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Coprimeness properties of nonlinear fractional system realizations[1]

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Abstract

In this paper, the relationship between the Bezout and the set-theoretic approaches to left coprimeness is studied. It is shown that left coprimeness in the set-theoretic sense implies left coprimeness in the Bezout sense. In addition to these results, we investigate whether some properties for linear left coprime realizations carry over to the nonlinear case, for example we examine the relations between two left coprime realizations of the same system. © 1998 Elsevier Science B.V. All rights reserved.

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1. Introduction

There are a number of fundamental results on coprime fractional descriptions of linear systems which are of great utility, and it is of interest to try to extend these to nonlinear systems. One use of linear coprime fractional representations has been in the characterization of all plants that can be stabilized by a given controller. This set of plants is parametrised by the Youla–Kucera parameter. Extensive work has been done in this area; see e.g. [18, 19]. The extension to nonlinear systems has been covered by [5, 14, 16, 17]. A discussion of characterization in the presence of noise can be found in [6].

Coprime linear fractional representations can also be used in an identification process. Suppose we have a controller that stabilizes both the plant and a nominal model of the plant. The problem of identifying the plant is equivalent to identifying the Youla–Kucera parameter with the advantage that the closed-loop identification problem has been turned into an open-loop problem. In [7], Hansen has examined this for linear systems. A number of iterative identification and control procedures based on this approach can be found in [8, 15]. The Hansen scheme has also been extended to the nonlinear case in [4, 9, 10].

These problems in both the linear and nonlinear cases have been approached predominantly using the Bezout definition of coprimeness. However, in [2, 3, 17], the "set theoretic" definition of right coprimeness for nonlinear systems has been used. In the linear case it is probably the easier tool to use in testing for coprimeness. The set-theoretic approach has the advantage that it is not necessary to introduce additional transfer functions or operators as is required in the Bezout identity approach. The two coprimeness definitions are equivalent for linear operators. Now, as the application of the ideas that utilize coprime factorizations has progressed to nonlinear systems it is of interest to see how the two definitions of coprimeness

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compare for nonlinear operators. Banos has examined the relationship between the coprimeness definitions for right coprime factors in [1]. This paper examines what happens in the left coprime case.

Consider right fractional descriptions $G(s)=N(s)D(s)^{-1}$, where $N(s)$, $D(s)\in \boldsymbol{M}(\boldsymbol{S})$ are matrices with entries in $\boldsymbol{S}$, the ring of proper stable rational transfer functions. Then coprimeness can be defined by the following requirement:

$$\begin{bmatrix} N(s_0) \\ D(s_0) \end{bmatrix} \text{ has full column rank } \forall s_0 \text{ with } \mathrm{Re}\,[s_0]\geqslant 0. \tag{1.1}$$

This is equivalent to the Bezout identity property; i.e. there exists $X(s)$, $Y(s)\in \boldsymbol{M}(\boldsymbol{S})$ with

$$X(s)N(s)+Y(s)D(s)=I \tag{1.2}$$

where I is the identity matrix. A further property is: if $G(s)=N_1(s)\,D_1(s)^{-1}=N_2(s)\,D_2(s)^{-1}$ with (N_1,D_1) coprime, then (N_2,D_2) is coprime if and only if there exists W a unit of $\boldsymbol{M}(\boldsymbol{S})$ (i.e. $W(s)$, $W(s)^{-1}\in \boldsymbol{M}(\boldsymbol{S})$) such that

$$N_2=N_1W, \qquad D_2=D_1W. \tag{1.3}$$

Finally, if $G(s)=N_1(s)D_1(s)^{-1}=N_2(s)\,D_2(s)^{-1}$ with (N_1,D_1) coprime, $\exists W\in \boldsymbol{M}(\boldsymbol{S})$ such that Eq. (1.3) holds. The above results apply to left fractional realizations, with obvious changes. Also, in Eq. (1.2), the identity matrix can be replaced by any unit without loss of generality.

The outline of our paper is as follows. Section 2 reviews the relationship between the Bezout and set-theoretic definitions of *right* coprimeness. Section 3 examines the relationship between the *left* coprimeness definitions. For a summary of the results of this paper the reader is referred to Fig. 1.

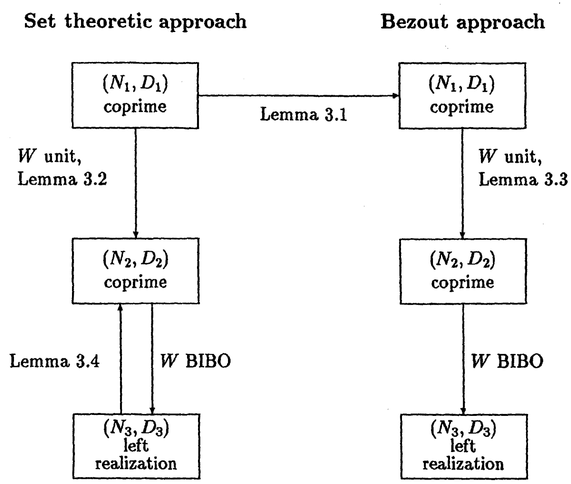

Fig. 1. Links between the "set-theoretic" and the "Bezout" approaches to left coprimeness.

2. Nonlinear right fractional realizations

We consider systems defined by a causal nonlinear operator $G:\mathscr{U}\to\mathscr{Y}$. The input and output spaces $\mathscr{U}$ and $\mathscr{Y}$ come equipped with a norm, and the subspaces $\mathscr{U}^{\mathrm{s}}$ and $\mathscr{Y}^{\mathrm{s}}$ comprise all stable (bounded norm) signals. Likewise, the subspaces $\mathscr{U}^{\mathrm{u}}$ and $\mathscr{Y}^{\mathrm{u}}$ comprise all unstable (unbounded norm) signals. An operator is termed BIBO (Bounded-Input–Bounded-Output stable) if it maps $\mathscr{U}^{\mathrm{s}}$ into $\mathscr{Y}^{\mathrm{s}}$. BIBO operators are very often finite norm operators, depending on the choice of norm. We shall assume that any BIBO operator G obeys $G(0)=0$. A right fractional representation is $G=ND^{-1}$ where N, D are BIBO.

Two different definitions of coprimeness are available in the literature. The "set-theoretic" approach has been used in e.g. [2, 3, 16, 17] and corresponds to Eq. (1.1); ND^{-1} is said to be coprime if $y=Nz$ and $u=Dz$ stable imply a stable partial state z. The "Bezout approach" corresponds to Eq. (1.2) and has been explored in, e.g. [5, 11]; ND^{-1} is said to be coprime if there exists BIBO X and Y with

$$XN+YD=V \tag{2.1}$$

where V is a unit, i.e. V and V^{-1} are BIBO. Roughly speaking, these are equivalent definitions, see, e.g. [1]. It is easy to demonstrate the following, which is at least implicit in [1].

Lemma 2.1. *If $G=N_1D_1^{-1}=N_2D_2^{-1}$ are two right fractional representations with (N_1,D_1) coprime, then (N_2,D_2) is coprime if and only if there exists a unit W such that*

$$N_2=N_1\,W, \qquad D_2=D_1\,W. \tag{2.2}$$

If (N_2,D_2) is not coprime, there exists a BIBO W such that Eq. (2.2) holds.

Proof. Adopt the Bezout definition of right coprimeness. Recall that $G=N_1D_1^{-1}$ has a right coprime factorization if and only if there exists a stable operator

L_F such that

$$L_F \begin{bmatrix} N_1 \\ D_1 \end{bmatrix} = I \tag{2.3}$$

where I is the identity operator. Now suppose that W is a unit operator and Eq. (2.2) holds, then we have that

$$L_F \begin{bmatrix} N_2 \\ D_2 \end{bmatrix} = L_F \begin{bmatrix} N_1 \\ D_1 \end{bmatrix} W = W, \tag{2.4}$$

i.e. (N_2, D_2) is coprime.

Conversely, suppose that (N_2, D_2) is a coprime realization of G. Since $G = N_1 D_1^{-1} = N_2 D_2^{-1}$ $N_2 = N_1 D_1^{-1} D_2$. Using Eq. (2.3) we have

$$L_F \begin{bmatrix} N_2 \\ D_2 \end{bmatrix} = L_F \begin{bmatrix} N_1 D_1^{-1} D_2 \\ D_2 \end{bmatrix}, \tag{2.5}$$

$$= L_F \begin{bmatrix} N_1 \\ D_1 \end{bmatrix} (D_1^{-1} D_2) = D_1^{-1} D_2. \tag{2.6}$$

The left side of this equation is BIBO stable, hence $W = D_1^{-1} D_2$ is also BIBO stable and satisfies Eq. (2.2). The reverse argument shows that $W^{-1} = D_2^{-1} D_1$ is BIBO stable, i.e. W is a unit operator.

Finally, if (N_2, D_2) is not coprime, the argument of the previous paragraph yields existence of a BIBO W satisfying Eq. (2.2). □

3. Nonlinear left fractional realizations

There is much less in the literature about *left* fractional representations of nonlinear systems, $G = D^{-1} N$, than about *right* fractional representations. For some key results, see [16]. Notice that while in the linear case, it is possible to obtain results for left realizations by transposition of those for right realizations, this is no longer possible in the nonlinear case. The distributivity property $A(B+C) = AB + AC$ is no longer valid, while the property $(B+C)A = BA + CA$ remains valid; this helps explain why the right coprime results are easier to obtain.

Refs. [12–14] show that for nonlinear systems a more useful and perhaps fundamental concept than a left realization is a stable kernel representation. In a sense, left factorizations are a special case of kernel representations.

Assumption. Consider the subset of input–output pairs (u, y) in which the output is unstable, i.e. $y \in \mathscr{Y}^{\mathrm{u}}$. Let $(\bar{u}, \bar{y})$ be an element in this set and let $U_{\bar{y}}$ denote the set of input signals u that map into $\bar{y}$. Then all the elements of $U_{\bar{y}}$ are either all stable or all unstable.

The previous assumption is a standing assumption for this section, and is common in the literature concerning nonlinear systems. This assumption is automatically satisfied in the linear case; see [16] for further details.

We can now state two distinct definitions of coprimeness, again "set theoretic" and "Bezout" in character. The two coprimeness definitions are as follows:

Set-theoretic approach: With $G = D^{-1} N$ and N, D BIBO, the pair (D, N) is said to be left coprime in the set-theoretic sense if the following property holds: if (u, y) is an input–output pair with $u \in \mathscr{U}^{\mathrm{u}}$ and $y \in \mathscr{Y}^{\mathrm{u}}$, then $z = Dy = Nu$ is also unstable, i.e. $z \in \mathscr{Z}_l^{\mathrm{u}}$.

Here, $\mathscr{Z}_l^{\mathrm{s}}$ and $\mathscr{Z}_l^{\mathrm{u}}$ are defined as the subspaces that, respectively, comprise all stable (bounded) and all unstable (unbounded) partial states z.

Bezout identity approach: With $G = D^{-1} N$ and N, D BIBO, the pair (D, N) is said to be left coprime in the Bezout sense if there exist BIBO X and Y with $DY + NX = V$ where V is a unit operator.

What is the connection between these two types of definitions? At present, all we can establish is the following result; see Fig. 1.

Lemma 3.1. *Coprimeness in the set-theoretic sense implies coprimeness in the Bezout sense.*

Proof. By assumption, $z \in \mathscr{Z}_l^{\mathrm{s}}$ implies that either the input $u \in \mathscr{U}^{\mathrm{s}}$ or the output $y \in \mathscr{Y}^{\mathrm{s}}$ where $z = Nu = Dy$. Equivalently, $z \in \mathscr{Z}_l^{\mathrm{u}}$ implies that $u \in \mathscr{U}^{\mathrm{u}}$ and $y \in \mathscr{Y}^{\mathrm{u}}$. We wish to use this to show that there exists a BIBO stable operator L_R such that

$$\begin{bmatrix} D & N \end{bmatrix} L_R = I. \tag{3.1}$$

The proof is constructive: we will consider the three different cases that can occur and, for each of these cases, construct an operator L_R for which Eq. (3.1) holds:

- If $z \in \mathscr{Z}_l^{\mathrm{s}}$ and $u \in \mathscr{U}^{\mathrm{s}}$ with $Nu = z$, define

$$L_R z = \begin{bmatrix} 0 \\ u \end{bmatrix}. \tag{3.2}$$

By substituting L_R in the left-hand term of Eq. (3.1), we observe that the Bezout identity holds:

$$[D\ N]\,L_R z = [D\ N]\begin{bmatrix}0\\ u\end{bmatrix} = Nu = z. \tag{3.3}$$

- If $z \in \mathscr{Z}_l^{\mathrm{s}}$, $u \in \mathscr{U}^{\mathrm{u}}$ and $y \in \mathscr{Y}^{\mathrm{s}}$ with $Dy = z$, define

$$L_R z = \begin{bmatrix}y\\ 0\end{bmatrix}. \tag{3.4}$$

By substituting L_R in the left-hand term of Eq. (3.1), we observe again that the Bezout identity holds:

$$[D\ N]\,L_R z = [D\ N]\begin{bmatrix}y\\ 0\end{bmatrix} = Dy = z. \tag{3.5}$$

- If $z \in \mathscr{Z}_l^{\mathrm{u}}$, define

$$L_R z = \begin{bmatrix}D^{-1}z\\ 0\end{bmatrix} \tag{3.6}$$

and substitution into the left-hand term of Eq. (3.1) yields

$$[D\ N]\,L_R z = [D\ N]\begin{bmatrix}D^{-1}z\\ 0\end{bmatrix} = z. \tag{3.7}$$

By collating these results, we obtain a discontinuous construction of L_R which shows that the definition of set theoretic coprimeness implies the Bezout definition of coprimeness. □

We remark that the construction is motivated by ideas of both [1,16]. The question whether the reverse result holds remains open. Notice also that the proof of Lemma 3.1 is constructive and the particular pair X, Y is constructed so that either Xz or Yz is zero for many z; in this sense, the construction is quite unlike any construction used in the linear case.

Next, we can address the question of the relationship between left coprime realizations.

Lemma 3.2. *Suppose $G = D_1^{-1}N_1 = D_2^{-1}N_2$ where (D_1, N_1) is coprime in the set-theoretic sense. Then (D_2, N_2) is coprime in the set-theoretic sense if there exists a unit W with*

$$N_2 = WN_1, \qquad D_2 = WD_1. \tag{3.8}$$

The only if part does not hold, i.e. (D_2, N_2) coprime with Eq. (3.8) *holding does not necessarily imply that W is a unit.*

Proof. Let us define $\gamma_{\mathrm{u}}^{\mathrm{u}}$ as the set of unbounded images of G that have unbounded pre-images. Similarly, $\gamma_{\mathrm{s}}^{\mathrm{u}}$ is defined as the set of unbounded images of G that have bounded pre-images. Now, we have $y = D_1^{-1}N_1\,u$ and $z_1 = D_1\,y = N_1\,u$ where (N_1, D_1) are coprime in a set-theoretic sense. If $y \in \gamma_{\mathrm{u}}^{\mathrm{u}}$, then $u \in \mathscr{U}^{\mathrm{u}}$ and by the coprimeness definitions, $z \in \mathscr{Z}_l^{\mathrm{u}}$. Hence $D_1(\gamma_{\mathrm{u}}^{\mathrm{u}}) \cap \mathscr{Z}_1^{\mathrm{s}} = \emptyset$. This is illustrated in Fig. 2. Since W is a unit operator, $W(D_1(\gamma_{\mathrm{u}}^{\mathrm{u}}))$ only contains unbounded signals. This in turn implies that $D_2(\gamma_{\mathrm{u}}^{\mathrm{u}}) \cap \mathscr{Z}_2^{\mathrm{s}} = \emptyset$, i.e. (N_2, D_2) are coprime in a set-theoretic sense.

Suppose that both (N_1, D_1) and (N_2, D_2) are coprime realizations of G, i.e. we have that $D_1(\gamma_{\mathrm{u}}^{\mathrm{u}}) \cap \mathscr{Z}_1^{\mathrm{s}} = \emptyset$ and $D_2(\gamma_{\mathrm{u}}^{\mathrm{u}}) \cap \mathscr{Z}_2^{\mathrm{s}} = \emptyset$. The operator W in Eq. (3.8) is defined by

$$W = D_2 D_1^{-1}. \tag{3.9}$$

Note that W is invertible by invertibility of both D_1 and D_2. Let us define the following sets:

$$\eta_{\mathrm{u}} = \mathscr{Y}^{\mathrm{u}} \backslash (\gamma_{\mathrm{u}}^{\mathrm{u}} \cup \gamma_s^{\mathrm{u}}), \tag{3.10}$$

$$\pi_1 = D_1(\eta_{\mathrm{u}}) \cap \mathscr{Z}_1^{\mathrm{s}}, \tag{3.11}$$

$$\pi_2 = D_2(\eta_{\mathrm{u}}) \cap \mathscr{Z}_2^{\mathrm{s}}. \tag{3.12}$$

Since π_1 is not necessarily mapped into π_2, we conclude that W is not necessarily a BIBO operator. Similarly, it is easy to see that W^{-1} is not necessarily a BIBO operator. This clearly shows that W above is not necessarily a unit operator and that the only if part does not hold. □

The above lemma is easy to establish. By contrast, the following result is comparatively difficult to establish.

Lemma 3.3. *Suppose a causal nonlinear operator G has fractional realizations $G = D_1^{-1}N_1 = D_2^{-1}N_2$ and suppose further there exist BIBO X and Y such that*

$$D_1 Y + N_1 X = V, \tag{3.13}$$

for some unit V. Define W by

$$N_2 = WN_1, \qquad D_2 = WD_1. \tag{3.14}$$

Suppose that Y is invertible (strict properness of G is effectively sufficient for this), that W is a unit, and that D_1Y, W and W^{-1} are Lipschitz continuous. Then (D_2, N_2) is coprime in the Bezout sense.

Proof. Consider the loop in Fig. 3.

Step 1: We shall show that satisfation of a Bezout identity involving N_1, D_1 is necessary and sufficient

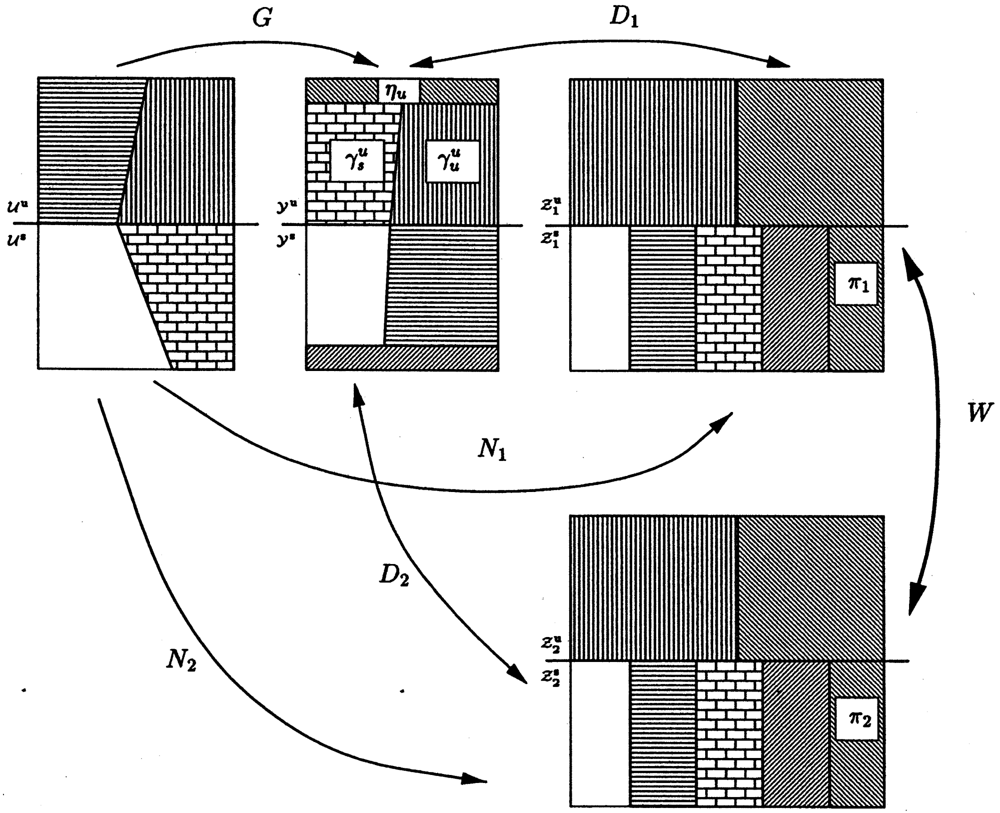

Fig. 2. Illustration of the proof of Lemma 3.2.

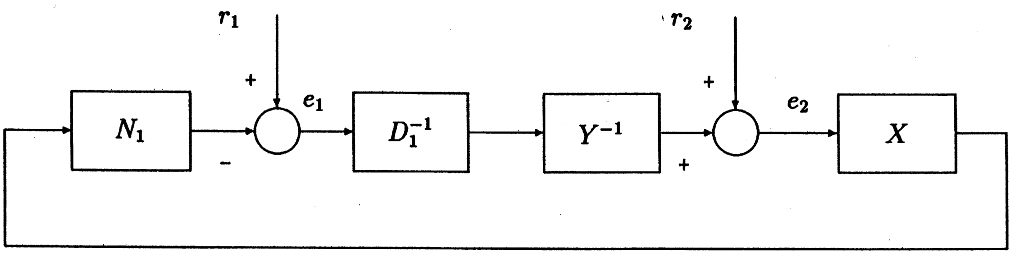

Fig. 3. Illustration of Lemma 3.3.

for the loop to exhibit BIBO behaviour. Suppose first that Eq. (3.13) holds. Then

$$e_1 = r_1 - N_1 X e_2, \tag{3.15}$$

$$e_2 = r_2 + Y^{-1} D_1^{-1} e_1 \tag{3.16}$$

or

$$D_1 Y[e_2 - r_2] = e_1. \tag{3.17}$$

Hence

$$r_1 - N_1 X e_2 = D_1 Y[e_2 - r_2]. \tag{3.18}$$

By the Lipschitz continuity of $D_1 Y$,

$$D_1 Y[e_2 - r_2] = D_1 Y e_2 + s(r_2) \tag{3.19}$$

for some s with $\|s\| \leqslant K \|r_2\|$. Then from Eqs. (3.18) and (3.19) we have

$$(D_1 Y + N_1 X) e_2 = r_1 - s(r_2) \tag{3.20}$$

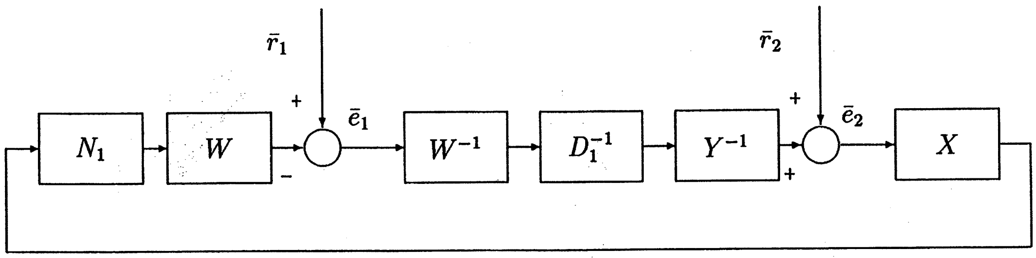

Fig. 4. Illustration of Lemma 3.3.

or

$$e_2 = V^{-1}[r_1 - s(r_2)]. \tag{3.21}$$

Thus bounded r_1, r_2 leads to bounded e_2 (and by Eq. (3.15) also bounded e_1).

Conversely, suppose BIBO behaviour is assumed. Take $r_2 = 0$. Then the above calculations show

$$(D_1Y + N_1X)e_2 = r_1 \tag{3.22}$$

(without actually invoking a Lipschitz condition). Since r_1 is arbitrary and bounded and e_2 is bounded by hypothesis, $(D_1Y + N_1X)^{-1}$ is a BIBO operator.

Now consider the new set-up of Fig. 4 where W is a unit operator; in effect, N_1 and D_1 of Fig. 3 have been replaced by N_1W and D_1W in Fig. 4.

Step 2: We shall show that if the loop of Fig. 3 is BIBO, then so is that of Fig. 4 and conversely. To begin, suppose the loop of Fig. 4 is BIBO, and let r_1, r_2 be two bounded inputs for Fig. 3. Define inputs $\bar{r}_1$, $\bar{r}_2$ for the scheme of Fig. 4 by

$$\begin{aligned} \bar{r}_1 &= W[r_1 - N_1X\,e_2] + WN_1X\,e_2, \\ \bar{r}_2 &= r_2. \end{aligned} \tag{3.23}$$

Since W is Lipschitz continuous, $\bar{r}_1$ is bounded, irrespective of e_2. Now for the loop of Fig. 4, we have

$$\bar{e}_1 = \bar{r}_1 - WN_1X\,\bar{e}_2 = WD_1Y[\bar{e}_2 - \bar{r}_2]. \tag{3.24}$$

Now use the expression for $\bar{r}_1$, $\bar{r}_2$ of Eq. (3.23) to obtain

$$\begin{aligned} &W[r_1 - N_1X\,e_2] + WN_1X\,e_2 - WN_1X\,\bar{e}_2 \\ &\quad = WD_1Y[\bar{e}_2 - r_2]. \end{aligned} \tag{3.25}$$

Compare this with the following consequence of Eq. (3.18):

$$W[r_1 - N_1X\,e_2] = WD_1Y[e_2 - r_2]. \tag{3.26}$$

Evidently, $\bar{e}_2 = e_2$ is a solution of Eq. (3.25), and by uniqueness, it is the only solution. To summarize, if r_1 and r_2 are inputs to Fig. 3, and $\bar{r}_1$, $\bar{r}_2$ are inputs to Fig. 4 generated using Eq. (3.23), there results $e_2 = \bar{e}_2$. Since r_1, r_2 bounded imply $\bar{r}_1$, $\bar{r}_2$ bounded (as already observed), which implies $\bar{e}_2$ bounded (by hypothesis that Fig. 4 is BIBO), we have e_2 bounded in Fig. 3, and then using Eq. (3.15) e_1 is bounded; i.e. Fig. 3 is BIBO.

The converse follows by interchanging the roles of W and W^{-1}, r_1 and $\bar{r}_1$.

Step 3: The proof is completed as follows:

$D_1Y + N_1X = V$ with V a unit operator
$\Leftrightarrow$ the loop of Fig. 3 is BIBO stable (Step 1),
$\Leftrightarrow$ the loop of Fig. 4 is BIBO stable (Step 2),
$\Leftrightarrow WD_1Y + WN_1X$ is stably invertible (Step 1),
$\Rightarrow (D_2, N_2) = (WD_1, WN_1)$ is left coprime. □

Remark 1. The question of whether an "only if" result holds remains open.

Remark 2. There is an apparently simple but actually erroneous approach to proving Lemma 3.3. Suppose X and Y are BIBO operators such that $D_1Y + N_1X = V$ where V is a unit. It is *not* true that this equation implies $WD_1Y + WN_1X = WV$ since W is nonlinear. Of course, if this equation were true, it would be an immediate consequence that $(D_2 = WD_1, N_2 = WN_1)$ is coprime in the Bezout sense, as WV will be a unit when W and V are separately units.

Our last comment concerns the incompleteness of yet another result. Suppose $G = D_2^{-1}N_2$ with (D_2, N_2) coprime in the set-theoretic sense. It is obvious that for any BIBO W with W^{-1} existing (but not necessarily BIBO), $G = D_3^{-1}N_3$ where $N_3 = WN_2, D_3 = WD_2$. However, it is not true that any realization gives rise to a W that is necessarily BIBO. [2]

[2] The proof of Lemma 3.2 can be easily varied to establish this claim.

Whereas, if $G = D_2^{-1}N_2$ with (D_2, N_2) coprime in the Bezout sense, then any BIBO W with W^{-1} existing defines another realization $G = D_3^{-1}N_3$ with $N_3 = WN_2$, $D_3 = WD_2$. Whether any realizations $G = D_3^{-1}N_3$ implies that $W = D_3D_2^{-1}$ is BIBO is unknown. (In contrast to the set-theoretic situation, an example with non-BIBO W is lacking.)

Despite the fact that we cannot relate via a BIBO W an arbitrary fractional representation of G to a coprime representation that is given a priori, we can make the following statement, which is implicit in [16]:

Lemma 3.4. *Suppose $G = D_3^{-1}N_3$ is a left fractional realization. Then there exists a left fractional realization $G = D_2^{-1}N_2$ which is coprime in the set-theoretic sense, with*

$$N_3 = WN_2, \qquad D_3 = WD_2 \tag{3.27}$$

where W is BIBO.

Proof. The authors of [16] show how to construct a W such that $G = D_2^{-1}N_2$ is left coprime. Although it is not stated in the proof of [16], it is fairly straightforward to see that this W is also BIBO stable by construction. □

4. Conclusions

In this paper, we have investigated the relationship between the "set-theoretic" and the "Bezout" approaches to left coprimeness. This research is motivated by dual results for right coprimeness in [1]. In particular, it is shown that coprimeness in the set-theoretic sense implies coprimeness in the Bezout sense. Another key result of this paper is that it is possible to construct a left coprime realization from another one using a unit operator. This result holds in general with coprimeness in the set-theoretic sense and under assumption of Lipschitz continuity of certain BIBO operators with coprimeness defined in the Bezout sense.

References

[1] A. Baños, Stabilization of nonlinear systems based on a generalized Bezout identity, Automatica 32 (1996) 591–595.

[2] G. Chen, R.J.P. de Figueiredo, On construction of coprime factorizations for nonlinear feedback control systems, IEEE Proc. on Circuits, Systems and Signal Processing 11 (1992) 285–307.

[3] R. Danow, G. Chen, A necessary and sufficient condition for right coprime factorization of nonlinear systems, IEEE Proc. on Circuits, Systems Signal Processing 12 (1993) 489–492.

[4] S. Dasgupta, B.D.O. Anderson, Closed loop identification of nonlinear time varying systems, Automatica 32 (1996) 1349–1360.

[5] J. Hammer, Nonlinear systems, stabilization, and coprimeness, Int. J. Control 42 (1985) 1–20.

[6] J. Hammer, Internally stable nonlinear systems with disturbances: a parametrization, IEEE Trans. Automat. Control 39 (1994) 300–315.

[7] F.R. Hansen, G.F. Franklin, R.L. Kosut, Closed-loop identification via the partial representation: experiment design, in: Proc. Am. Control Conf., 1989, pp. 386–391.

[8] W.S. Lee, B.D.O. Anderson, R.L. Kosut, I.M.Y. Mareels, A new approach to adaptive robust control, Int. J. Adaptive Control Signal Process. 7 (1993) 183–211.

[9] N. Linard, B.D.O. Anderson, Identification of nonlinear plants under linear control using Youla–Kucera parametrisations, in: Proc. 35th IEEE Conf. on Decision and Control, Kobe, Japan, 1996, pp. 1094–1099.

[10] N. Linard, B.D.O. Anderson, F. De Bruyne, Identification of a nonlinear plant under nonlinear feedback using left coprime fractional based representations, Automatica (1997) Submitted.

[11] A.D.B. Paice, J.B. Moore, R. Horowitz, Nonlinear feedback system stability via coprime factorization analysis, J. Math. Systems Estimation Control 2 (1992) 293–321.

[12] A.D.B. Paice, A.J. van der Schaft, Stable kernel representations as nonlinear left coprime factorizations, in: Proc. 33rd Conf. on Decision and Control, Orlando, Florida, 1994, pp. 2786–2791.

[13] A.D.B. Paice, A.J. van der Schaft, On the parametrization and construction of nonlinear stabilizing controllers, in: Proc. 3rd IFAC Sym. on Nonlinear Control Systems, 1995, pp. 509–513.

[14] A.D.B. Paice, A.J. van der Schaft, The Youla parametrization for nonlinear feedback systems with additive disturbances, in: Proc. 34th Conf. on Decision and Control, New Orleans, Louisiana, 1995, pp. 2976–2981.

[15] R.J.P. Schrama, P.M.J. Van den Hof, An iterative scheme for identification and control design based on coprime factorizations, in: Proc. Am. Control Conf., 1992, pp. 2842–2846.

[16] T.T. Tay, J.B. Moore, Left coprime factorizations and a class of stabilizing controllers for nonlinear systems, Int. J. Control 49 (1989) 1235–1248.

[17] M.S. Verma, Coprime fractional representations and stability of nonlinear feedback systems, Int. J. Control 48 (1988) 897–918.

[18] M. Vidyasagar, Control Systems Synthesis: A Factorization Approach, MIT Press, Boston, 1985.

[19] C.D. Youla, H.A. Jabr, J.J. Bongiorno Jr., Modern Wiener-Hopf design of optimal controllers, Part 1: the single-input–single-output case, IEEE Trans. Automat. Control 21 (1976) 3–13.