from (4.11) that $w(t)$ depends on u and y and it results that both $w(t)$ and $\phi(t,\theta)$ depend on the reference signal r. This shows that $w(t)$ and $\phi(t,\theta)$ are not independent and this causes the NL-CLOE algorithm to produce biased estimates.
- The situation is different in the linear case where a consistent estimate is obtained when the system is in the model set and the reference and noise signal are independent; see e.g. Landau and Karimi (1997). Indeed, it follows that (4.11) reduces to

  $$y_p = (I + PC_y)^{-1}v \tag{4.13}$$

  which is independent of the reference signal r. In the linear case and with the system in the model set, $w = y_p$ is therefore independent of $\phi(t,\theta)$.

5. Conclusion

The key contribution of this paper has been to show that the framework for a number of closed-loop output error identification algorithms can be pushed out from linear systems to nonlinear systems. An example of application of this approach can be found in Landau, Anderson, and de Bruyne (2000). Hence our results, not surprisingly, for the most part assume that the high-order terms can be neglected in certain Taylor series expansions, or we assume that they are at least small. Other than that, both the noisy and noiseless case are captured, as is the possibility that the true plant may not lie in the model set and that the parameters can be slowly time varying. Possible relationship with EKF and nonlinear observers deserves to be studied in the future.

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Appendix

Consider the system

$$y = Hu \tag{A.1}$$

and assume that it accepts a state space representation

$$\dot{x} = f(x, u, t), \tag{A.2}$$

$$y = h(x, t) \tag{A.3}$$

with $x \in \mathbf{R}^n$, $y \in \mathbf{R}^m$, $u \in \mathbf{R}^m$, f, h continuous in t and smooth in x. Suppose $f(0,0,t) = 0$ and $h(0,t) = 0$ for all $t \geq 0$.

Definition A.1. The system H is said to be *strongly strictly passive* if there exist a positive-definite (storage) function $V(x,t)$ which satisfies

$$\gamma_1(|x|) \leq V(x,t) \leq \gamma_2(|x|), \tag{A.4}$$

$$V(0,t) = 0, \quad \forall t \geq 0, \tag{A.5}$$

where $\gamma_1(|x|)$ and $\gamma_2(|x|)$ are class $\mathscr{K}_\infty$ functions, and there exists a positive-definite function (dissipation rate) $\psi(x) \geq \gamma_3(|x|)$; $\gamma_3(\cdot) \in \mathscr{K}_\infty$ such that

$$\int_{t_0}^{t} y^{\mathrm{T}}(\tau)u(\tau)\,\mathrm{d}\tau \geq V(x(t),t) - V(x(t_0),t_0) + \int_{t_0}^{t} \psi(x(\tau))\,\mathrm{d}\tau, \quad \forall t, t_0 \text{ with } t \geq t_0. \tag{A.6}$$

Definition A.2. A system S with input u, output y and state x (see (A.2) and (A.3)) is said to belong to the class $L(\Lambda)$ if it is strongly strictly passive and in addition the following strengthened version of (A.6) holds:

$$\int_{t_0}^{t} y^{\mathrm{T}}(\tau)u(\tau)\,\mathrm{d}\tau \geq V(x(t),t) - V(x_0,t_0) + \int_{t_0}^{t} \psi(x,\tau)\,\mathrm{d}\tau + \frac{1}{2}\int_{t_0}^{t} u^{\mathrm{T}}(\tau)^{\mathrm{T}}\Lambda(\tau)u(\tau)\,\mathrm{d}\tau, \quad \Lambda(t) > 0 \ \forall t \geq t_0. \tag{A.7}$$

Definition A.3. A system S with input u, output y and state x (see (A.2) and (A.3)) is said to belong to the class $N(\Gamma)$ if the integral of the input output product satisfies the following modified version of (A.6):

$$\int_{t_0}^{t} y^{\mathrm{T}}(\tau)u(\tau)\,\mathrm{d}\tau \geq V(x(t),t) - V(x_0,t_0) + \int_{t_0}^{t} \psi(x,\tau)\,\mathrm{d}\tau - \frac{1}{2}\int_{t_0}^{t} y^{\mathrm{T}}(\tau)\Gamma(\tau)y(\tau)\,\mathrm{d}\tau, \quad \Gamma(t) \geq 0 \ \forall t \geq t_0, \tag{A.8}$$

where V and ψ are non negative functions.

We now turn to some generalizations of the Positive Real Lemma (Anderson, 1967) to time-varying systems (Landau, 1979; Popov, 1973). Consider the linear time-varying multivariable system:

$$\dot{x} = A(t)x(t) + B(t)u, \tag{A.9}$$

$$y = C(t)x(t) + D(t)u \tag{A.10}$$

with $x \in \mathbf{R}^n$, $y \in \mathbf{R}^m$, $u \in \mathbf{R}^m$ and $A(t)$, $B(t)$, $C(t)$ and $D(t)$ continuous in t.

Lemma A.1 (Popov, 1973; Landau, 1979). *System* (A.9) and (A.10) *is passive if there exists a symmetric time-varying positive-definite matrix function $P(t)$ differentiable with respect to t, a symmetric time-varying semi-definite matrix $Q(t)$ and matrices $S(t)$ and $R(t)$ such that*

$$\dot{P}(t) + A^{\mathrm{T}}(t)P(t) + P(t)A(t) = -Q(t), \tag{A.11}$$

$$B^{\mathrm{T}}(t)P(t) - C(t) = S^{\mathrm{T}}(t), \tag{A.12}$$

$$D(t) + D^{\mathrm{T}}(t) = R(t), \tag{A.13}$$

$$\begin{bmatrix} Q(t) & S(t) \\ S^{\mathrm{T}}(t) & R(t) \end{bmatrix} \geq 0 \quad \text{for all } t \geq t_0. \tag{A.14}$$

The following lemma is trivial to prove.

Lemma A.2. *If the matrices $A(t)$, $B(t)$, $C(t)$, $D(t)$ satisfy the set of equations* (A.11)–(A.13) *for some matrices $P(t)$, $Q(t)$, $S(t)$, $R(t)$ with appropriate dimension, the integral of the input-output product can be expressed as*

$$\int_{t_0}^{t} y^{\mathrm{T}}(\tau)u(\tau)\,\mathrm{d}\tau = \frac{1}{2}x^{\mathrm{T}}(t)P(t)x(t) - \frac{1}{2}x^{\mathrm{T}}(t_0)P(t_0)x(t_0) + \frac{1}{2}\int_{t_0}^{t} [x^{\mathrm{T}}(\tau)Q(\tau)x(\tau) + 2u^{\mathrm{T}}(\tau)S(\tau)x(\tau) + u^{\mathrm{T}}(\tau)R(\tau)u(\tau)], \quad \forall t \geq t_0. \tag{A.15}$$

Theorem A.1. *Consider the feedback connection of two systems S_1 and S_2 with state-space realizations, containing state vectors x_1 and x_2, respectively. Suppose that S_1 is linear time varying and belongs to the class $L(\Lambda)$ and its storage function V_1 and dissipation rate ψ_1 are independent of x_2. Suppose that the system S_2 belongs to the class $N(\Gamma)$ and its storage function V_2 and dissipation rate ψ_2 are independent of x_1. Suppose that V_1 and V_2 are* *differentiable. Suppose that no external excitation is acting on this feedback system. Then, if*

$$\Lambda(t) - \Gamma(t) \geq \delta \quad \forall t \geq t_0 \text{ and some } \delta > 0, \tag{A.16}$$

- *the equilibrium state* $x^T = [x_1^T, x_2^T]$ *is globally uniformly stable (with* $x_1(t)$ *and* $x_2(t) \in \mathscr{L}_\infty$*),*
- *Also,*

$$\lim_{t \to \infty} x_1(t) = 0 \quad \text{and} \quad u_1 \in \mathscr{L}_2. \tag{A.17}$$

Proof. Follows the lines of Krstic, Kanellakopoulos, and Kokotovic (1995, p. 588). See also Landau et al. (1997, p. 525).

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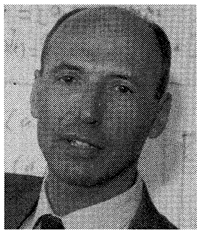

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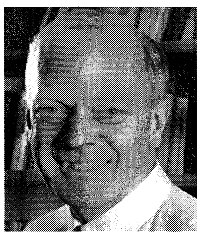

Anderson was born in Sydney, Australia in 1941. He is married and has three children. He took his undergraduate degrees in Mathematics and Electrical Engineering at Sydney University, and his doctoral degree in Electrical Engineering at Stanford University in 1966.

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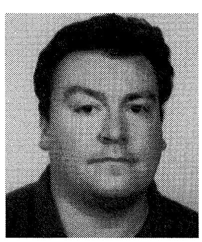
In 1998 he was elected President of the Australian Academy of Science.



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