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Gradient expressions for a closed-loop identification scheme with a tailor-made parametrization☆

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Abstract

In this paper, we present gradient expressions for a closed-loop parametric identification scheme. The method is based on the minimization of a standard identification criterion and a parametrization that is tailored to the closed-loop configuration. It is shown that for both linear and nonlinear plants and controllers, the gradient signals can be computed exactly.

Keywords: Identification for control; Nonlinear identification; Estimation

1. Introduction

In recent years, several new methods for the identification of approximate models of an open-loop plant on the basis of closed-loop data have been presented. This line of research follows from the fact that in reality, i.e. on many industrial processes, the data need to be collected in closed-loop either because the plant is unstable or because operating constraints do not allow one to open the control loop. Also there might be situations where it is wiser to identify the plant in closed-loop so that the identified model would capture the dynamical characteristics that are important for control design. We refer the reader to Gevers (1993) and Van den Hof (1997) for a discussion of this problem in the linear case.

Several methods have been proposed for the closed-loop identification of linear systems: see Van den Hof (1997) for a survey. One such method, first suggested by Ljung (1987) as an exercise in his textbook, and whose properties have been studied intensively recently in a linear framework, is a method based on a "tailor-made parametrization": see Egardt (1997), Landau and Karimi (1997), Ljung (1987, 1997) and Van Donkelaar and Van den Hof (1996). The method uses knowledge of the controller; it minimizes an error between the closed-loop transfer functions of the true closed-loop and the model closed-loop, using a parametric model of the open-loop model only. The main result of Van Donkelaar and Van den Hof (1996) is to show that, provided the model order is higher than the order of the controller, the parameter set is connected. Their paper also provides consistency results and gradient expressions. The reference of Sjöberg et al. (1995) is an excellent survey for nonlinear identification techniques.

In this note we use the same closed-loop matching criterion as in Van Donkelaar and Van den Hof (1996) with a tailor-made parametrization, but we extend the results to the case of nonlinear systems and/or systems with nonlinear controllers. Our contribution is to show how the gradient of the identification criterion with respect to the model parameters can be computed in this nonlinear framework.

The ideas in this paper heavily rely on data-driven model-free control design methods that have recently been proposed in De Bruyne, Anderson, Gevers and Linard (1997), Hjalmarsson, Gevers, Gunnarsson and Lequin (1998) and Sjöberg and Agarwal (1996). Indeed,

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our solution method for the minimization of the closed-loop identification criterion w.r.t. the open-loop model parameters is dual to the method used in Hjalmarsson et al. (1998) for the optimization of the control performance criterion w.r.t. the controller parameters.

The organization of the paper is as follows. In Section 2, we describe the problem at hand. In Section 3 we present expressions of the gradient signals. We conclude in Section 4.

2. General problem setting

For ease of notation, we will omit the time argument of the signals. Let us assume that the true system is the single-input–single-output (SISO) nonlinear time-invariant system described by

$$\mathscr{S}: y = P_o(u, v), \tag{2.1}$$

where P_o is an unknown causal nonlinear operator. The restriction to scalar plants is inessential, but notationally convenient. Here u is the control input signal, y is the achieved output signal and v is a process disturbance signal which is assumed to be generated by filtering of a white noise sequence using a stable linear filter. Note that the disturbance signal v is allowed to enter the system nonlinearly. The input signal is determined according to a known controller

$$\mathscr{C}: u = C(r, y), \tag{2.2}$$

where r is an external reference which is assumed to be quasi-stationary and uncorrelated with v. The controller C is a causal nonlinear operator of both r and y. The closed-loop operator from measured reference signal r to measured output signal y, as defined in Fig. 1, can be written as follows:

$$y = T_o(r, v). \tag{2.3}$$

We require that the closed-loop system is bounded-input–bounded-output (BIBO) stable. In the sequel we often make use of linearizations of some nonlinear operators around their operating trajectories. We therefore make the following assumption.

Assumption 1. *The plant, the controller and all closed-loop operators are smooth functions of the reference signal, the input signal, the output signal and the disturbance signal. This means that if the closed-loop operator is linearized around any (stable) trajectory, the resulting linear system is BIBO stable.*

We refer the reader to Desoer and Vidyasagar (1975) for more details on such smoothness assumptions and a full treatment of the linearization problem. The basic idea is that the closed-loop operator from the reference signal r to the output signal y is identified using a parametrized output predictor

$$y(\theta) = T(\theta, r), \tag{2.4}$$

obtained from the feedback interconnection of an open-loop plant model

$$\mathscr{M}: y(\theta) = P(\theta, u) \tag{2.5}$$

for P_o, parametrized by a vector $\theta \in D_\theta \subset \mathbb{R}^n$ where D_θ is some prescribed domain, and the possibly nonlinear controller C in (2.2).

Assumption 2. *The output predictor* (2.4) *or, equivalently, the loop in Fig. 2 has the BIBO and smoothness properties of the true closed-loop system, for all values of $\theta \in D_\theta$; see Assumption* 1.

Note that it is not assumed that the true system (even without noise) is in the model set.

Suppose that a data set $\{r, y\}$ has been collected on the actual system of Fig. 1. The problem that is addressed in this paper is that of selecting the model for P_o in (2.5) that best explains this data set in a closed-loop sense.

We make use of the identification criterion

$$V_N(\theta) = \frac{1}{2N} \sum_{t=1}^{N} [L(y - y(\theta))]^2. \tag{2.6}$$

Here L can be any causal BIBO stable design operator. Besides the intuitively reasonable aspect of (2.6), it is shown in Ljung (1987), Söderström and Stoica (1989), Van Donkelaar and Van den Hof (1996) that this criterion allows a consistent identification of a linear plant under linear feedback, when the input–output dynamics are in the model set. This result does not generalize when both the plant and the controller are allowed to be nonlinear; this is further investigated in De Bruyne et al., (1998). In any case, the linear consistency result adds greater weight to the selection of the identification criterion (2.6). We also refer the reader to Gevers, Ljung and Van den Hof (1997) for variance considerations in the linear case.

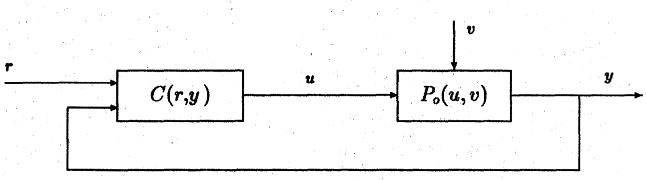

Fig. 1. The actual loop.

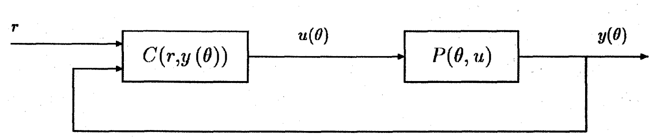

Fig. 2. The simulation loop.

Note that, provided the input signal u is measured, the generalization to the nonstandard identification criterion

$$\mathcal{V}_N(\theta) = \frac{1}{2N}\sum_{t=1}^{N} \{[L_y(y - y(\theta))]^2 + \lambda[L_u(u - u(\theta))]^2\},$$

is straightforward. Again, L_y and L_u are causal BIBO stable design operators.

The preceding parameter estimation problem is typically solved using gradient search techniques such as Gauss Newton; we refer the reader to Ljung (1987) for a discussion on initial estimates, convergence, local minima, etc. We refer to Van Donkelaar and Van den Hof (1996) for a discussion on the connectedness of the set of all models (2.5) stabilized by the controller (2.2) in the linear case. To minimize (2.6) with respect to the model parameter vector θ, it is standard that one can iteratively seek a solution for θ to

$$V'_N(\theta) = -\frac{1}{N}\sum_{t=1}^{N} [(y - y(\theta))y'(\theta)] = 0, \tag{2.7}$$

by taking steps in the negative gradient direction

$$\theta[i+1] = \theta[i] - \gamma_i R_i^{-1} V'_N(\theta[i]), \tag{2.8}$$

where $V'_N(\theta)$ and $y'(\theta)$, respectively, denote the gradient of $V_N(\theta)$ and $y(\theta)$ with respect to θ, and where R_i is some appropriate positive-definite matrix, typically an estimate of the Hessian of V_N. The update equation (2.8) is a batch mode type of adjustment.

Assumption 3. *The stability of the predictor is preserved during the iterations.*

This is a very reasonable assumption since the step size γ_i can be used effectively to control how much the model is allowed to change per iteration; the identified model is therefore stabilized by the known controller.

The key technical step in this iterative algorithm is the computation of the gradient $y'(\theta)$. Our contribution here is to show that this gradient computation can be performed by feeding the signal $u(\theta)$ in Fig. 2 as the input of a closed-loop simulation system.

3. Gradient expressions

It follows from (2.2) and (2.5) that the closed-loop model is described by

$$y(\theta) = P(\theta, u(\theta)), \qquad u(\theta) = C(r, y(\theta)). \tag{3.1}$$

As a tool for obtaining the gradient of V_N w.r.t. θ, we seek the gradients of $u(\theta)$ and $y(\theta)$ w.r.t. θ_j. If one of the parameter vector entries, say θ_j, is perturbed by a small $\delta\theta_j$, we obtain

$$\begin{aligned} u(\theta_1, \dots, \theta_j + \delta\theta_j, \dots, \theta_n) &= C(r, y(\theta_1, \dots, \theta_j + \delta\theta_j, \dots, \theta_n)) \\ &\simeq C(r, y(\theta) + y'_{\theta_j}(\theta)\delta\theta_j) \\ &\simeq C(r, y(\theta)) + \partial C_y(r, y(\theta))y'_{\theta_j}(\theta)\delta\theta_j \\ &= u(\theta) + \partial C_y(r, y(\theta))y'_{\theta_j}(\theta)\,\delta\theta_j, \end{aligned} \tag{3.2}$$

where $\partial C_y(r, y(\theta))$ is the linearization of C in response to a perturbation in y around the trajectory produced by r and by $y(\theta)$, i.e. the trajectory around which C is linearized depends on θ. The derivative of $y(\theta)$ w.r.t. θ_j is denoted $y'_{\theta_j}(\theta)$ and it is the jth component of the vector $y'(\theta)$. It is straightforward to see that (3.2) yields

$$u'_{\theta_j}(\theta) = \partial C_y(r, y(\theta))y'_{\theta_j}(\theta), \tag{3.3}$$

where $u'_{\theta_j}(\theta)$ is defined in a similar fashion as $y'_{\theta_j}(\theta)$. A similar reasoning yields

$$y'_{\theta_j}(\theta) = P'_{\theta_j}(\theta, u(\theta)) + \partial P_u(\theta, u(\theta))\,u'_{\theta_j}(\theta), \tag{3.4}$$

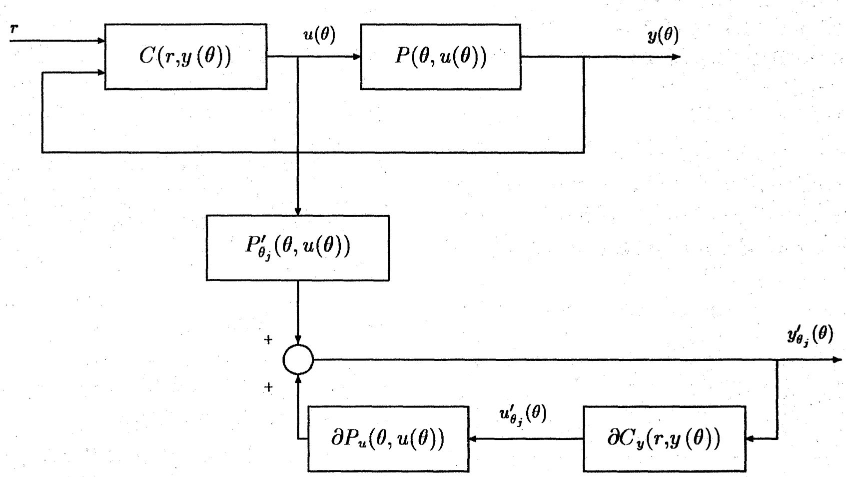

Fig. 3. Generation of $u'_{\theta_j}(\theta)$ and $y'_{\theta_j}(\theta)$ in the nonlinear case.

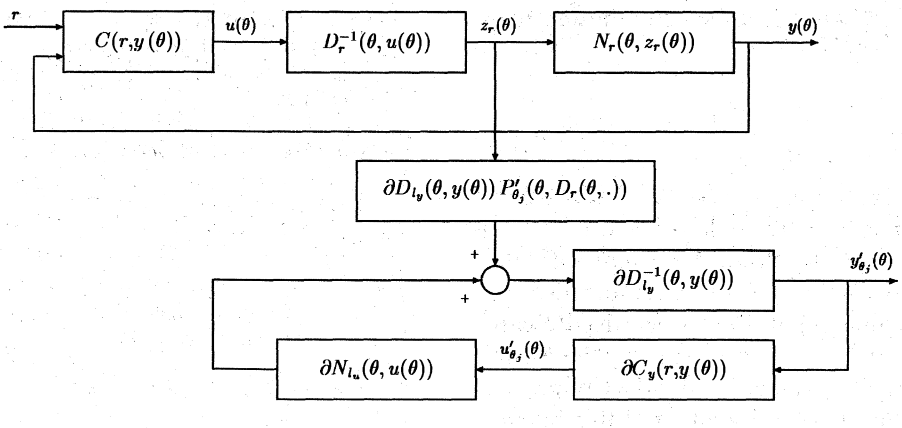

Fig. 4. Alternative implementation of Fig. 3.

where $\partial P_u(\theta, u(\theta))$ is the linearization of $P(\theta)$ in response to a perturbation in u around the trajectory produced by $u(\theta)$. The partial derivative of $P(\theta)$ w.r.t. θ_j is denoted by $P'_{\theta_j}(\theta, u(\theta))$. It can easily be obtained since $P(\theta)$ has a known structure.

In the nonlinear case, there are no compact expressions for the gradient signals like the ones derived at the end of the section for the linear case; see (3.12) and (3.13) below. Indeed, the exact gradient signals can be obtained by feeding the signal $u(\theta)$ generated in the loop of Fig. 2, filtered through $P'_{\theta_j}(\theta, u(\theta))$, as input of the (linear time-varying) linearized closed-loop system of Fig. 3. The stability of the lower loop follows from the smoothness assumption on the nonlinear closed-loop operator (Assumption 2) and the stability of the simulation loop at each iteration (Assumption 3). These two assumptions are equivalent to a small signal BIBO stability assumption, i.e. we assume that a small perturbation in the reference signal produces a small perturbation in the output signal.

The loop shown in Fig. 3 generates stable gradient estimates provided $P'_{\theta_j}(\theta, u(\theta))$ is a BIBO operator. In the contrary case, the signal entering the lower loop of Fig. 3 is unbounded. One can recover a stable estimate of the gradient by replacing Fig. 3 with Fig. 4. This is always possible provided one can construct

$$y(\theta) = N_r(\theta, z_r(\theta)), \qquad u = D_r(\theta, z_r(\theta)) \tag{3.5}$$

and

$$z_l(\theta) = D_l(\theta, y(\theta)) = N_l(\theta, u), \tag{3.6}$$

respectively, as stable right and left coprime descriptions of (2.5); see Hammer (1987) for further details. Here $\partial D_{l_y}(\theta, y(\theta))$ and $\partial N_{l_u}(\theta, u(\theta))$ are, respectively, the linearizations of $D_l(\theta, y(\theta))$ and $N_l(\theta, u(\theta))$ around their trajectory.

The stability of Fig. 4 follows from the stability of the simulation loop (Assumption 3), the smoothness assumption on the closed-loop system (Assumption 2) and the fact that

$$\partial D_{l_y}(\theta, y(\theta)) P'_{\theta_j}(\theta, D_r(\theta, .)), \tag{3.7}$$

is a stable operator for $j = 1, \ldots, n$ and $\forall \theta \in D_\theta$, i.e. even if $P(\theta, u(\theta))$ is an unstable operator. Indeed, it follows from (3.5) and (3.6) that

$$z'_{l_{\theta_j}}(\theta) = N'_{l_{\theta_j}}(\theta, u) = D'_{l_{\theta_j}}(\theta, y(\theta)) + \partial D_{l_y}(\theta, y(\theta)) y'_{\theta_j}(\theta), \tag{3.8}$$

$$y'_{\theta_j}(\theta) = P'_{\theta_j}(\theta, u). \tag{3.9}$$

Using the preceding equations, one obtains

$$\partial D_{l_y}(\theta, y(\theta)) P'_{\theta_j}(\theta, u) = N'_{l_{\theta_j}}(\theta, u) - D'_{l_{\theta_j}}(\theta, y(\theta)). \tag{3.10}$$

It is now straightforward to see that

$$\begin{aligned} &\partial D_{l_y}(\theta, y(\theta)) P'_{\theta_j}(\theta, D_r(\theta, z_r(\theta))) \\ &\quad = N'_{l_{\theta_j}}(\theta, D_r(\theta, z_r(\theta))) - D'_{l_{\theta_j}}(\theta, N_r(\theta, z_r(\theta))) \end{aligned} \tag{3.11}$$

which shows that (3.7) is a BIBO operator.

In the simplified case where both the real system and the controller are linear, (2.1), (2.2) and (2.5) reduce to

$$\mathscr{S}: y = P_0 u + v, \quad \mathscr{C}: u = C_r r - C_y y, \quad \mathscr{M}: y(\theta) = P(\theta) u.$$

Also,

$$\partial P_u(\theta, u(\theta)) = P(\theta) \quad \text{and} \quad \partial C_y(r, y(\theta)) = C_y.$$

It is now straightforward to see from Fig. 3 that

$$y'_{\theta_j}(\theta) = \frac{P'_{\theta_j}(\theta)}{(1 + P(\theta) C_y)} u(\theta) = \frac{P'_{\theta_j}(\theta) C_r}{(1 + P(\theta) C_y)^2} r, \tag{3.12}$$

$$u'_{\theta_j}(\theta) = -C_y y'_{\theta_j}(\theta). \tag{3.13}$$

The compact expressions of the gradients presented in (3.12) and (3.13) are equivalent to those developed in Van Donkelaar and Van den Hof (1996).

4. Conclusions

In this paper, we have presented gradient expressions for a closed-loop identification scheme with tailor-made parametrization. The main novelty of these gradient expressions is that they extend the current literature to nonstandard identification criteria and that the plant, the parametric model and the controller are allowed to be nonlinear.

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References

De Bruyne, F., Anderson, B., Gevers, M., & Linard, N. (1997). Iterative controller optimization for nonlinear systems. *Proceedings of the conference on decision and control* (pp. 3749–3754). San Diego, USA.

De Bruyne, F., Anderson, B., Gevers, M., & Linard, N. (1998). On closed-loop identification with a tailor-made parametrization. *Proceedings of the American control conference* (pp. 3177–3181). Philadelphia, USA.

Desoer, C., & Vidyasagar, M. (1975). *Feedback systems: input and output properties.* New York: Electrical Science Series, Academic Press.

Egardt, B. (1997). On the role of noise models for approximate closed-loop identification. *CD proceedings of the European control conference*, Brussels, Belgium. TH-A-F3 (634).

Gevers, M. (1993). Towards a joint design of identification and control? In H. L. Trentelman, & J. C. Willems, *Essays on control: perspectives in the theory and its applications* (pp. 111–151). Basel: Birkhäuser.

Gevers, M., Ljung, L., & Van den Hof, P. (1997). Asymptotic variance expressions for closed-loop identification and their relevance in identification for control. *Proceeding of the symposium on system identification* (pp. 1449–1454). Fukuoka, Japan.

Hammer, J. (1987). Fraction representations of nonlinear systems: a simplified approach. *International Journal of Control*, 46, 455–472.

Hjalmarsson, H., Gevers, M., Gunnarsson, S., & Lequin, O. (1998). Iterative feedback tuning: theory and applications. *IEEE Control Systems Magazine*, *18*, 26–41.

Landau, I., & Karimi, A. (1997). An output error recursive algorithm for unbiased identification in closed-loop. *Automatica*, *33*, 933–938.

Ljung, L. (1987). *System identification: Theory for the user*. Englewood Cliffs, NJ: Prentice-Hall.

Ljung, L. (1997). Identification in closed-loop: some aspects on direct and indirect approaches. *Proceedings of the symposium on system identification* (pp. 141–146). Fukuoka, Japan.

Sjöberg, J., & Agarwal, M. (1996). Model-free repetitive control design for nonlinear systems. *Proceedings of the conference on decision and control* (pp. 2824–2829). Kobe, Japan.

Sjöberg, J., Zhang, Q., Ljung, L., Benveniste, A., Deylon, B., Glorennec, P. -Y., Hjalmarsson, H., & Juditsky, A. (1995). Non-linear black-box modeling in system identification: a unified overview. *Automatica*, *31*, 1691–1724.

Söderström, T., & Stoica, P. (1989). *System identification.* Hemel Hempstead, Hertfordshire: Prentice-Hall International.

Van den Hof, P. (1997). Closed-loop issues in system identification. *Proceedings of the symposium on system identification* (pp. 1651–1664). Fukuoka, Japan.

Van Donkelaar, E., & Van den Hof, P. (1996). Analysis of closed-loop identification with a tailor-made parametrization. *Selected Topics in Identification, Modelling and Control*, *9*, 17–24.