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A Parametrization for the Closed-loop Identification of Nonlinear Time-varying Systems*

SOURA DASGUPTA† and BRIAN D. O. ANDERSON‡

We convert the closed-loop identification of nonlinear time-varying plants, with high signal-to-noise ratio, to an open-loop identification problem of an associated Youla–Kucera parameter. The plant must be stabilizable by a known linear, possibly time-varying controller.

Key Words—Closed-loop identification; nonlinear; time-varying; coprime factorization.

Abstract—It has recently been shown that the identification of a linear time-invariant plant using closed-loop measurements with a known linear controller can be effectively tackled by regarding the unknown plant as a member of the set of all plants stabilized by the known linear controller. This set is parametrized by a Youla–Kucera parameter, itself a stable transfer function which, it turns out, can be identified using open-loop techniques. This paper generalizes the parametrization idea to nonlinear plants, and treats identification of nonlinear plants. Copyright © 1996 Elsevier Science Ltd.

1. INTRODUCTION AND PROBLEM MOTIVATION

This paper considers the closed-loop identification of nonlinear time-varying (NLTV) plants operating under linear, possibly time-varying, feedback. More precisely, the setting is one of Fig. 1, where G is the NLTV plant to be identified, K is the linear time-varying (LTV) controller, H is a linear stable output measurement noise-generating system, affected in turn by the zero-mean, white, stationary noise process $w(t)$. We shall assume that K internally stabilizes the unknown plant G.

We note that for control systems design, closed-loop identification of the plant is often of far greater value than is its open-loop identification. It is easy to construct examples where a plant estimate obtained through closed-loop identification provides good control performance despite a potentially large open-loop identification error. Equally, there are examples where controllers obtained through open-loop identification with low error fail to meet the standards of good closed-loop performance (Zhang *et al.*, 1991). Thus in this paper closed-loop, as opposed to open-loop, identification is considered.

The fact that the class of controllers under consideration is linear is also well grounded in engineering practice. Even though all plants are nonlinear, the controllers employed are frequently linear. Furthermore, the fact that the controllers we consider may be time-varying significantly expands the class of plants relative to which they exhibit good, stable performance.

Having thus motivated the general setting of this paper, we note that there are, however, several technical difficulties associated with identification in the closed loop. Consider, for example, identification of the plant in Fig. 1. While in an open-loop setting one may reasonably assume the input process $u(t)$ to be uncorrelated with the measurement noise $v(t)$, in the closed-loop framework of Fig. 1 such an assumption is clearly unwarranted. The measurement noise $v(t)$ is correlated to $u(t)$, and, what is more, this correlation, being dependent on the unknown plant G, cannot be determined a priori.

A second problem is that if G is unstable then some identification methods, especially those based on using u and y may not be usable. If one seeks to deal with this difficulty by identifying the stable closed-loop operator relating r_1 to y, where of course r_1 is independent of v, one has the new problem of having to unravel the closed-loop operator to obtain G. Even when K, G and H are all linear, G appears in a nonlinear way in the closed-loop quantities.

In a series of papers on single-input

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† Department of Electrical and Computer Engineering, The University of Iowa, Iowa City, IA 52242, U.S.A.

‡ Research School of Information Sciences and Engineering, Australian National University, ACT 0200, Australia.

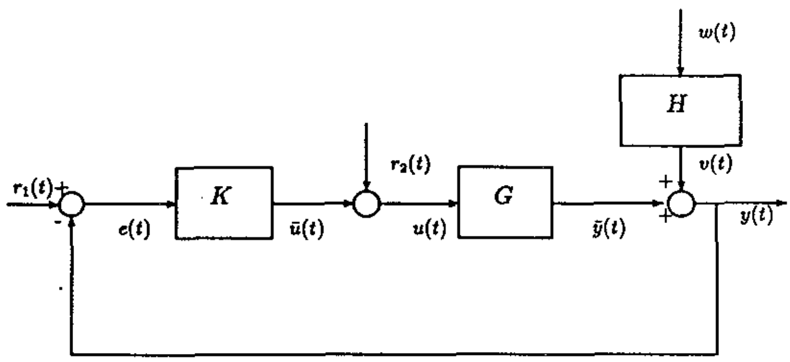

Fig. 1. The closed-loop setting.

single-output (SISO), linear time invariant (LTI) plants, Hansen (1989), Hansen and Franklin (1988) and Hansen *et al.* (1989), have approached this problem through the use of coprime factorization theory (Vidyasagar, 1985). They show that instead of identifying G itself, if one identifies directly the Youla–Kucera parameter (Vidyasagar, 1985) associated with K and G then the identification problem reduces to an open-loop one.

This ingenious technique has been further exploited in the so-called windsurfer adaptive control approach of Lee *et al.* (1993), which under a particular SISO, LTI setting involves an iterative scheme of interlaced plant identification and controller update. A key fact of the Hansen approach that is exploited in Lee *et al.* (1993) is the relation between the errors in the identification of the Youla–Kucera parameter and the respective errors in the closed- and open-loop system identification. Van den Hof and Schrama (1994) reproduce the approach of Hansen (1989) using a dual coprime factorization though for linear multivariable plants.

This paper extends the theory underlying Hansen (1989), Hansen and Franklin (1988), Hansen *et al.* (1989), Lee *et al.* (1993) and Van den Hof and Schrama (1994) to NLTV plants. As in these references the starting point is the assumption that one knows a controller, in this paper LTV, that stabilizes the unknown plant to be identified; equivalently that, given a known LTV controller K, the plant model set is the class of all NLTV plants stabilized by K.

Section 2 is devoted to the characterization of this model set in a noise-free setting. Characterizations on the basis of both left- and right-coprime factorization descriptions of the controller are presented. The former are stated without proof, being as they are straightforward extensions of the work of Verma (1988), which provides right-coprime factorization-based characterization of all NLTV controllers that stabilize a given LTV plant. We note that Verma (1988) does not deal with left-coprime factors of the controller. Nor does it address the question of incorporating noise terms, central to the issues underlying this paper.

A novelty of this section is the characterization of the plant model set in terms of the left-coprime factors of the controller. To elaborate, consider the previous work on left-coprime factor-based Youla–Kucera parametrization of the class of NLTV systems stabilized by a given controller. This work has been conducted in the main by Tay and Moore (1989) and Paice and Moore (1990) (see also the references therein). In a comprehensive set of papers these authors provide such a characterization for *all plants admitting a left-coprime factorization* that can be stabilized by a given NLTV controller. However, it is known that *not every stabilizable NLTV plant has a left-coprime factorization.* Thus the parametrization obtained in Tay and Moore (1989) and Paice and Moore (1990) covers only a subset of plants that can be stabilized by a given controller. By contrast, we provide here a Youla–Kucera parametrization of *all* plants stabilized by a given LTV controller in terms of the *left-coprime factors* of the controller. Obviously, in view of what has been stated above, not every plant in this set has left-coprime factorizations. However, as will be demonstrated here, the underlying premise of the approach of Hansen *et al.*, namely that the direct identification of the Youla–Kucera parameter underlying the plant description converts closed-loop identification to an open-loop problem, remains valid for the characterization which we obtain here.

While the two characterizations of Section 2 assume zero measurement noise, Section 3 modifies each to incorporate the measurement noise term. Section 4 demonstrates how the models of Section 3 can be exploited to meet our identification objectives under a high signal-to-noise ratio (SNR) assumption. One of the facts demonstrated in Section 4 is that consistent, unique identification requires that *both* r_1 *and* r_2 be present in the system. The absence of r_1 may preclude consistent estimation, while the absence of r_2 may make the unique identification of the Youla–Kucera parameter impossible.

It is worth noting that in a recent paper Hammer (1994) has provided right-coprime factor based descriptions of nonlinear plants incorporating measurement noise. The results of Section 3 differ from those of Hammer (1994) in two respects. First, we consider both left- and right-coprime factor descriptions. Second, even for right-coprime factorization based descriptions, the generality of the result of Hammer (1994) prevents ready specializations that facilitate the resolution of the identification problem considered here. Section 5 contains concluding remarks.

2. THE CLASS OF PLANTS: NOISE-FREE CASE

In this section, given an LTV controller K, we provide a characterization of all NLTV plants stabilized by K. At this point v in Fig. 1 will be assumed zero.

In the sequel an operator A will be called *well posed* if with

$$z = Ax$$

the time function z can be causally, uniquely determined from the time function x for all bounded x. The particular choice of norm does not affect future development. One may choose any among $\|\cdot\|_1$, $\|\cdot\|_2$, $\|\cdot\|_\infty$. We shall say that A^{-1} exists, i.e. A is *invertible*, if A^{-1} is well posed. We shall call A *stable* if for all bounded x,z is bounded. Closed loops such as in Fig. 1, with $w=0$, will be called, (i) well posed if $[e, \bar{u}, u, y]$ can be causally, uniquely determined from r_1 and r_2 for all bounded r_1 and r_2; (ii) *internally stable* if $[e, \bar{u}, u, y]$ is bounded for all bounded r_1 and r_2. When $w \neq 0$, r_1 and r_2 in the foregoing must be replaced by r_1, r_2 and w.

It is worth recounting that with A and B NLTV operators and x and z suitable signals, whereas by definition

$$(A+B)x = Ax + Bx,$$

in general

$$A(x+z) \neq Ax + Az.$$

The following standing assumptions apply throughout the paper (see Khargonekar and Rotea, 1989).

Assumption 2.1. The controller K is linear and has left- and right-coprime factors (U_l, V_l) and (U_r, V_r) respectively, i.e. (i) with U_l, V_l, U_r and V_r all linear, stable and well posed, one has

$$K = U_r V_r^{-1} = V_l^{-1} U_l; \tag{1}$$

and (ii) there exist linear, stable well-posed operators N_l, D_l, N_r and D_r, with D_l and D_r invertible, such that

$$U_l N_r + V_l D_r = I, \tag{2}$$

$$N_l U_r + D_l V_r = I \tag{3}$$

and

$$G_0 = N_r D_r^{-1} = D_l^{-1} N_l. \tag{4}$$

Observe that (4) introduces no loss of generality. To see this, note that the fact that K has the coprime factorizations as in (1) implies that there exists a LTV system

$$G_0 = D_l^{-1} N_l,$$

with N_l and D_l linear, stable and well posed and D_l invertible, for which (3) holds (Khargonekar and Rotea, 1989). In other words, K stabilizes the linear plant G_0, whence G_0, being internally stabilizable, has right-coprime, stable, well-posed factors $\bar{N}_r$ and $\bar{D}_r$ with $\bar{D}_r$ invertible, such that

$$G_0 = \bar{N}_r \bar{D}_r^{-1}.$$

Further, since K stabilizes G_0, there exists a *stably invertible* linear operator X such that

$$U_l \bar{N}_r + V_l \bar{D}_r = X.$$

Then choosing

$$N_r = \bar{N}_r X^{-1},$$

$$D_r = \bar{D}_r X^{-1},$$

one finds that N_r and D_r are as in Assumption 2.1.

In a sense, one may view G_0 as a *nominal linear* plant stabilized by K. In a practical setting it is G_0 that one initially has an estimate of, and K is designed on the basis of this knowledge. Thus henceforth we shall refer to G_0 as the nominal plant.

The second assumption we need is as follows.

Assumption 2.2. The nonlinear plant G is well posed.

The main results of this section demonstrate that all NLTV plants stabilized by K can be represented by the settings of Figs 2 and 3, with R any NLTV-stable, well-posed operator, also known as the Youla–Kucera parameter.

Observe that both figures involve the coprime factors of K and G_0. Since Fig. 2 (respectively 3)

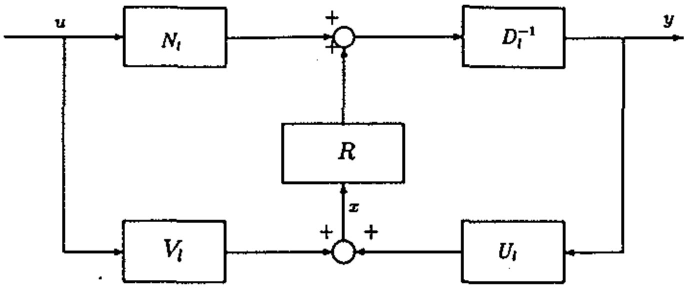

Fig. 2. Left-coprime factorization-based description of G.

involves the left (respectively right) factors, it will be called a left- (respectively right-) coprime factorization-based description of G. At the same time, as will be evident in the sequel, while G as expressed in Fig. 3 always has a right-coprime factorization, G as expressed in Fig. 2 need not always have a left-coprime factorization.

In particular, the scheme of Fig. 3 ensures that

$$y = (N_r + V_r R)x,$$
$$D_r x = u + U_r Rx.$$

Thus

$$y = (N_r + V_r R)(D_r - U_r R)^{-1}u,$$

at least formally. On the other hand, the scheme of Fig. 2 yields

$$D_l y = N_l u + R(V_l u + U_l y).$$

If R is linear then

$$y = (D_l - RU_l)^{-1}(N_l + RV_l)u,$$

at least formally. However, this formula fails for NLTV R (recall in the present setting that R is in general NLTV) and a left-coprime factorization, if it exists at all, does not follow directly. When $R = 0$, either figure reduces to the nominal system G_0 (see (4)).

Sections 2.1 and 2.2 discuss Figs 2 and 3 respectively.

2.1. *Left-coprime factor-based description*

The principal result derived here is as follows.

Theorem 2.1. Under Assumptions 2.1 and 2.2, with $w = 0$, the closed loop in Fig. 1 is well posed and internally stable iff G has a description of the form of Fig. 2, with R a well-posed stable operator.

Observe, that unlike in Tay and Moore (1989) and Paice and Moore (1990), the 'only if' part of the theorem makes no a priori assumption regarding the description of G. Thus we are not starting with the assumption that G has a representation of the form in Fig. 2. Of course, the setting considered in Tay and Moore (1989) and Paice and Moore (1990) is far more general in that all operators in the equivalent of Fig. 2 are permitted to be NLTV. In fact, the setting of Tay and Moore (1989) and Paice and Moore (1990) differs further from the most general form of Fig. 2, since these references require G to have a left-coprime factorization. As noted earlier, an arbitrary plant as in Fig. 2 may not have a left-coprime factorization.

The proof of the theorem requires several intermediate results. In preparation for proving the 'if' part of the theorem, first consider Fig. 4, where, with $w = 0$, the plant in Fig. 1 has been replaced by the description of Fig. 2, in accord with the 'if' statement hypothesis.

Now in this figure, from (1) and the linearity of V_l and U_l,

$$\begin{aligned} x &= V_l u + U_l y \\ &= V_l[r_2 + K(r_1 - y)] + U_l y \\ &= V_l r_2 + U_l(r_1 - y) + U_l y \\ &= V_l r_2 + U_l r_1. \end{aligned} \quad (5)$$

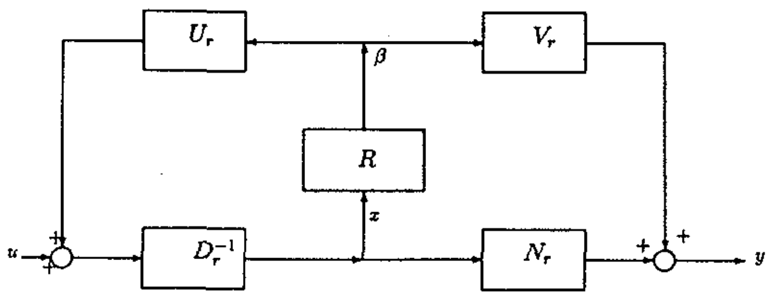

Fig. 3. Right-coprime factorization-based description of G.

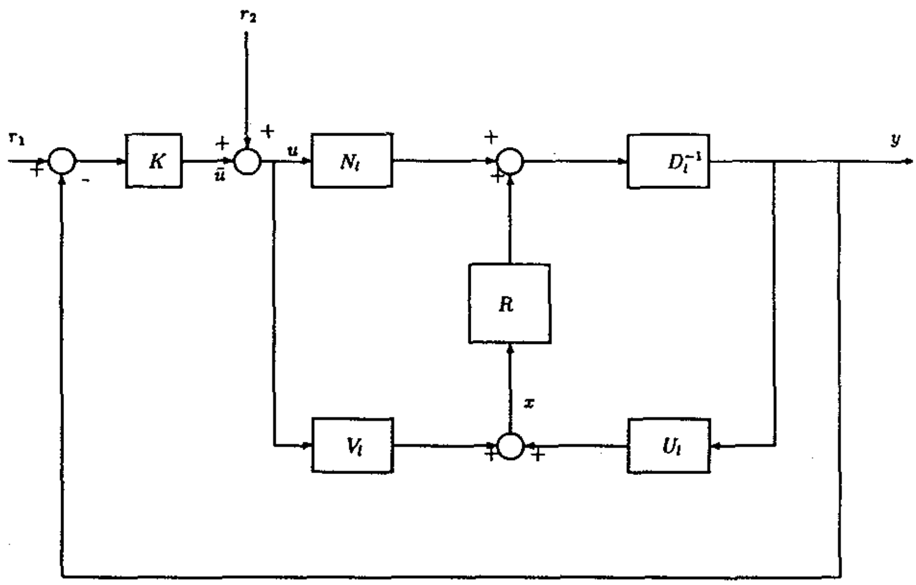

Fig. 4. Closed loop with G as in Fig. 2.

Observe for all bounded r_1 and r_2, that x is bounded and can be causally uniquely determined from r_1 and r_2. Further, from the linearity of N_l and K,

$$\begin{aligned} D_l y &= Rx + N_l u \\ &= Rx + N_l r_2 + N_l K r_1 - N_l K y. \end{aligned} \quad (6)$$

Thus, using (3) and the linearity of N_l, K, D_l and V_r, we obtain

$$(N_l K + D_l) y = Rx + N_l r_2 + N_l K r_1,$$

or

$$(N_l U_r + D_l V_r) V_r^{-1} y = Rx + N_l r_2 + N_l K r_1.$$

Hence

$$\begin{aligned} y &= V_r Rx + V_r N_l r_2 + V_r N_l U_r V_r^{-1} r_1 \\ &= V_r Rx + V_r N_l r_2 + V_r (I - D_l V_r) V_r^{-1} r_1 \\ &= V_r Rx + V_r N_l r_2 + r_1 - V_r D_l r_1. \end{aligned} \quad (7)$$

Similarly,

$$\begin{aligned} \bar{u} &= K(r_1 - y) \\ &= Kr_1 - KV_r Rx - KV_r N_l r_2 - Kr_1 + KV_r D_l r_1 \\ &= -U_r Rx - U_r N_l r_2 + U_r D_l r_1. \end{aligned} \quad (8)$$

Then we have the following lemma.

Lemma 2.1. Under Assumption 2.1, the closed loop in Fig. 4 is well posed and internally stable iff R is a well-posed stable operator.

Proof. Assume R is well posed and stable. Then the result follows from (5), (7) and (8) and the fact that

$$\bar{u} = u - r_2,$$
$$e = r_1 - y.$$

Conversely, suppose the loop in Fig. 4 is well posed and stable. From (7), (8) and (3),

$$D_l y - N_l \bar{u} - N_l r_2 = Rx.$$

Now suppose, to obtain a contradiction, that $R\hat{x}$ either cannot be causally uniquely determined from or is unbounded for some bounded $\hat{x}$. Choose

$$r_1 = N_r \hat{x}, \quad r_2 = D_r \hat{x}.$$

Then, from (5),

$$x = V_l r_2 + U_l r_1 = (V_l D_r + U_l N_r)\hat{x} = \hat{x}.$$

It follows that $D_l y - N_l \bar{u} - N_l r_2 = R\hat{x}$ is either unbounded or cannot be causally determined from $\hat{x}$, thereby leading to a contradiction. □

We have in fact proved the 'if' part of Theorem 2.1. To prove the 'only if' part, first consider the following lemma.

Lemma 2.2. Under Assumptions 2.1 and 2.2, with $w = 0$, suppose the closed loop in Fig. 1 is well posed. Then $(V_l + U_l G)^{-1}$ exists.

Proof. Assume $w(t) = 0$. Suppose for some bounded signal $\alpha(t)$,

$$r_1(t) = N_r \alpha(t), \quad (9)$$
$$r_2(t) = D_r \alpha(t). \quad (10)$$

Since the closed loop and N_r and D_r are well posed, the signal

$$u(t) = Ke(t) + D_r \alpha(t) \quad (11)$$

is causally, uniquely determinable from $\alpha(t)$. Now, in Fig. 1, from (9) and (10), one has

$$\begin{aligned} e &= N_r\alpha - y \\ &= N_r\alpha - G(Ke + D_r\alpha). \end{aligned} \tag{12}$$

Thus for all bounded $\alpha(t)$,

$$Ke + D_r\alpha = D_r\alpha + KN_r\alpha - KG(Ke + D_r\alpha),$$

whence

$$(I + KG)(Ke + D_r\alpha) = D_r\alpha + KN_r\alpha,$$

or

$$V_l^{-1}(V_l + U_lG)u = D_r\alpha + V_l^{-1}U_lN_r\alpha,$$

or

$$(V_l + U_lG)u = (V_lD_r + U_lN_r)\alpha \tag{13}$$

$$= \alpha. \tag{14}$$

Thus, since u can be uniquely, causally determined from α, the result holds. □

The next lemma is relevant to the 'only if' part of Theorem 2.1 and proves the existence of a well-posed R through which Fig. 2 describes G. Boundedness of R is dealt with in Lemma 2.4.

Lemma 2.3. Under Assumptions 2.1 and 2.2, with $w = 0$, suppose the closed loop in Fig. 1 is well posed and internally stable. Then there exists a well-posed R given by

$$R = (D_lG - N_l)(V_l + U_lG)^{-1} \tag{15}$$

such that in Fig. 2

$$y = Gu.$$

Proof. By Lemma 2.2, R as in (15) is well posed. It remains to show that the setting of Fig. 2, with R as in (15), leads to $y = Gu$. From the figure,

$$D_ly = N_lu + R(V_lu + U_ly). \tag{16}$$

Hence

$$y = G_0u + D_l^{-1}R(V_lu + U_ly),$$

or

$$V_lu + U_ly = V_lu + U_lG_0u + U_lD_l^{-1}R(V_lu + U_ly),$$

or

$$(I - U_lD_l^{-1}R)(V_lu + U_ly) = (V_l + U_lG_0)u. \tag{17}$$

Now,

$$\begin{aligned} I &- U_lD_l^{-1}R \\ &= I - U_lD_l^{-1}(D_lG - N_l)(V_l + U_lG)^{-1} \\ &= I - U_l(G - G_0)(V_l + U_lG)^{-1} \\ &= [V_l + U_lG - U_l(G - G_0)](V_l + U_lG)^{-1} \\ &= (V_l + U_lG_0)(V_l + U_lG)^{-1} \\ &= (V_l + U_lN_rD_r^{-1})(V_l + U_lG)^{-1} \\ &= (V_lD_r + U_lN_r)D_r^{-1}(V_l + U_lG)^{-1} \\ &= D_r^{-1}(V_l + U_lG)^{-1}. \end{aligned} \tag{18}$$

Substituting into (17), we have

$$\begin{aligned} V_lu + U_ly &= (V_l + U_lG)D_r(V_l + U_lG_0)u \\ &= (V_l + U_lG)D_r(V_lD_r + U_lN_r)D_r^{-1}u \\ &= (V_l + U_lG)u. \end{aligned}$$

Thus, in (16)

$$\begin{aligned} D_ly &= N_lu + R(V_l + U_lG)u \\ &= [N_l + (D_lG - N_l)(V_l + U_lG)^{-1}(V_l + U_lG)]u \\ &= D_lGu. \end{aligned} \tag{19}$$

Hence, since D_l is invertible, $y = Gu$. □

Thus the well-posedness of the closed loop in Fig. 1 with $w = 0$ has been shown to imply the existence of a well-posed R such that G is described by Fig. 2. It remains to show that internal stability of Fig. 1 with $w = 0$ implies the stability of R.

Lemma 2.4. Under Assumptions 2.1 and 2.2, suppose the closed loop in Fig. 1 with $w = 0$ is well posed and internally stable. Then R in (15) is a stable operator.

Proof. From Lemma 2.3, under $w = 0$, one can indeed describe Fig. 1 by Fig. 4. Notice that y, $\bar{u}$ and x are bounded, by (5), (7) and (8). Hence the scheme of Fig. 4 is both well posed and internally stable. Then, by Lemma 2.1, R is stable. □

Lemmas 2.1–2.4 thus prove Theorem 2.1.

We conclude this subsection by expressing G in terms of R. In Fig. 2

$$D_ly = N_lu + R(V_lu + U_ly).$$

It follows that

$$y = G_0u + D_l^{-1}R(V_lu + U_ly), \tag{20}$$

or

$$V_lu + U_ly = V_lu + U_lG_0u + U_lD_l^{-1}R(V_lu + U_ly).$$

Then, using (18), we note that $I - U_l D_l^{-1} R$ is invertible, whence

$$V_l u + U_l y = (I - U_l D_l^{-1} R)^{-1}(V_l + U_l G_0)u.$$

Hence, from (20),

$$G = G_0 + D_l^{-1} R (I - U_l D_l^{-1} R)^{-1}(V_l + U_l G_0) \quad (21)$$

$$= G_0 + D_l^{-1} R (I - U_l D_l^{-1} R)^{-1} D_r^{-1}. \quad (22)$$

As far as we can tell, this expression cannot be reduced to a left-coprime factor description for G, though in Section 4 a further minor simplification is given. When R is linear, one can of course obtain a left-coprime factor description, exploiting initially the fact that

$$R(I - U_l D_l^{-1} R)^{-1} = (I - R U_l D_l^{-1})^{-1} R,$$

something that is not in general guaranteed for nonlinear R. Observe though that (21) provides a description for G that involves only the left factors of K and G_0. In this sense it is left-coprime factorization-based.

2.2. *Right-coprime factor-based description*

The principal result derived here is the following.

Theorem 2.2. Under Assumptions 2.1 and 2.2, with $w = 0$, the closed loop in Fig. 1 is well posed and internally stable iff G has a description of the form of Fig. 3, with R a well-posed stable operator.

As noted earlier, this theorem follows from Verma (1988) by simply exchanging the controller and the plant. Here we give formulas for R and G in terms of each other. In the sequel, whenever an inverse appears, using techniques in Verma (1988) one can show that its existence follows from the well-posedness of the underlying closed loop.

As noted earlier, in Fig. 3 one readily obtains

$$G = (N_r + V_r R)(D_r - U_r R)^{-1}, \quad (23)$$

which is in the right-coprime factor form. Also, from (23),

$$N_r + V_r R = G(D_r - U_r R), \quad (24)$$

whence

$$R = -V_r^{-1} N_r + V_r^{-1} G(D_r - U_r R). \quad (25)$$

Also, (24), $K = U_r V_r^{-1}$ and minor algebra yield

$$D_r - U_r R = D_r + K N_r - K G(D_r - U_r R),$$

whence

$$D_r - U_r R = (I + KG)^{-1}(D_r + K N_r).$$

Substituting into (25), we thus have

$$R = -V_r^{-1} N_r + V_r^{-1} G(I + KG)^{-1}(D_r + K N_r). \quad (26)$$

Again a further minor simplification to this expression is suggested in Section 4.

2.3. *Connection between the right- and left-coprime factor-based description*

The two characterizations of Sections 2.1 and 2.2 both involve a well-posed stable system described by an operator R. The content of Theorem 2.3 below is that R is the same for both descriptions. Before presenting and proving this theorem, we provide a lemma useful in its proof.

Lemma 2.5. Under Assumption 2.1, it also holds that

$$D_r V_l + U_r N_l = I, \quad (27)$$

$$N_r U_l + V_r D_l = I, \quad (28)$$

$$D_r U_l = U_r D_l \leftrightarrow U_l D_l^{-1} = D_r^{-1} U_r, \quad (29)$$

$$N_r V_l = V_r N_l \leftrightarrow N_l V_l^{-1} = V_r^{-1} N_r. \quad (30)$$

Proof. Under (1)–(4), and the linearity of the operators appearing in these equations,

$$\begin{bmatrix} V_l & U_l \\ -N_l & D_l \end{bmatrix}\begin{bmatrix} D_r & -U_r \\ N_r & V_r \end{bmatrix} = I.$$

Thus

$$\begin{bmatrix} D_r & -U_r \\ N_r & V_r \end{bmatrix}\begin{bmatrix} V_l & U_l \\ -N_l & D_l \end{bmatrix} = I;$$

hence the result follows. □

We now present the promised theorem.

Theorem 2.3. Assume that Assumptions 2.1 and 2.2 hold. Then the operator labelled R in Fig. 2 is identical to the operator labelled R in Fig. 3.

Proof. For the right-coprime factor-based characterization of Fig. 3, we have from (26) that

$$R = -V_r^{-1} N_r + V_r^{-1} G(I + KG)^{-1}(D_r + K N_r)$$

$$= -V_r^{-1} N_r + V_r^{-1} G(I + V_l^{-1} U_l G)^{-1}(D_r + V_l^{-1} U_l N_r)$$

$$= -V_r^{-1} N_r + V_r^{-1} G(V_l + U_l G)^{-1}(V_l D_r + U_l N_r).$$

$$= V_r^{-1}[-N_r(V_l + U_l G) + G](V_l + U_l G)^{-1}$$

$$= V_r^{-1}[-N_r V_l + (I - N_r U_l) G](V_l + U_l G)^{-1}.$$

Now it follows from (30) and (28) that R is as in (15), the expression for R in Fig. 2.

3. INCORPORATION OF MEASUREMENT NOISE

In this section the structures considered in Section 2 will be expanded to include the measurement noise in such a way as to convert

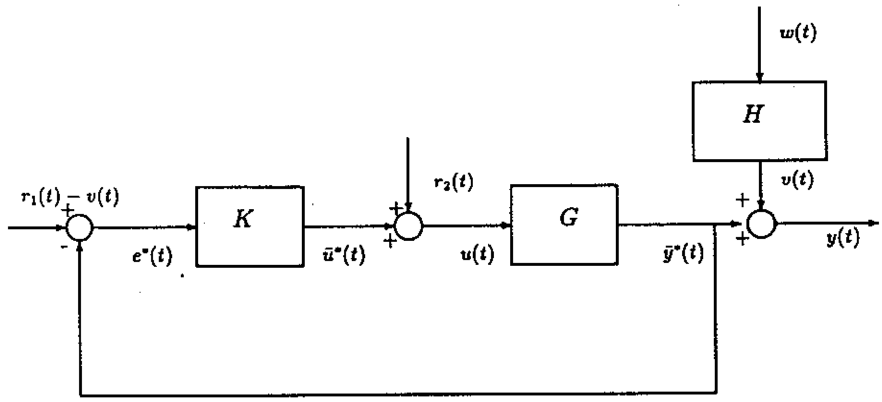

Fig. 5. Redrawn Fig. 1.

the closed-loop identification problem into an open loop one. This next section reveals how the noise incorporation carried out here facilitates the identification process.

Before proceeding further, we show how the various closed-loop signals in the noise-free case compare with those in the noisy case.

Compare Figs 1 and 5. Focus on the way r_1 and v enter in Fig. 1, as compared with the way $r_1 - v$ enters in Fig. 5. It is not hard to see that

$$\begin{aligned} \bar{u}^* &= \bar{u}, \\ u^* &= u, \\ \bar{y}^* &= \bar{y}, \\ e^* &= e. \end{aligned} \tag{31}$$

Also,

$$y = \bar{y}^* + v. \tag{32}$$

Using this relationship, we shall show how to modify Figs 2 and 3 to introduce noise in Sections 3.1 and 3.2 respectively.

3.1. *Noise in the left-coprime factor-based description*

In keeping with the results of Section 2.1, we shall assume that the open-loop NLTV plant has the description of Fig. 2. Equations (5) and (7) then apply, with r_1 replaced by $r_1 - v$ and y replaced by $y - v$ (the latter in light of (32)). Accordingly, the output in Fig. 1 is given by

$$\begin{aligned} y = V_r R[V_l r_2 + U_l(r_1 - v)] \\ + V_r N_l r_2 + (I - V_r D_l)(r_1 - v) + v. \end{aligned} \tag{33}$$

Then the principal result of this section is as follows.

Theorem 3.1. Consider the closed loop of Fig. 1, with G as in Fig. 2 and Assumptions 2.1 and 2.2 in force. Then the signals in Fig. 6 relate to those of Fig. 1 by $\hat{u} = u$, $\hat{e} = e$, $\hat{\bar{u}} = \bar{u}$ and $\hat{y} = y$.

Evidently the setting of Fig. 6 captures

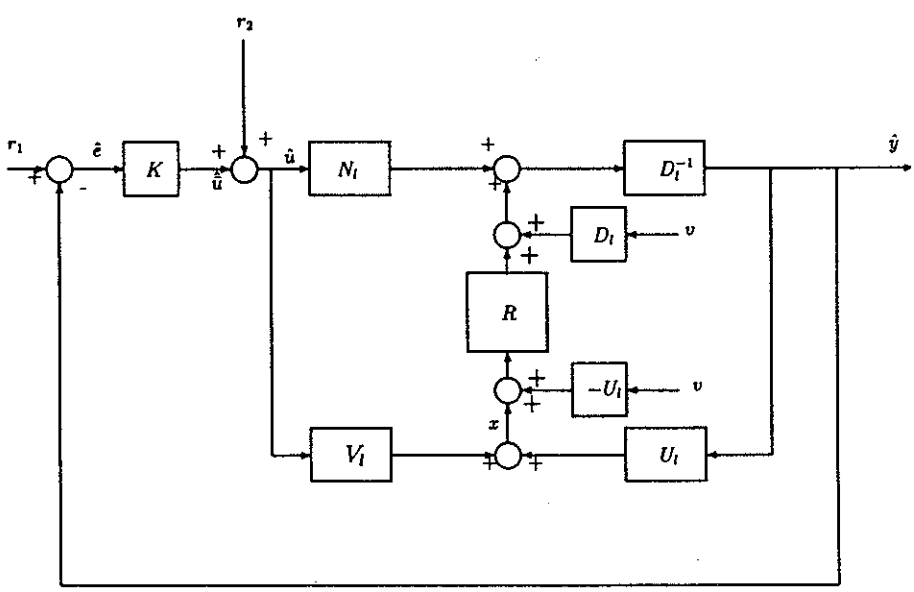

Fig. 6. Noise-incorporated left-coprime factorization-based description.

completely the setting of Fig. 1. Further, it represents a variant of Fig. 2 in that the noise v enters at two points. It is interesting to note that for linear plants, the noise v enters through only one block, namely one directly above R (Hansen, 1989; Hansen and Franklin, 1988; Hansen *et al.*, 1989). The nonlinearity of R in general necessitates entry also via the second block below R. Of course, with linear R, simple manipulation of the block diagram of Fig. 6 allows the two blocks to be replaced by a single block.

Proof. We prove Theorem 3.1 somewhat informally. Notice that to prove the theorem, we need *only* show that

$$\hat{y} = y,$$

since the remaining equalities will then follow very simply. Now, in Fig. 6

$$\begin{aligned} x &= V_l[r_2 + K(r_1 - \hat{y})] + U_l\hat{y} \\ &= V_l r_2 + U_l(r_1 - \hat{y}) + U_l\hat{y} \\ &= V_l r_2 + U_l r_1. \end{aligned} \tag{34}$$

Then

$$D_l\hat{y} = D_l v + R(x - U_l v) + N_l[r_2 + K(r_1 - \hat{y})].$$

Thus

$$(D_l + N_l U_r V_r^{-1})\hat{y} = D_l v + R(x - U_l v) + N_l r_2 + N_l K r_1.$$

Hence, using the Bézout identity (3), we obtain

$$V_r^{-1}\hat{y} = D_l v + R(x - U_l v) + N_l r_2 + N_l U_r V_r^{-1} r_1.$$

Thus, from (3),

$$\begin{aligned} \hat{y} &= V_r D_l v + V_r R(x - U_l v) + V_r N_l r_2 \\ &\quad + V_r(I - D_l V_r)V_r^{-1} r_1 \\ &= V_r D_l v + V_r R(V_l r_2 + U_l r_1 - U_l v) \\ &\quad + V_r N_l r_2 + (I - V_r D_l) r_1 \\ &= y, \end{aligned}$$

where the last equality follows from (33). This proves Theorem 3.1. □

3.2. *Noise in the right-coprime factor-based description*

We assume now that the open-loop NLTV plant G has the description of Fig. 3.

Before proceeding further, we need an expression for y in Fig. 1 based on this description of G. Consider first the noiseless closed loop as depicted in Fig. 7. Subsequently, we shall allow for the noise $v(\cdot)$.

Then

$$\begin{aligned} D_r x &= U_r\beta + u \\ &= U_r V_r^{-1}(y - N_r x) + r_2 + K(r_1 - y) \qquad (35) \\ &= -KN_r x + r_2 + K r_1. \qquad (36) \end{aligned}$$

Hence

$$(KN_r + D_r)x = r_2 + V_l^{-1}U_l r_1.$$

Using (2), this readily yields

$$x = V_l r_2 + U_l r_1. \tag{37}$$

This is the same expression as available in (5) for the input of the R-block in the right-coprime based description of G used in Fig. 4.

Now, from (37), in Fig. 7

$$\begin{aligned} y &= (N_r + V_r R)x \\ &= (N_r + V_r R)(V_l r_2 + U_l r_1). \end{aligned} \tag{38}$$

Now, allowing for the presence of noise in the manner described in Section 3.1, in Fig. 1 we must have

$$y = v + (N_r + V_r R)[V_l r_2 + U_l(r_1 - v)]. \tag{39}$$

Then the main result of this Section, i.e. the dual to Theorem 3.1, is as follows.

Theorem 3.2. Consider the closed loop of Fig. 1, with G as in Fig. 3 and Assumptions 2.1 and 2.2

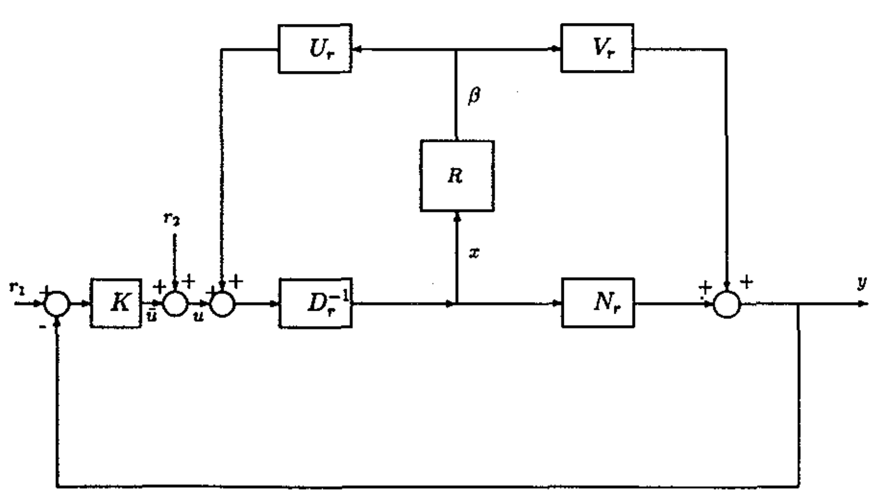

Fig. 7. Noise-free closed loop for right-coprime factorization-based description.

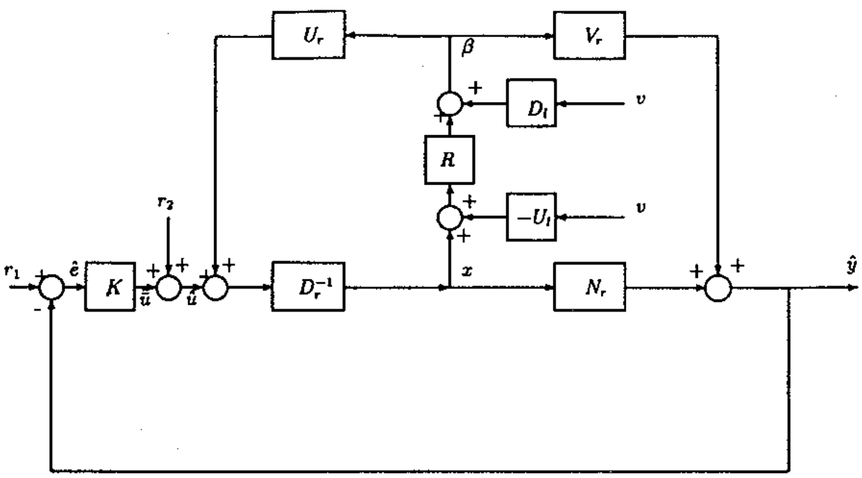

Fig. 8. Noise-incorporated right-coprime factorization-based description.

in force. Then the signals in Fig. 8 relate to those of Fig. 1 by $\hat{u} = u$, $\hat{e} = e$, $\hat{\bar{u}} = \bar{u}$ and $\hat{y} = y$.

Proof. We need only show that $\hat{y}$ in Fig. 8 equals the y of (39). Observe in Fig. 8 that

$$D_r x = U_r V_r^{-1}(\hat{y} - N_r x) + r_2 + K(r_1 - \hat{y}).$$

Then, using considerations similar to those used before, we have

$$x = V_l r_2 + U_l r_1. \tag{40}$$

Again, this is the same expression as applies for the scheme of Fig. 6, based on left-coprime factorization-based description of the noisy plant. Then

$$\begin{aligned}\hat{y} &= N_r x + V_r[D_l v + R(x - U_l v)] \\ &= N_r(V_l r_2 + U_l r_1) \\ &\quad + V_r D_l v + V_r R[V_l r_2 + U_l(r_1 - v)] \\ &= (N_r U_l + V_r D_l)v \\ &\quad + (V_r R + N_r)[V_l r_2 + U_l(r_1 - v)] \\ &= v + (V_r R + N_r)[V_l r_2 + U_l(r_1 - v)], \end{aligned} \tag{41}$$

where the last equality exploits (28). Then comparing (41) with (39) proves the result. □

As with Section 3.1, the linearity of R would obviate the need for having v enter at two separate places in Fig. 8. It is, moreover, important and intriguing that the blocks through which v enters in Fig. 8, the operator R and the signals x and β are identical in both Figs 6 and 8.

4. IMPLICATIONS FOR CLOSED-LOOP IDENTIFICATION

Our basic premise in this section is that the difficulties associated with closed-loop system identification can be circumvented through the structures proposed in the previous sections. The approach advocated here is that in both the left- and right-coprime factorization based characterizations that we have presented, instead of identifying G directly, one should identify the Youla–Kucera parameter R. That such an identification is tantamount to identifying G is evident from the results of the previous section. Indeed, for the left-coprime factorization case, (22) provides the connection. In fact, using (29), (22) reduces further to

$$G = G_0 + D_l^{-1}R(D_r - U_r R)^{-1}. \tag{42}$$

Likewise, for the right-coprime factorization case, G is as in (23).

Also important (see Section 4.2) is the conversion in the reverse direction, either through (15) in the left-coprime factorization case or through (26) in the right case. Indeed, (26) can be re-expressed as

$$R = -V_r^{-1}N_r + V_r^{-1}G(V_l + U_l G)^{-1}. \tag{43}$$

As an interesting aside, observe the correspondence between (42) and (43); and (15) and (23).

More compelling, however, is the dependence on R of the closed-loop model error. Observe from (5) and (7), in the noise-free case of the left-coprime factorization-based characterization, that with R_1 an estimate of the true Youla parameter R, and y_1 the corresponding output,

$$y - y_1 = V_r(R - R_1)(U_l r_1 + V_l r_2). \tag{44}$$

Similarly, in the dual case, from (37) and (38),

$$y - y_1 = V_r(R - R_1)(U_l r_l + V_l r_2), \tag{45}$$

an expression identical to (44). Thus in both cases the error in the closed-loop response is directly proportional to that in the estimation of R.

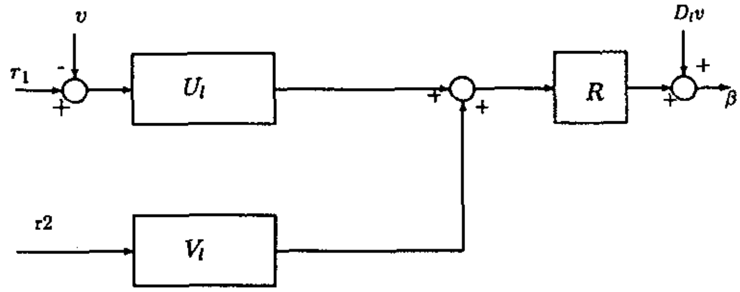

Fig. 9. Depiction of (46).

Thus it is indeed appropriate that one focuses on the estimation of R. To this end, observe that in both Figs 6 and 8

$$\beta = D_l v + R[U_l(r_1 - v) + V_l r_2]. \qquad (46)$$

Observe that the r_i are obviously uncorrelated with v. Essentially then, one obtains the setting depicted in Fig. 9.

Observe that β can be measured using u and y, and is bounded provided the closed loop is stable, e.g., for Fig. 6, with $\hat{y} = y$,

$$\beta = D_l y - N_l u. \qquad (47)$$

Likewise, in Fig. 8, with $\hat{u} = u$ and $\hat{y} = y$,

$$\beta = V_r^{-1}(y - N_r x).$$

Thus

$$V_r \beta = y - N_r D_r^{-1}(u + U_r \beta);$$

whence

$$(V_r + N_r D_r^{-1} U_r)\beta = y - N_r D_r^{-1} u,$$

or

$$(V_r + N_r D_r^{-1} U_r)\beta = y - D_l^{-1} N_l u,$$

or

$$D_l^{-1}\beta = y - D_l^{-1} N_l u,$$

the last equality following from (3). Thus again (47) holds.

If one focuses for the moment on identifying the blocks RU_l and RV_l in Fig. 9, one notices that the setting of Fig. 9 still involves correlation of the input and the measurement noise processes, though this correlation is now a priori known, since D_l is known. Nonetheless, a conversion to a pure open-loop identification would require *no correlation* between the input and the output.

On the other hand, suppose one is prepared to make a small-signal assumption on v. Then, with δR the linearization of R around the operating trajectory produced by x, (46) becomes

$$\begin{aligned}\beta &= (D_l - \delta R U_l)v + R(U_l r_1 + V_l r_2)\\ &= (D_l - \delta R U_l)Hw + R(U_l r_1 + V_l r_2). \qquad (48)\end{aligned}$$

Now one has the setting of Fig. 10, where the input to the system we must identify is uncorrelated with the measurement noise. This is indeed a true open-loop setting. If one assumes further that w is zero-mean then any identification procedure that involves the correlation of β (or delayed versions there of) with the r_i and their delayed versions will automatically remove the effect of w, thereby providing an estimate of the noise-free part of Fig. 10.

Notice, however, one additional difficulty. All this leads directly to the identification of RU_l (when $r_2 = 0$) and RV_l (when $r_1 = 0$), as opposed to that of R. As such, U_l need not be invertible and V_l^{-1} may not be stable. Thus a unique determination of R cannot directly be concluded.

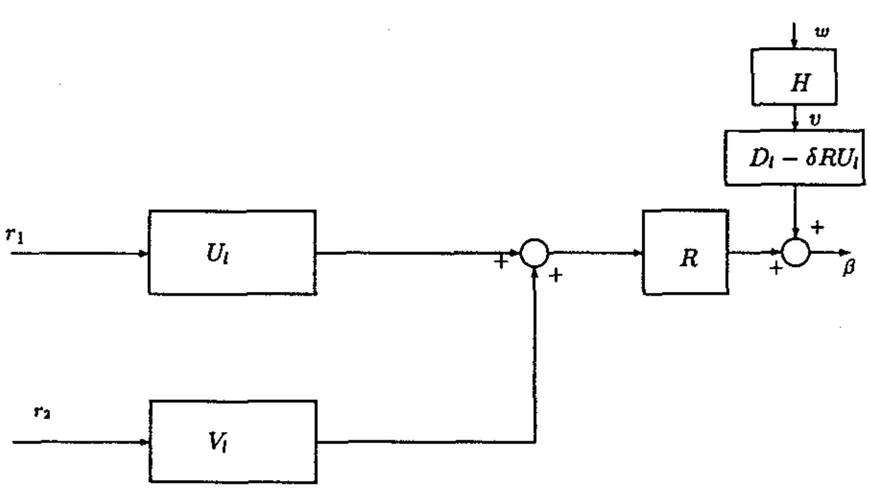

Fig. 10. Open-loop identification with small w.

This difficulty can, however, be circumvented by a suitable design of the reference inputs r_i. As an example, choose, for a signal α independent of w,

$$r_1 = N_r\alpha,$$

$$r_2 = D_r\alpha.$$

Then, from (2), the noise-free part of β is simply $R\alpha$.

An interesting consequence of the foregoing is that to stably identify R, *both* r_1 *and* r_2 *must be nonzero*. The absence of the latter and the possible noninvertibility of U_l will render the recovery of R impossible. The absence of r_1 and the possible instability of V_l^{-1} will preclude consistent estimation of R.

Under these conditions, one can use Wiener-kernel-based (see Wiener, 1958; Schetzer, 1980) identification techniques to identify R.

5. CONCLUSIONS

We have considered the closed-loop identification of nonlinear plants stabilized by linear, albeit time-varying, feedback. The main focus has been on deriving factorization-based structures that help convert the underlying closed-loop identification problem to one that is essentially one of open-loop identification. To what extent these results generalize to more general plants, i.e. those not necessarily stabilizable by LTV controllers, is an interesting open issue.

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REFERENCES

Hammer, J. (1994). Internally stable nonlinear systems with disturbances: a parametrization. *IEEE Trans. Autom. Control*, **AC-39**, 300–315.

Hansen, F. (1989). A fractional representation approach to closed loop system identification and experimental design. PhD thesis, Stanford University.

Hansen, F. and G. Franklin (1988). On a fractional representation approach to closed loop experiment design. In *Proc. American Control Conf.*, Atlanta, GA, pp. 1319–1320.

Hansen, F., G. Franklin and R. L. Kosut (1989). Closed loop identification via the fractional representation approach: experiment design. In *Proc. American Control Conf.*, Pittsburg, PA, pp. 1422–1427.

Khargonekar, P. P. and M. Rotea (1989). Coprime factorization for linear time varying systems. In *Proc. American Control Conf.*, Atlanta, GA, pp. 848–851.

Lee, W. S., B. D. O. Anderson, R. L. Kosut and I. M. Y. Mareels (1993). A new approach to adaptive robust control. *Int. J. Adaptive Control Sig. Process.*, **7**, 183–211.

Paice, A. D. B. and J. B. Moore (1990). Robust stabilization of nonlinear plants via left coprime factorizations. *Syst. Control Lett.*, **15**, 1235–1248.

Schetzer, M. (1980). *The Volterra and Wiener Theories of Nonlinear Systems.* Wiley, New York.

Tay, T. T. and J. B. Moore (1989). Left coprime factorizations and a class of stabilizing controllers for nonlinear systems. *Int. J. Control*, **49**, 1235–1248.

Van den Hof, P. M. J. and R. J. P. Schrama (1994). Identification and control-closed loop issues. In *Preprints 10th IFAC Symp. on System Identification*, Copenhagen.

Verma, M. S. (1988). Coprime fractional representations and stability of nonlinear feedback systems. *Int. J. Control*, **48**, 897–918.

Vidyasagar, M. (1985). Control Systems Synthesis: A Factorization Approach. MIT Press, Cambridge, MA.

Wiener, N. (1958). *Nonlinear Problems in Random Theory.* MIT Press, Cambridge, MA.

Zhang, Z., R. R. Bitmead and M. Gevers (1991). H_2 iterative model refinement and control robustness enhancement. In *Proc. 30th IEEE Conf. on Decision and Control*, Brighton, U.K., pp. 279–284.