Impossibility of a Class of Gyrator Realizations

In this correspondence, we show that it is impossible to realize a gyrator using time-variable transformers, and passive time-invariant resistors and capacitors.

We point out that it has earlier been demonstrated that a gyrator realization is possible which uses time-varying transformers, together with positive *and negative* time-invariant capacitors.[1] Thus here it is demonstrated that it is impossible to improve on this realization, in the sense that it is impossible to dispense with the negative capacitors. In other words, it is both impossible to realize a gyrator with time-varying transformers alone, or a combination of time-varying transformers and positive capacitors.

It is easy to observe that if a realization exists using time-varying transformers, and passive time-invariant resistors and capacitors, then the resistors can be removed and replaced by open circuits without altering the port behavior. This is true because the gyrator is a lossless device, and if the resistors in an all-passive-element realization carried current, the lossless property would be destroyed. Since then any resistors in the realization carry no current, they may evidently be removed.

Consequently, we are led to hypothesizing a realization of the type shown in Fig. 1, where, it will be noticed, transformer normalizations allow us to assume all the capacitors are one farad. Gyrator behavior is assumed at ports 1 and 2.

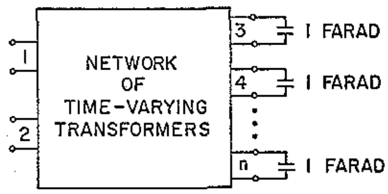

Fig. 1. Tentative gyrator realization.

Now suppose port 2 is short-circuited while to port 1 there is connected an ideal source v_1. Then zero current must flow at port 1 because the gyrator action of the network requires port 1 to appear as an open circuit when port 2 is short-circuited. There must, moreover, be zero voltage at ports 3 through n and zero current there, for otherwise the terminating capacitors would be storing energy at some time, despite the fact that none is supplied through ports 1 and 2. Similar arguments apply for the case when port 1 is short-circuited and excitation is applied at port 2.

By linearity, we conclude that the currents and voltages at ports 3 through n will be identically zero irrespective of the excitations at ports 1 and 2, and that, consequently, the capacitors in the gyrator equivalent circuit play no part and may be removed. We are then led to suppose the existence of a gyrator equivalent circuit involving only interconnected time-varying transformers.

But this is evidently impossible. The transformers themselves in such an interconnection and the Kirchhoff voltage and current laws may force typically

$$\alpha_1(t)v_1(t) + \alpha_2(t)v_2(t) = 0 \qquad (1)$$

but cannot force "mixed" relations like

$$\alpha_1(t)v_1(t) + \alpha_2(t)i_1(t) = 0 \qquad (2)$$

or

$$\alpha_1(t)v_1(t) + \alpha_2(t)i_2(t) = 0 \qquad (3)$$

for time-varying functions $\alpha_1(.)$ and $\alpha_2(.)$.

With (2) and (3) ruling out a connection between v_1 and i_1 or v_1 and i_2, it is evident that if the port-1 behavior is to be dependent on the port-2 termination, a relation like (1) must hold. Losslessness then requires

$$\alpha_2(t)i_1(t) - \alpha_1(t)i_2(t) = 0. \qquad (4)$$

But from (1) and (4) it is evident then that gyrator behavior is not obtained, but rather transformer behavior with turns ratio $-\alpha_2/\alpha_1$.

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[1] B. D. Anderson and R. W. Newcomb, "A capacitor transformer gyrator realization," *Proc. IEEE*, vol. 53, p. 1640, October 1965.

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