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Iterative identification and two step control design for partially unknown unstable plants

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In this paper we shall extend the applications of iterative identification and control design to partially unknown unstable plants. We show that by employong a *two step approach*, where an unstable plant is first stabilized by a parallel feedback stabilizer, it is possible to design *systematically* an overall closed-loop system that has good step responses with little overshoots by using the iterative identification and control design methodology. Furthermore, this approach easily preserves the simplicity of an IMC design through tuning the overall designed closed-loop bandwidth with a single design parameter. Specifically, similarly to situations where the plant is stable (apart from possibly including a simple integrator), we can design a system with a small initial overall designed closed-loop bandwidth (after the plant is stabilized by a known parallel feedback stabilizer) such that high frequency unmodelled dynamics of the plant are not overly excited. Through iterative applications of a control-relevant closed-loop system identification procedure and the standard IMC design method to the stabilized plant, the overall designed closed-loop bandwidth of the system can be widened progressively while maintaining good step responses with little overshoots. Two simulation examples are employed to illustrate the method. These examples show that, irrespective of the presence of adverse unstable real pole-zero structures, the expected results are achievable by the method described.

1. Introduction

It has been shown in Lee *et al.* (1993) that the IMC method (Morari and Zafiriou 1989) is a simple and effective technique for designing the underlying control law in a new approach of iterative identification and control design when the plant is stable (apart from possibly including an integrator). The control objective is to achieve, in the face of model uncertainties, a specified (or a largest possible) closed-loop bandwidth and a closed-loop step response with little overshoot. In this connection, it is obviously desirable to have a single design parameter which can be interpreted as the designed closed-loop bandwidth. In the case of stable plants, the IMC method is found to have this desirable attribute. Specifically, the bandwidth of the designed closed-loop system is determined by a simple IMC filter with a single design parameter (see Morari and Zafiriou 1989). However, if the plant is unstable, the aforementioned single design parameter can no longer be interpreted as the designed closed-loop bandwidth. This poses a problem if the IMC method is to be used in the iterative identification and control design method when the plant is unstable. Motivated by the problems discussed above, Campi *et al.* (1994) has studied the design of a new IMC filter in situations where the plant involved has one or two unstable poles and has no unstable zeros.

Although the method proposed in Campi *et al.* (1994) results in improved step responses for the IMC filter, it requires the tuning of two parameters to achieve a specified designed closed-loop bandwidth, and the trade-off between the magnitude of the *inevitable* overshoot and the settling time of the step response. Furthermore we will show later that, if the model has unstable poles and zeros, additional unstable zeros at *undesirable* locations may be introduced into some performance determining transfer functions irrespective of whether the IMC design method of Morari and Zafiriou (1989) or Campi *et al.* (1994) is employed to design controllers *directly* for unstable plants.

It is well known (Freudenberg and Looze 1985) that open-loop unstable zeros impose a fundamental limit on closed-loop control performance. It is therefore important that control design methods do not introduce unnecessary performance-limiting open-loop unstable zeros through the controllers. Furthermore it was also reported in Lee *et al.* (1995) that unstable zeros in the designed closed-loop transfer function may hinder closed-loop system identification. Hence the above-mentioned precaution has special significance in iterative identification and control design procedures which attempt to improve the closed-loop performance over an extended frequency range.

In this paper we study *an iterative identification and two step control design* for unstable plants. The resulting closed-loop system will take the structure depicted in

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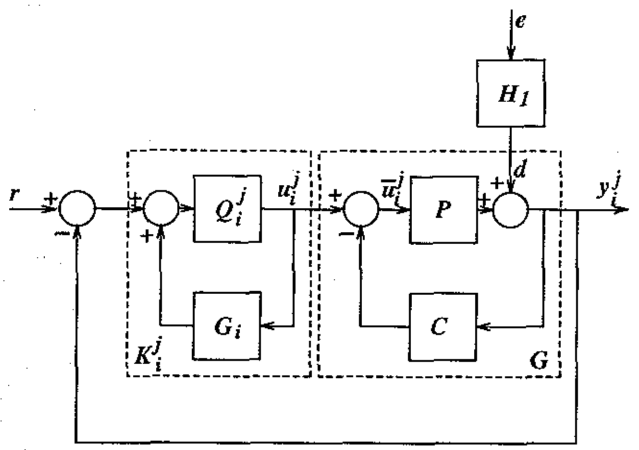

Figure 1. Closed-loop system structure for the two step approach.

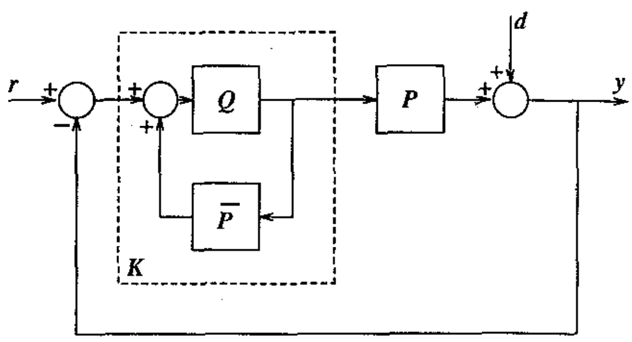

Figure 2. Internal model control structure.

figure 1, where r is the reference input and e is the unknown noise disturbance. The objective is to increase the overall closed-loop bandwidth progressively while good step responses are maintained. In the first step, the unstable plant P is stabilized by a parallel feedback stabilizer C. In the second step, we apply the iterative identification and control design procedure to the stabilized plant $G = P/(1 + CP)$. That is, a sequence of series controllers, $\{K_i^j; j = 0, 1, 2, \ldots\}$ is designed on the basis of a sequence of improving stable models $\{G_i; i = 0, 1, 2, \cdots\}$, obtained by identifying the *stabilized* plant G. Since the IMC design method is employed only in the second step, where controllers are designed on the basis of stable models for G, the iterative identification and two step control design approach completely avoids the problems that plague the one step control design approach.

The paper is structured as follows. In § 2 we highlight difficulties related to the iterative identification and one step control design approach where the IMC method is applied directly to the unstable plant. An iterative identification and two step control design approach for overcoming these difficulties is presented in § 3. For the purpose of easy reference, guidelines for designing parallel feedback stabilizers (the first step) are listed in § 3.2. Two simulation examples are presented in § 4. We conclude the paper in § 5.

2. One step control design approach for unstable plants

In § 2.1 we shall highlight the key features of the iterative identification and *one step* control design approach, where the IMC design method is employed to design controllers *directly* for unstable plants. In § 2.2 we show by simple examples that there are some problems associated with the one step control design approach.

2.1. *Outline of the one step control design approach*

In this subsection we outline the IMC methods employed by Morari and Zafiriou (1989) and Campi *et al.* (1994) to design controllers for unstable plants directly. To differentiate these from methods where the unstable plant is first stabilized by a compensator (like for example, the method to be discussed in § 3), we shall call any such method a *one step control design approach.*

In the one step approach we are concerned with the design of a series controller, K, situated within a control loop which involves a partially known (not necessarily stable) plant P. Let $\bar{P}$ be a model of the plant P. The controller K is parametrized in terms of a stable transfer function Q (as shown in figure 2) via

$$K = \frac{Q}{1 - \bar{P}Q} \tag{1}$$

The associated designed closed-loop transfer function is easily evaluated as $\bar{T} = \bar{P}Q$.

Since the plant P is only partially known in terms of its model $\bar{P}$, any practical design method must take into account the discrepancies between P and $\bar{P}$. In the IMC design method, this is achieved by specializing Q as

$$Q = \tilde{Q}F$$

such that $\tilde{Q}$ and F can be designed separately. If the *unknown but bounded* multiplicative model uncertainties are sufficiently small in the low frequency region (where the controller must have sufficiently large gain for the stabilization of $\bar{P}$), we can secure robust stability of the closed-loop system by specifying an *appropriate* bandwidth for F (which has to be smaller than the frequencies where the effect of the multiplicative unstructured model uncertainties is significant).

General results related to the design of $\tilde{Q}$ and F were given in Chapter 5 of Morari and Zafiriou (1989). The facts we need for the subsequent discussions are summarized in the next theorem. Note that the discussions will involve an unstable plant, the corresponding stabilized plant, and their respective models, it is therefore necessary to adopt the following system of notations. We use P and $\bar{P}$ to denote the unstable plant and its model; G and G_i are reserved, respectively, for the stabilized plant and its ith model.

Theorem 1: *With reference to figure 2, suppose that $\bar{P}$ has no poles on the imaginary axis, except those at the origin, and has no zeros on the imaginary axis. Let $\bar{P}$ have k poles, $p_1 \dots, p_k$, in the open right half-plane and a pole of multiplicity l at the origin. Assume that the collection of open right half-plane poles in the Laplace transform of the generalized input, $\nu(t) = r(t) - d(t)$, is a subset of $\{p_1, \dots, p_k\}$.*[1] *Denote these as $p_1, \dots, p_{k'}$, with $0 \leq k' \leq k$. Furthermore assume that $\nu(s)$ has at least l poles at the origin.*[2]

Define

$$B_{\bar{P}} = \prod_{i=1}^{k} \frac{p_i - s}{p_i^* + s} \tag{2}$$

and factor $\bar{P}$ into an all-pass factor $\bar{P}_a$ (which contains all the zeros of $\bar{P}$ in the open right half-plane) and a minimum-phase factor $\bar{P}_m$ (which includes all the poles of $\bar{P}$ in the open right half-plane and at the origin) such that

$$\bar{P}(s) = \bar{P}_m(s)\bar{P}_a(s)$$

Similarly, define

$$B_\nu = \prod_{i=1}^{k'} \frac{p_i - s}{p_i^* + s} \tag{3}$$

and factor $\nu(s)$ such that

$$\nu(s) = \nu_m(s)\nu_a(s)$$

where $\nu_m(s)$ is a minimum-phase factor (which includes all the poles of $\nu(s)$ in the open right half-plane and at the origin), and $\nu_a(s)$ is an all-pass factor (which contains all the zeros of $\nu(s)$ in the open right half-plane). Then the controller parameterization $\tilde{Q}$ which minimizes the objective

$$\int_0^\infty [r(t) - y(t)]^2 \mathrm{d}t = \frac{1}{2\pi}\int_{-\infty}^{\infty} |r(j\omega) - y(j\omega)|^2 \mathrm{d}\omega$$

is given by

$$\tilde{Q} = B_{\bar{P}}(\bar{P}_m B_\nu \nu_m)^{-1}\{(B_{\bar{P}}\bar{P}_a)^{-1} B_\nu \nu_m\}_* \tag{4}$$

where the operator $\{\cdot\}_*$ denotes that after a partial fraction expansion of the operand all terms involving the poles of $\bar{P}_a^{-1}$ are omitted.

[1] As noted in Morari and Zafiriou (1989), this assumption is necessary to make a well posed problem.

[2] As noted in Morari and Zafiriou (1989), this assumption is necessary for the closed-loop system to handle plant input disturbances whose Laplace transform may have poles at the origin.

It was shown in Campi *et al.* (1994) that, *when the model is unstable*, the standard IMC filter F (which is discussed in detail by Morari and Zafiriou 1989) will have a very large overshoot in its step response. Furthermore, the designed closed-loop bandwidth is no longer directly specified by the single design parameter of the standard IMC filter. A new method for designing IMC filters that can alleviate these difficulties was proposed by Campi *et al.* (1994). However, the new IMC filter requires an additional tuning parameter (other than the one for specifying the designed closed-loop bandwidth) to trade-off the magnitude of the inevitable overshoot and the settling time in its step response. Furthermore, the design method described in Campi *et al.* (1994) assumed that the unstable plant and its model have no unstable zeros.

We will show in Example 1 that, when the model has unstable real poles and zeros, controllers designed by the one step control design approach (using either the standard or the new IMC filter design method) may introduce additional unstable zeros (other than those of the model) into the key transfer functions of the designed closed-loop systems. However, this is still not the main problem with the one step control design approach. More strikingly, we will show in Example 2 that, even in the absence of unstable zeros in the model $\bar{P}$, closed-loop step responses resulted from the one step control design approach for unstable models can have *unacceptably large* overshoot when the designed closed-loop bandwidth is limited by the presence of high frequency unmodelled dynamics.

2.2. *Difficulties with the one step control design approach*

Example 1 (Additional unstable zeros): Consider an unstable model with the transfer function

$$\bar{P} = \frac{s-1}{(s+0.5)(s-2)}$$

According to the procedure outlined in §2.1, we can calculate, for a step reference input

$$\tilde{Q} = \bar{P}_m^{-1}\tilde{Q}_{nm}$$

where

$$\bar{P}_m = \frac{s+1}{(s+0.5)(s-2)}$$

is the minimum-phase factor of $\bar{P}$, and

$$\tilde{Q}_{nm} = \frac{7s-2}{s+2}$$

is a non-minimum-phase factor introduced by the $\{\cdot\}_*$ operation.

Note that $\tilde{Q}_{nm}$ has a gain magnitude greater than one for all non-zero frequencies and has a phase lag approaching 2π radians for high frequencies. Furthermore, irrespective of the method by which the IMC filter is designed, the unstable zeros of $\tilde{Q}_{nm}$ must appear as the unstable zero of $Q = \tilde{Q}F$, $K = Q/(1 - \bar{P}Q)$, the designed open-loop transfer function $L_O = K\bar{P}$, and the designed closed-loop transfer function $\bar{T} = F\tilde{Q}_{nm}\bar{P}_a$. This implies that, even though the IMC filter proposed in Campi *et al.* (1994) provides good step response, the designed closed-loop system may still have small stability margins and will have poor transient response if the designed closed-loop bandwidth is comparable or larger than 2/7 rad/s (corresponding to the unstable zero in $\tilde{Q}_{nm}$).

Remark 1: It was indicated in Lee *et al.* (1995) (for cases where $\tilde{Q}_{nm}$ is unity) that the undesirable phase lag of the all-pass factor $\bar{P}_a$ may hinder closed-loop system identification through increasing the magnitude of the designed sensitivity function within the designed closed-loop bandwidth. By comparing the frequency characteristics of $\tilde{Q}_{nm}$ and $\bar{P}_a$ in the above example, we observe that $\tilde{Q}_{nm}$ could have more adverse effects than $\bar{P}_a$ on closed-loop system identification.

It is important to observe that the additional unstable zero in $\tilde{Q}$ comes from the operation $\{\cdot\}_*$ when the model has a certain distribution of unstable poles and zeros. We can overcome this problem if the operation $\{\cdot\}_*$ is performed only on stable models. This prompted us to study in § 3 a two step control design approach, where the standard IMC design method is applied after an unstable plant is stabilized by a parallel feedback stabilizer.

We shall now present an example which is representative of a more practical situation than that of Example 1. This example will show that, even in benign situations where the unstable model has no unstable zeros, closed-loop step responses resulting from the one step control design approach can have *unacceptably large* overshoot when the designed closed-loop bandwidth is limited by the presence of high frequency unmodelled dynamics.

Example 2 (Excessive overshoot): Consider the model

$$\bar{P}(s) = \frac{0.1}{-s + 0.1}$$

of an unstable plant

$$P(s) = \frac{0.1(s-4)}{(s-0.1)(s^2+0.2s+4)}$$

We shall design the controller K (refer to figure 2) by the one step control design approach of Campi *et al.* (1994), which will result in an IMC filter with better characteristics than those designed by the method of Morari and Zafiriou (1989).

By Theorem 1, we can calculate, on the basis of $\bar{P}$

$$\tilde{Q} = \frac{-s + 0.1}{0.1}$$

According to Campi *et al.* (1994), in order for $Q = \tilde{Q}F$ to be strictly proper, the IMC filter should take the form

$$F(s) = \frac{\mu(s+\alpha)}{(s+\gamma)(s+\lambda)(s+10\lambda)}$$

in this case. There are two tuning parameters λ and γ in this filter. A good trade-off between overshoot and settling time of the closed-loop step response can usually be achieved for some $\gamma \in [0.02\lambda, 0.2\lambda]$. The constants μ and α are determined from the two interpolation constraints: $F(p) = 1$, where $p = 0.1$ is the unstable pole of $\bar{P}$, and $F(0) = 1$ for a step reference input.

In order to keep the overshoot in the step response of the closed-loop system to not more than 10% of the magnitude of the step input, we attempt to design F for $\lambda = 5p$ (see Middleton and Goodwin (1990) for a discussion of this choice). We have discovered that it is impossible to design K for this choice of $\lambda = 0.5$ with $\gamma \in [0.02\lambda, 0.2\lambda]$ such that P is stabilized (although $\bar{P}$ is stabilized by K). After searching over the two dimensional parameter space of λ and γ, we found that robust stabilization can be secured with $\lambda \leq 0.26$ and $\gamma \in [0.02\lambda, 0.2\lambda]$. When P is controlled by K (designed for $\lambda = 0.26$ to give minimum possible overshoot), the step repsonses are shown in figures 3 and 4. These highly oscillatory actual step responses clearly demonstrate that the closed-loop system resulting from the one step control design approach is *highly sensitive* to high frequency unmodelled dynamics even when λ is well below 0.5 (the value of λ that is expected to keep the overshoot of the designed step response to within 10%).

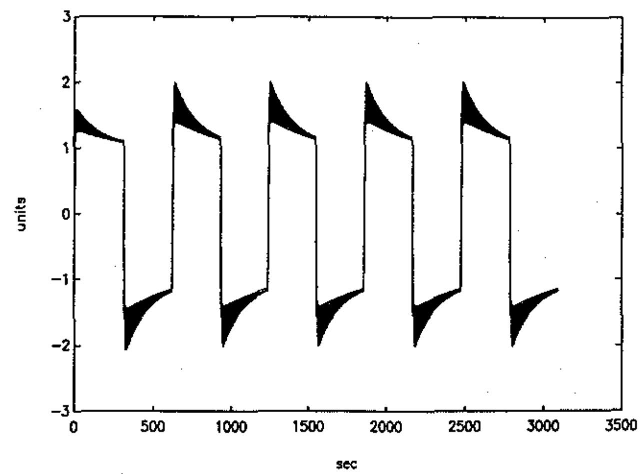

Figure 3. Actual closed-loop system response for a square wave input ($\lambda = 0.26, \gamma = 0.0052$).

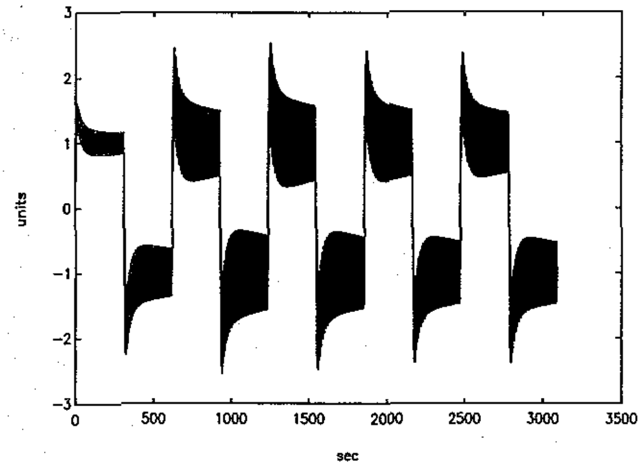

Figure 4. Actual closed-loop system response for a square wave input ($\lambda = 0.26, \gamma = 0.0312$).

By using the same plant and model, we will demonstrate in § 4 that a two step control design approach (to be described in § 3) can give better results.

3. An iterative identification and two step control design approach for unstable plants

In § 3.1 we outline an iterative identification and two step control design approach for unstable plants. Since the second step involves a direct application of the iterative identification and control design procedure described in detail in Lee *et al.* (1995), the emphasis is on developing design guidelines for the first step where the unstable plant P is stabilized by a parallel feedback stabilizer C. In § 3.2, we summarize the design guidelines for easy reference. The unstable model discussed in Example 1 will then be employed to illustrate the design procedure.

3.1. *Outline of a two step control design approach*

In this subsection we shall outline an interative identification and two step control design approach for unstable plants. The relevant system structure is depicted in figure 1.

In the first step, we shall stabilize the unstable plant P by a parallel feedback stabilizer C. Obviously we have to design C on the basis of an approximate, presumably unstable model, $\bar{P}$, of P. For this purpose, we shall assume that P and $\bar{P}$ have the same number of poles in the open right-half plane, and have no poles on the $j\omega$-axis. This assumption is consistent with the conditions required for a robust stabilizing IMC controller to exist. In the second step we employ the standard IMC design method to design a sequence of series controllers, $\{K_i^j; j = 0, 1, 2, \ldots\}$, on the basis of a sequence of identified stable models, $\{G_i; i = 0, 1, 2, \ldots\}$, for the stabilized plant $G = P/(1 + CP)$. The objective of this iterative identification and control design procedure is to achieve the desired closed-loop bandwidth and good step response. Since each of these IMC controllers is designed on the basis of a stable model, the iterative identification and two step control design approach completely avoids the problems that plague the one step control design approach.

Remark 2: The idea of a two step control design approach (namely, employing a minor loop to stabilize an unstable plant before the major loop that includes the stabilized plant is designed for achieving performance) is not new. It was suggested as early as 1957 in Newton *et al.* (1957), and more recently in Callier and Desoer (1982) and Middleton and Goodwin (1990). What is new are the introduction of a pole-placement procedure (albeit not thoroughly studied here) that takes into account the effects of high frequency modelling errors in the first design step and the application of the iterative identification and control design procedure in the second design step.

In the two step control design approach, since we do not like the outcome of the first step to cause unnecessary difficulties to the second step, we would like the parallel feedback stabilizer to have the following properties:

(1) Its introduction does not cause the level of the noise disturbance at the output of the stabilized plant G to be higher than the level of the noise disturbance at the output of the unstable plant P, so that the system identification procedure in the second step does not become more difficult.

(2) Its employment does not cause the magnitude of the multiplicative model uncertainties associated with the initial stable model G_0 to be larger than the magnitude of the multiplicative model uncertainties associated with the unstable model $\bar{P}$, so that the initial control design in the second step does not become more difficult.

We shall deal with the effects of disturbance (the first point) now. Considerations with respect to model uncertainties (the second point) will be discussed later.

With reference to figure 5, we consider the simple situation where a perfectly known unstable plant

$$P(s) = \frac{1}{s - a}, \qquad a > 0$$

is to be stabilized by the parallel output feedback controller

$$C(s) = k$$

The transfer function between the disturbance d and the closed-loop output y is given by

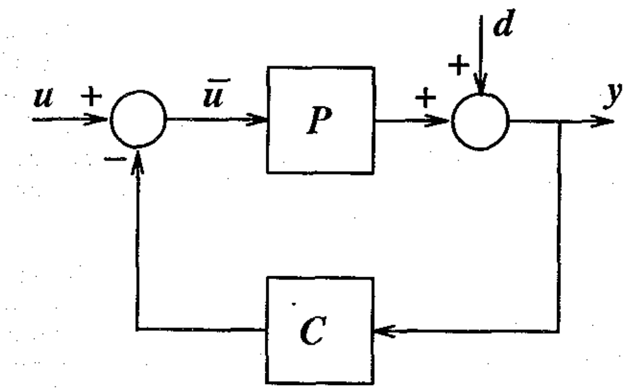

Figure 5. Parallel output feedback stabilization.

$$Z(s) = \frac{s-a}{s+k-a}$$

and the transfer function between the disturbance d and the input to the plant $\bar{u}$ is

$$L(s) = -\frac{k(s-a)}{s+k-a}$$

Apparently, $Z(s)$ and $L(s)$ are stable if and only if $k > a$.

Firstly, we consider $Z(s)$. The magnitudes of its asymptotic frequency responses for various values of k are shown in figure 6. We observe from figure 6(a) that the effect of noise disturbance at the output of the plant is amplified at low frequencies for $a < k < 2a$. When the value of k approaches a, this noise amplification effect may become very serious. The resulting poor signal-to-noise ratio at the plant output may hamper the system identification process in the second step.

Next we consider $L(s)$. The magnitudes of its asymptotic frequency responses for various values of k are shown in figure 7. We observe from figure 7(c) that the effect of noise disturbance at the input of the plant will be amplified at high frequencies for $k > 2a$. Furthermore, this noise amplification effect may become very serious when the value of k is much larger than $2a$. This may cause the actuator of the plant to saturate.

From the above discussions it appears that a reasonable trade-off is to let $k = 2a$. Observe that this will lead to a stabilized plant G with a pole at $s = -a$, which is the mirror image of the unstable pole of P at $s = a$ (about the $j\omega$-axis). Under this condition, the magnitude of the frequency response for $G = 1/(s+a)$ is the same as the magnitude of the frequency response for $P = 1/(s-a)$. In the sequel, we will use the above observation as one of the main guidelines for designing parallel feedback stabilizers. Specifically, given an unstable model $\bar{P}$, we shall design the parallel feedback stabilizer such that the

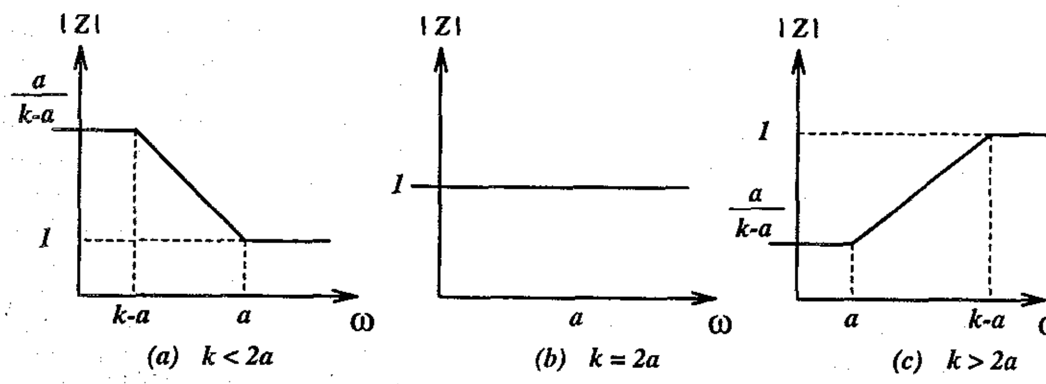

Figure 6. Magnitudes of $|Z|$ for various values of feedback gain k.

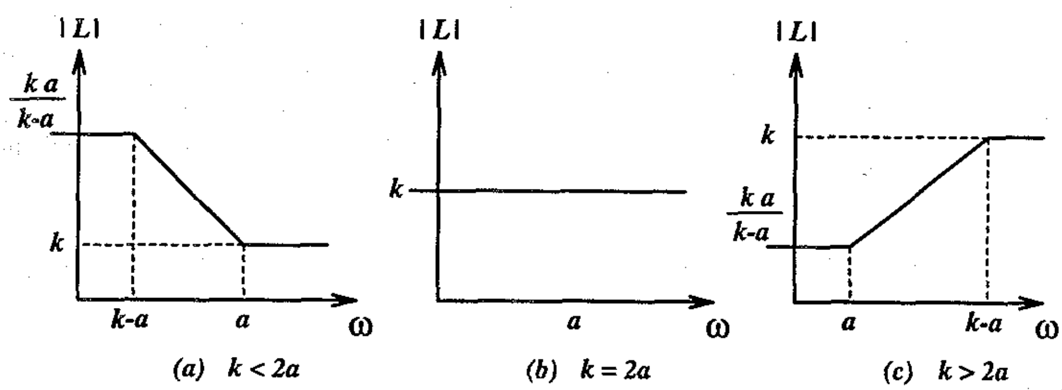

Figure 7. Magnitudes of $|L|$ for various values of feedback gain k.

model $G_0 = \bar{P}/(1 + C\bar{P})$ of the stabilized plant G retains the poles of $\bar{P}$ in the *open* left-half plane and will have a pole at each of the mirror image locations (with respect to the $j\omega$-axis) for the *open* right-half plane poles of $\bar{P}$.

We shall next estimate the multiplicative model uncertainties associated with the initial model G_0 that is induced by the multiplicative model uncertainties associated with $\bar{P}$ under the influence of the parallel feedback stabilizer. In order to secure robustness against high frequency unstructured model uncertainties when $P \neq \bar{P}$, we shall use a *strictly proper* parallel *output feedback* compensator to stabilize $\bar{P}$.

By expressing the unstable plant transfer function as $P = \bar{P}(1 + \Delta_P)$, where Δ_P denotes the multiplicative modelling error associated with $\bar{P}$, we can write the transfer function of the stabilized plant $G = P/(1 + CP)$ as

$$G = G_0(1 + \Delta_G)$$

where

$$\Delta_G = \frac{\Delta_P}{1 + C\bar{P}(1 + \Delta_P)}$$

is the multiplicative modelling error associated with G_0 induced by Δ_P under the influence of C.

Assume that the strictly proper parallel feedback stabilizer C has sufficiently large gain in the low frequency region to stabilize P and $\bar{P}$. Let Ω denote the high frequency region where Δ_P is significant and large controller gain is not necessary for securing stabilization. Observe that if C is designed such that the condition

$$\left| \frac{C(j\omega)\bar{P}(j\omega)}{1 + C(j\omega)\bar{P}(j\omega)} \Delta_P(j\omega) \right| \ll 1, \qquad \forall \omega \in \Omega$$

is satisfied through requiring

$$|C(j\omega)\bar{P}(j\omega)| \ll 1, \qquad \forall \omega \in \Omega$$

then

$$|\Delta_G(j\omega)| \approx |\Delta_P(j\omega)|, \qquad \forall \omega \in \Omega$$

Remark 3: Roughly speaking, the above discussion shows that, to prevent Δ_G from impinging upon the design of K_0^j more seriously than the manner that Δ_P has impinged upon the design of C, it is sufficient that

$$|C(j\omega)\bar{P}(j\omega)| \ll 1, \qquad \forall \omega \in \Omega$$

In other words, outside the frequency range where $|C(j\omega)|$ has to be sufficiently large for the purpose of stabilizing $\bar{P}$, we would like $|C(j\omega)|$ to roll off quickly.

We shall next outline a pole-placement technique for computing C. For details, see Kailath (1980).

Remark 4: It is important to emphasize that the following pole-placement technique will not be employed as a stand alone procedure for the synthesis of C. It will instead be integrated into a design procedure, with additional guidelines to be developed shortly.

Consider the inner loop of the system structure shown in figure 1. Assume that the model of P is given by $\bar{P} = B/A$, where A is a monic polynomial of order n and B is a polynomial of order $n - 1$. We also assume that A and B are coprime. We would like to find a parallel feedback stabilizer $C = N/D$ such that the compensated model $G_0 = (\bar{P}/(1 + C\bar{P})$ has the *monic* characteristic polynomial W. This can be achieved by finding polynomials N and D that satisfy the Diophantine equation

$$A(s)D(s) + B(s)N(s) = W(s) \qquad (5)$$

Recall that we want C to be *strictly proper*. However we do not like C to have unnecessarily high order. Let C have order m and a relative degree of one and let the polynomial D be monic, then equating the coefficients in equation (5) leads to $m + n$ linear algebraic equations in the $2m$ unknown coefficients of polynomials D and N.

Remark 5: In general, a strictly proper C could have relative degree q for $1 \leq q \leq m$. For a solution of the corresponding Diophantine equation to exist such that C has the minimum possible order, it is necessary that $m = n + q - 1$, where n is the order of the model $\bar{P}$. This will result in a closed-loop characteristic polynomial $W(s)$ with a degree $2n + q - 1$. Observe that if we increase the relative degree for C, its order has to be increased correspondingly. This means a higher order C and more degrees of freedom in specifying the poles of $W(s)$.

We shall now develop further design guidelines for C. In the second design step, since we are going to use the IMC method for plants that are stable (other than possibly having a simple pole at the origin), it is necessary that G_i and G do not have poles on the imaginary axis other than possibly having one at the origin. Furthermore, in view of the ease of applying the IMC method to design the series controller K_i^j for step reference input, we suggest that:

- If $\bar{P}$ has poles at the origin, one of these poles may be retained if so desired (for example, for the purpose of rejecting step disturbances that may enter the plant output). The remaining poles at the origin are to be assigned according to the next guideline for poles on the imaginary axis.
- Corresponding to poles of $\bar{P}$ on the imaginary axis (say at $\pm j\omega_2$; $\omega_2 \neq 0$), $-\sigma_2 \pm j\omega_2$; $\sigma_2 > 0$ are to be assigned as the poles of G_0 to achieve a desirable degree of stability (measured by σ_2) deemed

appropriate by the designer's experience for the situation at hand.

Remark 6: Note that in the last guideline, using a larger value for σ_2 has the advantage of improving the relative stability for the pole (or pair of complex-conjugate poles) in question. However, this usually comes with an increase in the controller gain. In the presence of high frequency unstructured model uncertainties, increasing the controller gain may induce destabilizing effects. Therefore it is important to caution against increasing σ_2 too aggressively.

From the point of view of designing series controllers K_i^j (in the second step), it is desirable that the bandwidth of the overall closed-loop transfer function $GK_i^j/(1+GK_i^j)$ is not constrained by unnecessary unstable open-loop zeros. Therefore C should not unnecessarily introduce these unstable zeros into G and G_0. It is easy to see from

$$G_0 = \frac{BD}{AD+BN} \tag{6}$$

that, other than inheriting all the unstable zeros of $\bar{P}$, G_0 will have additional unstable zeros at the locations where C has unstable poles. These additional unstable zeros in G_0 may impose limitations on the achievable overall closed-loop performance when series controllers are designed in the second step. Therefore it is important to design a stable C, *if possible*. It is well known (Youla *et al.* 1974, Vidyasagar 1985, Doyle *et al.* 1992) that, if the order of C is not constrained to some fixed, preselected value, then a stable stabilizing C exists if and only if the unstable real poles and the unstable real zeros of $\bar{P}$ possess the parity interlacing property. We have the following observations:

(1) In the absence of the parity interlacing property, the stabilizing compensator C is unstable.
(2) If *the order of C is not constrained*, then the unstable real poles in a C that are necessary for the stabilization of $\bar{P}$ are those which, when augmented with the unstable real poles of $\bar{P}$, satisfy the parity interlacing property. Therefore the unstable real zero introduced into G_0 (and G) by the smallest necessary unstable real pole of C is always larger than the smallest unstable real zero of $\bar{P}$ inherited by G_0. Hence, if C has unstable real poles only for satisfying the parity interlacing property, it is automatically guaranteed that no unnecessary limitations are imposed on the overall closed-loop performance by the unstable poles of C.

On the basis of the above observations, we suggest that:

- After C is designed, examine whether its unstable poles are only those that, when taken together with the unstable poles of $\bar{P}$, are necessary for satisfying the parity interlacing property.

Remark 7: Since we do not usually like the order of C to be excessively large (so that it can be implemented without too much difficulty using current technologies), we may not be able to find a reasonably low order C that has the above desirable property. In this situation, the locations of the offensive real unstable zeros introduced into G_0 by C may be used to estimate the extent to which we may push the overall closed-loop bandwidth through iterative identification and control design in the second design stage.

After we have assigned n poles for G_0 (recall that n is the order of $\bar{P}$) according to the above considerations, there are still n more poles in G_0 to be assigned. It was shown in Leon de la Barra (1992) that if the stable zeros of P are located well to the right of the dominant poles of the transfer function $CP/(1+CP)$, step-like disturbances at the input of P may cause the control signal at the actuator input of P to have large excursions. This may cause actuator saturation. Assuming that these stable zeros of P are also the zeros of $\bar{P}$, the problem can then be alleviated by cancelling these zeros by the poles of C. Summarizing, we suggest the following:

- It is desirable to cancel, via the poles of C, the *well-damped* stable zeros of $\bar{P}$ that are located within the expected overall designed closed-loop bandwidth. This can be accomplished by assigning the well-damped stable zeros of $\bar{P}$ as the zeros of the characteristic polynomial $W = AD + BN$.

Remark 8: Since attempting to cancel poorly damped (although stable) zeros of $\bar{P}$ by the poles of C may, due to modelling error, lead to an unstable G, we should only attempt to cancel well-damped stable zeros of $\bar{P}$.

Remark 9: Note that $G_0 = \bar{P}/(1 + C\bar{P})$ may still have zeros at the locations where $\bar{P}$ has well-damped stable zeros. However, these stable zeros in G_0 are easily handled by the IMC method in the second design stage.

Finally, before we can assign the remaining poles for G_0 (if there are any more left), it is helpful to consider the following. So far, the high frequency unmodelled dynamics associated with $\bar{P}$ have not been taken into account. In order to achieve robust stabilization, such that P is stabilized by the C designed on the basis of $\bar{P}$, we would like the open-loop gain $|C(j\omega)\bar{P}(j\omega)|$ to become sufficiently small in the frequency range where the high frequency unmodelled dynamics may be significant (see also Remark 3 regarding the other advantage of having a quick high frequency roll off for $|C(j\omega)\bar{P}(j\omega)|$). This implies that, from robust stability point of view, we would like $|G_0(j\omega)|$ to roll off at a sufficiently low frequency. On the other hand, from the point of view of keeping $|G_0(j\omega)|$ reasonably large for frequencies up to approximately twice the expected overall closed-loop bandwidth, it is desirable that $|G_0(j\omega)|$ does not start to roll off at too low a frequency. Hence we need to compromise. This may be achieved by placing the remaining poles of G_0 at locations between $-2\omega_b$ and $-10\omega_b$, where ω_b is the expected overall closed-loop bandwidth.

3.2. *Design guidelines for step one and a stabilization example*

In this subsection, we first list the guidelines for designing C. We then give a stabilization example to illustrate the first step of the two step control design method.

Guidelines for designing C:

(1) Retain all the poles of $\bar{P}$ in the open left half-plane as the poles of G_0.

(2) Corresponding to the poles of $\bar{P}$ at $\sigma_1 \pm j\omega_1$; $\sigma_1 > 0$, assign $-\sigma_1 \pm j\omega_1$ as the poles of G_0.

(3) If $\bar{P}$ has poles at the origin, one of these poles may be retained if so desired. The remaining poles at the origin are to be assigned according to the next guideline for poles on the imaginary axis.

(4) Corresponding to the poles of $\bar{P}$ at $\pm j\omega_2$, assign $-\sigma_2 \pm j\omega_2$; $\sigma_2 > 0$ as the poles of G_0 to achieve a desirable degree of stability (measured by σ_2) deemed appropriate by the designer's experience for the situation at hand.

(5) Cancel well-damped, stable zeros of $\bar{P}$ by the poles of C. This is accomplished by assigning the well-damped stable zeros of $\bar{P}$ as the zeros of the characteristic polynomial, W, of G_0.

(6) Assign the remaining poles of G_0 at locations between $-2\omega_b$ and $-10\omega_b$, where ω_b is the expected overall closed-loop bandwidth.

(7) Examine whether C is stable if the unstable real zeros and the unstable real poles of the strictly proper model $\bar{P}$ possess the parity interlacing property. If this requirement is not met, it is necessary to modify the locations of the poles assigned on the basis of guidelines (4) to (6) and/or increase the assumed order of C and repeat the design procedure.

(8) If the unstable real poles and the unstable real zeros of the strictly proper model $\bar{P}$ do not possess the parity interlacing property, examine whether C only has unstable poles for satisfying the parity interlacing property. If this requirement is not satisfied, it is necessary to modify the locations of the poles assigned on the basis of guidelines (4) to (6) and/or increase the assumed order of C and repeat the design procedure.

(9) If the designed C does not stabilize P (due to high frequency model uncertainties), it is necessary to modify the locations of the poles assigned on the basis of guidelines (4) to (6) and repeat the design procedure. This is usually accomplished by moving those poles of G_0 that correspond to the poles of $\bar{P}$ on the $j\omega$-axis nearer to the $j\omega$-axis (so that the controller gain is reduced), or by moving those poles of G_0 between $-2\omega_b$ and $-10\omega_b$ nearer to the origin (so that the magnitude of the complementary-sensitivity function, CG_0, starts to roll off at a lower frequency).

Remark 10: It is important to emphasize that what we have described are *neither synthesis procedures nor rigid design rules.* As in any design method, *trial and error may very well be necessary* in the process of designing a C which stabilizes $\bar{P}$ and P.

Stabilization Example: To illustrate the design method described, we shall use the model

$$\bar{P} = \frac{s-1}{(s+0.5)(s-2)}$$

discussed in Example 1. Observe that the unstable real pole and the unstable real zero of $\bar{P}$ do not possess the parity interlacing property. Therefore an unstable C is necessary for the stabilization of $\bar{P}$.

Since the order of $\bar{P}$ is two, the minimum required order for a strictly proper C is two. This will result in a fourth order characteristic polynomial W. Following the guidelines listed above, we shall retain the pole of $\bar{P}$ at $s = -0.5$ in G_0. Corresponding to the pole of $\bar{P}$ at $s = 2$, we assign a pole of G_0 to $s = -2$. To cancel the effect of the unstable zero (of both $\bar{P}$ and G_0) at $s = 1$ on $|G_0(j\omega)|$, we assign a pole of G_0 to $s = -1$. Since the unstable zeros in $\bar{P}$ and G_0 are at $s = 1$, under the assumption that the unstable poles of C that we are going to design are further than $s = 1$ is from the origin, we would design an overall closed-loop system to have a bandwidth not exceeding 2 rad/s. We therefore assign the remaining pole of G_0 to $s = -4$. Therefore the desired characteristic polynomial of G_0 become

$$W(s) = (s+0.5)(s+2)(s+1)(s+4)$$

In this case, it turns out that

$$C(s) = \frac{72(s+0.5)}{(s-3.2621)(s+12.2621)}$$

and

$$G_0(s) + \frac{(s-3.2621)(s+12.2621)}{(s+0.5)(s+2)(s+4)} \left[\frac{s-1}{s+1}\right]$$

Remark 11: It is apparent that the unstable real pole of C at $s = 3.2621$ is further away from the origin than the unstable real zero of $\bar{P}$ at $s = 1$ is away from the origin. Therefore the unstable real zero of G_0 introduced by C at $s = 3.2621$ does not impose unnecessary limits on the overall closed-loop performance, as compared to the unstable real zero at $s = 1$ inherited from $\bar{P}$.

4. Simulation results

We present the results of two simulation examples in this section. Simulation Example 1 illustrates the situation where the effect of high frequency unmodelled dynamics associated with the initial model of an unstable plant is the main obstacle to achieving a large closed-loop bandwidth. By using the iterative identification and two step control design approach described in § 3, we successfully increased the bandwidth of the overall closed-loop system. In Simulation Example 2, the model of the unstable plant P is identical to the transfer function $\bar{P}$ used in Example 1. It should be realized that in this situation, where $\bar{P}$ and P have unstable real poles larger than unstable real zeros, serious fundamental limitations (Freudenberg and Looze 1985, Middleton and Goodwin 1990, Doyle *et al.* 1992) are imposed on the achievable closed-loop performance. The system response will be sensitive to noise disturbances when the closed-loop bandwidth is smaller than the largest magnitude of the unstable poles, and will have large undershoot when the closed-loop bandwidth is larger than the smallest magnitude of the unstable zeros. Furthermore, because the unstable real poles are larger than the unstable real zeros, it is impossible to achieve low sensitivity to noise disturbances while having a small undershoot in the system response. It is important to emphasize that we *do not* claim to be able to overcome the above-mentioned fundamental limitations. The *sole objective* of Simulation Example 2 is to show that, under this adverse situation, it is still possible to alleviate the effect of initial modelling errors through the iterative identification and two step control design approach where the second step involves the iterative identification and control design procedure.

4.1. *Simulation Example 1 (Model of unstable plant does not capture high frequency resonance)*

In this example we consider an unstable plant with

$$P(s) = \frac{0.1(s-4)}{(s-0.1)(s^2+0.2s+4)}$$

and

$$H_1(s) = 1$$

The noise disturbance e is zero mean and has a constant energy density of 0.0025 within the bandwidth of interest.

It is given that a model of $P(s)$ is

$$\bar{P}(s) = \frac{0.1}{-s+0.1}$$

The frequency responses of P and $\bar{P}$ are shown in figure 8.

Assuming that it is desirable to have a closed-loop bandwidth of at least 0.5 rad/s. We shall show that this design objective can be achieved by the iterative identification and two step control design approach.

In the first step we stabilize $\bar{P}$ with a strictly proper parallel feedback stabilizer C. Following the design guidelines listed in § 3.2, we assign the characteristic polynomial of $G_0 = \bar{P}/(1 + C\bar{P})$ as $(s+0.1)(s+1)$. The required C and G_0 are found to be, respectively

$$C(s) = -\frac{2.2}{s+1.2}$$

and

$$G_0(s) = -\frac{0.1(s+1.2)}{(s+0.1)(s+1)}$$

Note that C is stable.

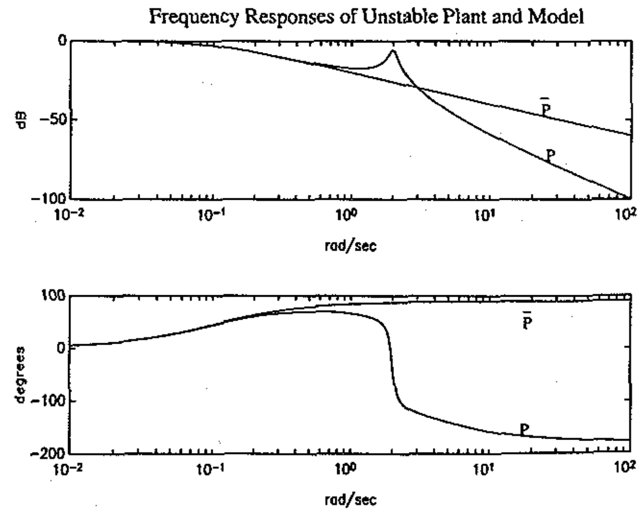

Figure 8. Frequency responses of P and $\bar{P}$.

It can easily be shown that the transfer function of the stabilized plant is

$$G(s)=\frac{0.1(s^2-2.8s-4.8)}{s^4+1.3s^3+4.1s^2+4.156s+0.4}$$

with poles at $s=-0.1072$, $s=-0.9892$ and $s=-0.1018\pm j1.939$, and zeros at $s=-1.2$ and $s=4.0$. Note that the unstable zero at $s=4.0$ is inherited from P. The corresponding noise transfer function $H=H_1/(1+CP)$ at the output of the stabilized plant is

$$H(s)=\frac{s^4+1.3s^3+4.1s^2+4.376s-0.48}{s^4+1.3s^3+4.1s^2+4.156s+0.4}$$

Since the negative signs in G_0 and G can be eliminated easily by cascading an inverter with each of these transfer functions, we shall omit them in the following discussions and assume that

$$G_0(s)=\frac{0.1(s+1.2)}{(s+0.1)(s+1)}$$

and

$$G(s)=\frac{0.1(-s^2+2.8s+4.8)}{s^4+1.3s^3+4.1s^2+4.156s+0.4}$$

The frequency responses of G and G_0 are shown in figure 9.

To begin the second design step, we apply the standard IMC design method to design, on the basis of the initial model G_0 for the stabilized plant G, a sequence of series controllers $\{K_0^j; j=0.1,\ldots,f\}$ (corresponding to a sequence of increasing overall designed closed-loop bandwidth $\{\lambda_0^j; j=0,1,\ldots,f\}$, where λ_0^f is the final achievable bandwidth before a better model G_1 is necessary). To avoid exciting high frequency modelling errors associated with G_0, we start with an overall designed closed-loop bandwidth of $\lambda_0^0=0.1$ rad/s. For such a small bandwidth, we cannot find any perceptible effects of high frequency modelling errors associated with G_0 on the closed-loop step responses. We then progressively increase the overall designed closed-loop bandwidth by re-designing K_0^j. When the overall designed closed-loop bandwidth reaches $\lambda_0^f=0.75$ rad/s, it was found, by using the model valuation methods described in Lee *et al.* (1995), that effects of high frequency modelling errors associated with G_0 on the closed-loop step response have become significant. The actual closed-loop step response is shown in figure 10.

By using the control-relevant closed-loop system identification, model reduction and validation procedure (see Lee *et al.* 1995), a model of G is estimated and validated as

$$G_1(s)=\frac{-0.0102s^2-0.0939s+0.4537}{s^3+0.3879s^2+3.805s+0.3657}$$

The frequency responses of G and G_1 are compared in figure 11.

On the basis of G_1 we design the sequence of controllers $\{K_1^j; j=0,1,2,\ldots\}$. We start with $\lambda_1^0=0.75$ rad/s. The resulting actual step response is presented in figure 12. When the overall designed closed-loop bandwidth is increased to $\lambda_1^f=3.0$ rad/s, it was found that a more accurate model is necessary for increasing the closed-loop bandwidth further. At this stage, the actual step response is shown in figure 13.

By repeating the closed-loop system identification, model reduction and validation procedure, a model of G is now estimated and validated as

$$G_2=\frac{-0.0106s^4-0.1086s^3+0.3092s^2+0.3612s+0.0125}{s^5+1.0896s^4+4.0596s^3+3.2852s^2+0.3924s+0.0096}$$

Its frequency response is compared to that of G in figure 14. At $\lambda_2^0=3.0$, the actual step response for the closed-

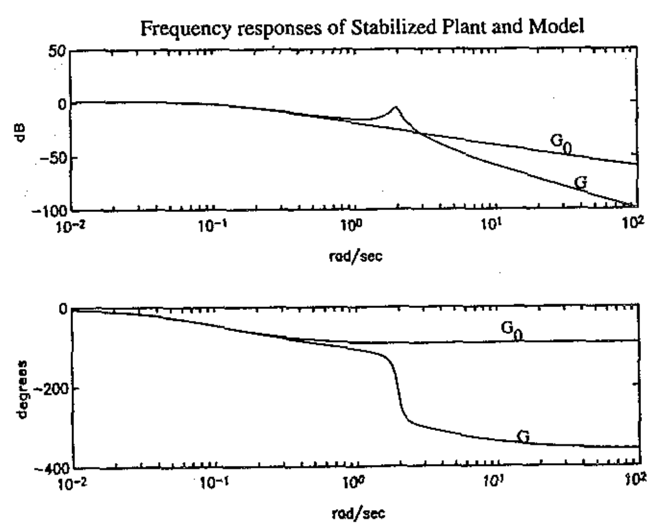

Figure 9. Frequency responses of G and G_0.

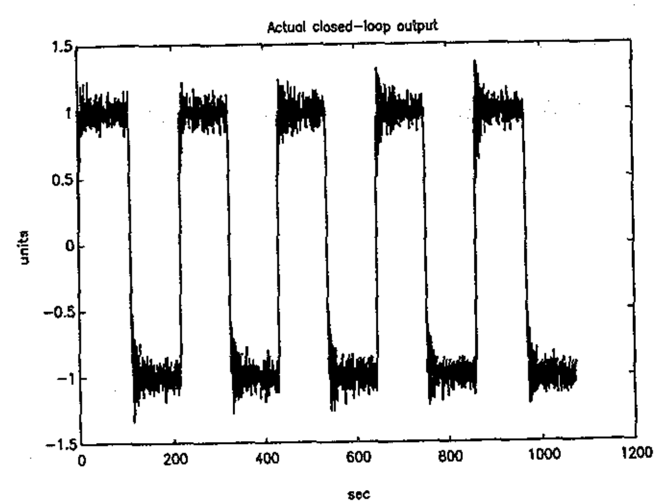

Figure 10. Noisy response of actual closed-loop for a square wave input ($\lambda_0^f=0.75$).

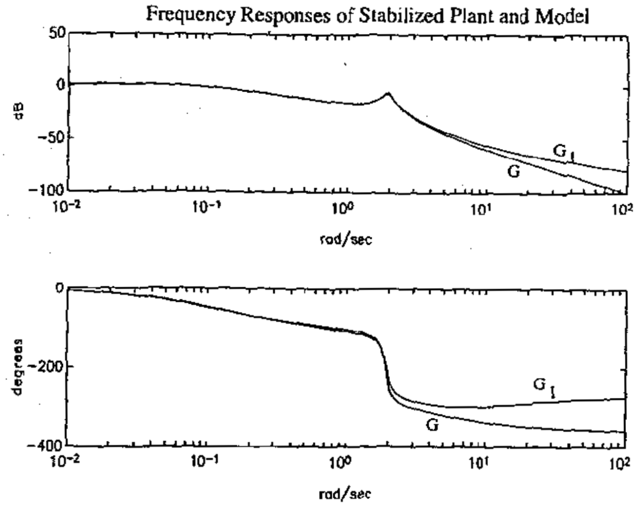

Figure 11. Frequency responses of G and G_1.

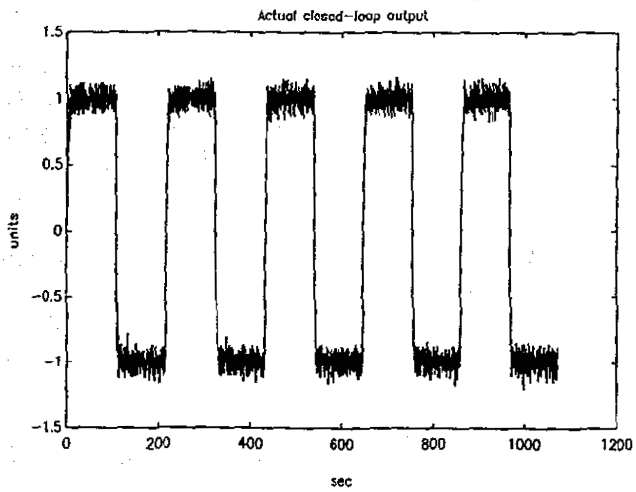

Figure 12. Noisy response of actual closed-loop for a square wave input ($\lambda_1^0 = 0.75$).

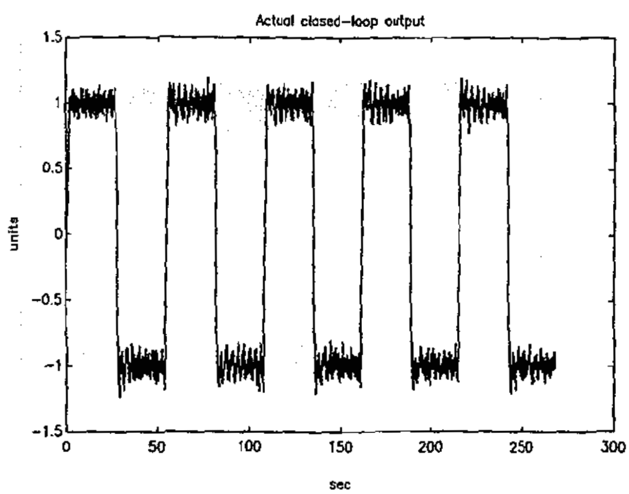

Figure 13. Noisy response of actual closed-loop for a square wave input ($\lambda_1^f = 3.0$).

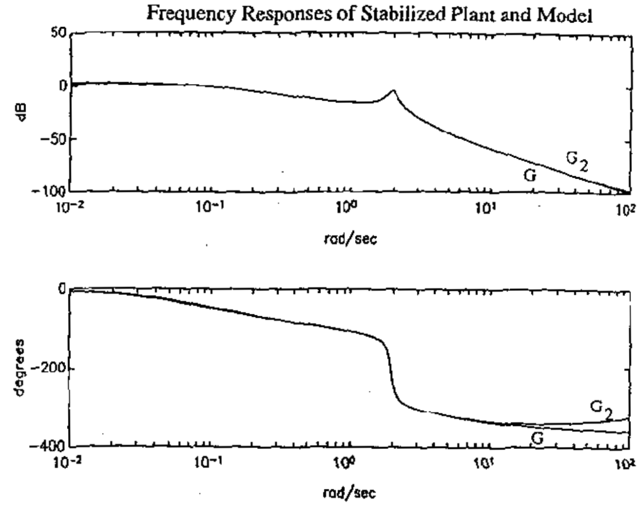

Figure 14. Frequency responses of G and G_2.

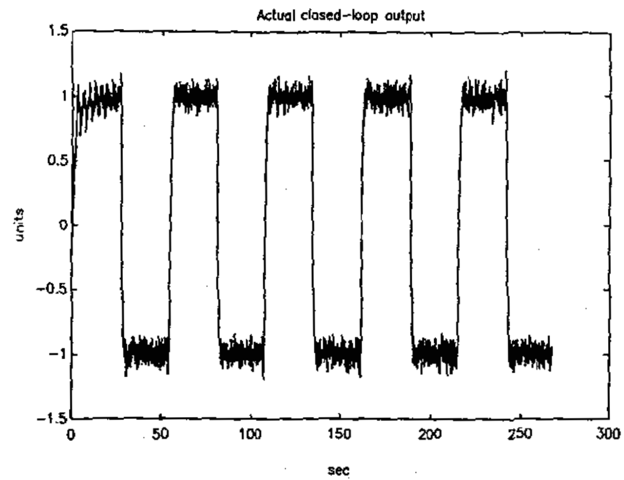

Figure 15. Noise response of actual closed-loop for a square wave input ($\lambda_2^0 = 3.0$).

loop system designed on the basis of G_2 is shown in figure 15.

With G_2, the overall closed-loop bandwidth can easily be increased to 6.0 rad/s. The actual closed-loop step response when the overall closed-loop bandwidth is 6.0 rad/s is shown in figure 16.

4.2. *Simulation Example 2 (Adversely positioned unstable real pole and zero)*

In this example, the unstable plant is described by

$$P(s) = \frac{400(s-1)}{(s+0.5)(s^2+25s+400)(s-2)}$$

and

$$H_1(s) = 1$$

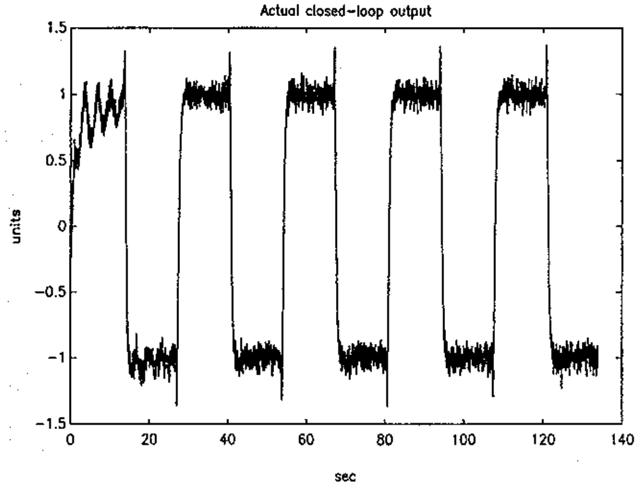

Figure 16. Noisy response of actual closed-loop for a square wave input ($\lambda_2 = 6.0$).

The noise disturbance e is zero mean and has a constant energy density of 0.0025 within the bandwidth of interest.

A given model of P is

$$\bar{P}(s) = \frac{s-1}{(s+0.5)(s-2)}$$

The frequency responses of P and $\bar{P}$ are shown in figure 17.

Note that $\bar{P}$ is the same as the unstable model employed in Example 1. In § 3.2 we have found that a strictly proper parallel feedback stabilizer for $\bar{P}$ is

$$C(s) = \frac{72(s+0.5)}{(s-3.2621)(s+12.2621)}$$

The resulting stabilized plant and its initial model are given, respectively, by

$$G(s) = \frac{400(s^3+8s^2-49s+40)}{s^6+32.5s^5+533s^4+1688.5s^3+8315s^2+7000s+1600}$$

and

$$G_0(s) = \frac{(s-3.2621)(s+12.2621)}{(s+0.5)(s+2)(s+4)}\left[\frac{s-1}{s+1}\right]$$

The noise transfer function $H = H_1/(1+CP)$ at the output of the stabilized plant is given by

$$H(s) = \begin{pmatrix} s^6+32.5s^5+533s^4+1688.5s^3 \\ -20485s^2+21400s+16000 \end{pmatrix} \div \begin{pmatrix} s^6+32.5s^5+533s^4+1688.5s^3 \\ +8315s^2+7000s+1600 \end{pmatrix}$$

The frequency responses of G and G_0 are shown in figure 18.

Following the idea of the iterative identification and two step control design approach, we start with a small overall designed closed-loop bandwidth of $\lambda_0^0 = 0.1$ rad/s. A series controller K_0^0 is designed on the basis of G_0 by the standard IMC design method. The actual step response of the resulting closed-loop system ae shown in figure 19. It was verified by the methods described in Lee *et al.* (1995) that the model errors associated with G_0 have negligible effects on the closed-loop response for $\lambda_0^0 = 0.1$ rad/s. In fact it was not until $\lambda_0^f = 4.0$ rad/s that the model errors associated with G_0 has significant effects on the closed-loop response. The highly oscillatory actual closed-loop step response at this stage is presented in figure 20. By using the closed-loop system identification, model reduction and validation procedure, a model of G is estimated and validated as

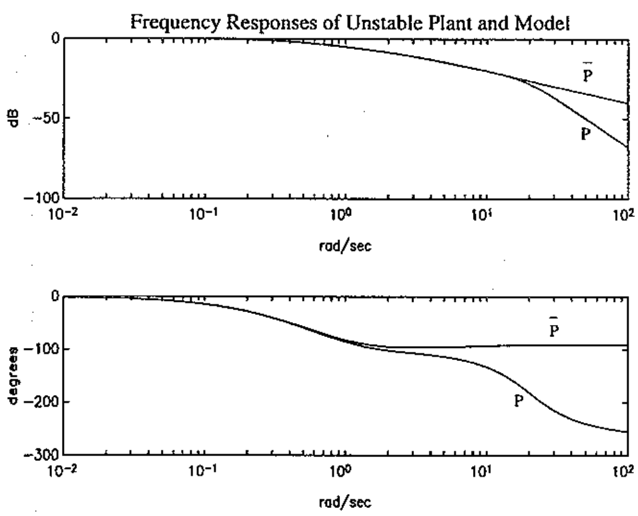

Figure 17. Frequency responses of P and $\bar{P}$.

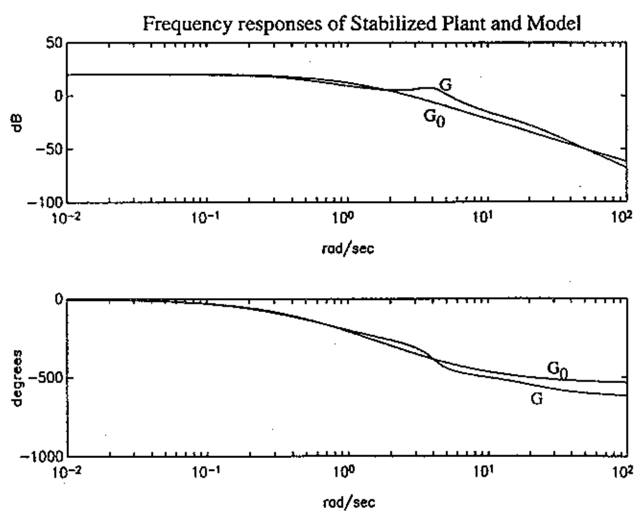

Figure 18. Frequency responses of G and G_0.

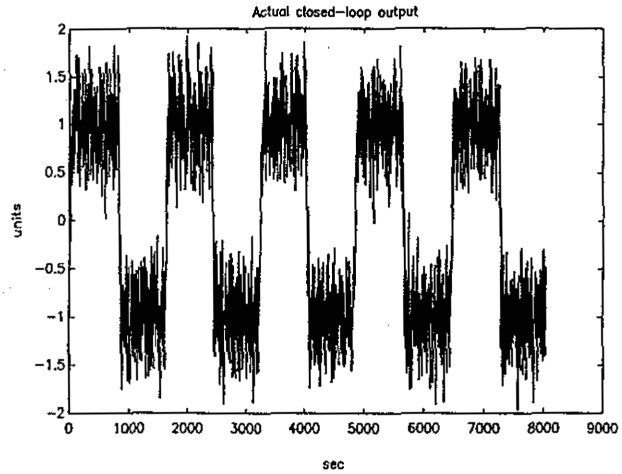

Figure 19. Noisy response of actual closed-loop for a square wave input ($\lambda_0^0 = 0.1$).

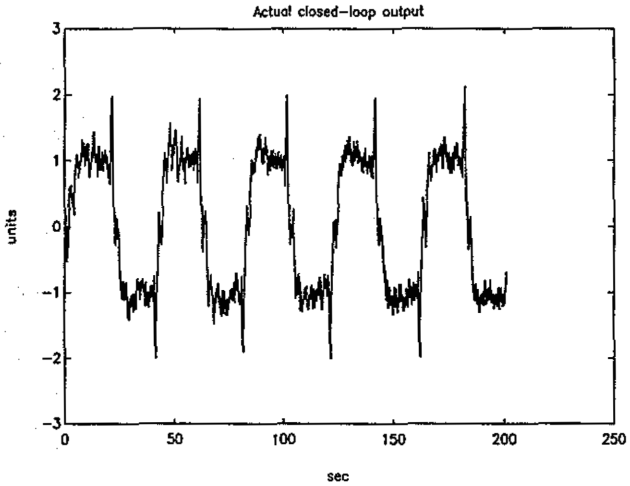

Figure 20. Noisy response of actual closed-loop for a square wave input ($\lambda_0^f = 4.0$).

$$G_1(s) = \begin{pmatrix} 0.3785s^{11} + 14.58s^{10} \\ +290.521s^9 + 2114.88s^8 \\ +11796.6s^7 + 4867.14s^6 \\ -48554.3s^5 - 309692s^4 \\ -581936.1s^3 - 195981.1s^2 \\ -340906.2s + 1210046.8 \end{pmatrix}$$

$$\div \begin{pmatrix} s^{12} + 22.156s^{11} + 312.05s^{10} \\ +2556.3s^9 + 16457s^8 + 66104.6s^7 \\ +248183s^6 + 536823.3s^5 \\ +1135468s^4 + 1227317.3s^3 \\ +1378618.5s^2 + 569837.6s \\ +133434 \end{pmatrix}.$$

The frequency responses of G and G_1 are compared in figure 21.

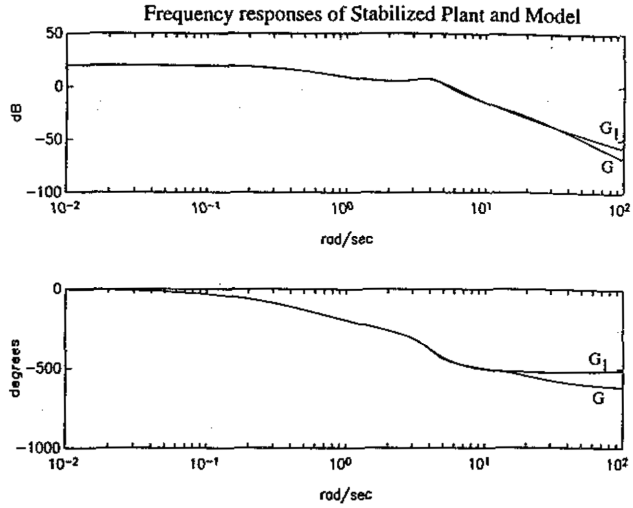

Figure 21. Frequency responses of G and G_1.

On the basis of G_1, we design K_1^0 while keeping the overall designed closed-loop bandwidth as $\lambda_1^0 = 4.0$ rad/s. The resulting actual step response is shown in figure 22. Notice that the oscillations in the actual responses are almost eliminated. Once again, the practicality of the overall approach is validated.

5. Summary and discussion

In this paper we have reviewed some difficulties of designing closed-loop systems for partially unknown unstable plants by the one step control design methods discussed in Campi *et al.* (1994) and Morari and Zafiriou (1989) in association with the interative identification and control design of Lee *et al.* (1993). To overcome these difficulties, we have presented an iterative identification and two step control design approach for unstable plants. We have shown that, after stabilizing the unstable plant with a strictly proper parallel feed-

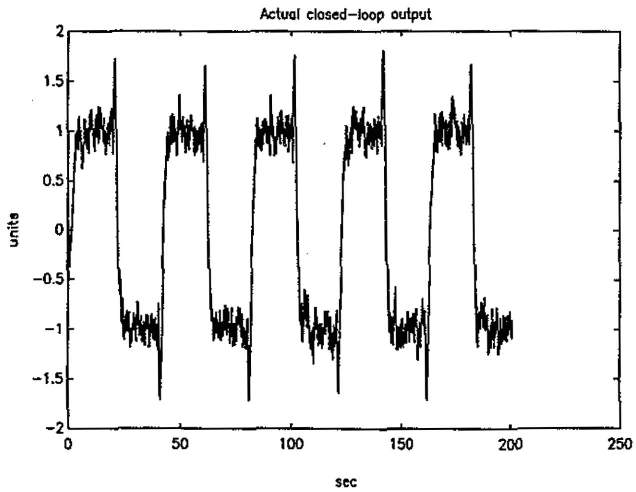

Figure 22. Noisy response of actual closed-loop for a square wave input ($\lambda_1^0 = 4.0$).

back stabilizer, it is possible to apply the iterative identification and control design methodology to extend the overall closed-loop bandwidth progressively. The proposed approach is illustrated with two simulation examples. Simulation Example 1 has shown that the approach produces very encouraging results when high frequency modelling errors associated with the initial model are the main constraints to a large overall closed-loop bandwidth. Simulation Example 2 indicated that, although *fundamental limitations* imposed on the closed-loop performance by the plant's undesirable unstable pole-zero structure cannot be overcome by any control design method (including our iterative identification and two step control design approach), the adverse conditions do not prevent the iterative identification and control design methodology from successfully alleviating the oscillatory behaviour in the closed-loop response that is due to high frequency modelling errors.

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