On the Robustness of FIR Channel Identification from Fractionally Spaced Received Signal Second-Order Statistics

T. J. Endres, B. D. O. Anderson, C. R. Johnson, Jr., and L. Tong

***Abstract*— A number of recent papers have studied blind channel identification/equalization algorithms relying purely on second-order statistics of the fractionally sampled received signal. A class of cyclic allpass channels is shown to be the only class of IIR channels whose cascade with a core channel results in no change in the second-order-statistics indicators used by several of these algorithms. FIR approximations to the poles of these allpass channels result in robustness concerns for the second-order-statistics algorithms, even when FIR identifiability conditions are well satisfied. This sensitivity suggests that excluding only those channels with common subchannel roots is insufficient when applying these second-order-statistics algorithms.**

I. Introduction

The algorithm in [1] and numerous others ([2], [3], [5]–[7], for example) rely for the identification of a $T/2$-spaced channel transfer function $H(z)$ on the uniqueness of the $\Gamma^0(z), \Gamma^1(z)$ pair, where the $\Gamma^i(z)$ are derived from the power spectral density of the received data, yielding the identification equations

$$\Gamma^0(z) = H(z)H(z^{-1}) \quad \text{and} \quad \Gamma^1(z) = H(z)H(-z^{-1}). \quad (1)$$

Excluding $T/2$-spaced channels with zeros reflected through the origin, an FIR channel may be estimated from the common zeros of the $\Gamma^0(z), \Gamma^1(z)$ pair. The issue addressed in this letter is whether or not the exclusion of just FIR channels with such undesirable reflected zero patterns (or their close approximation) guarantees a robust estimation task in ascertaining the roots of $H(z)$ as the common roots of $\Gamma^0(z)$ and $\Gamma^1(z)$. This letter shows, by simple construction, that even when such FIR identifiability conditions are well satisfied, serious robustness problems may arise due to close approximation of nonidentifiable IIR models by an identifiable FIR model without nearly reflected zeros. A general method to construct channels with such sensitivity concerns is presented by example.

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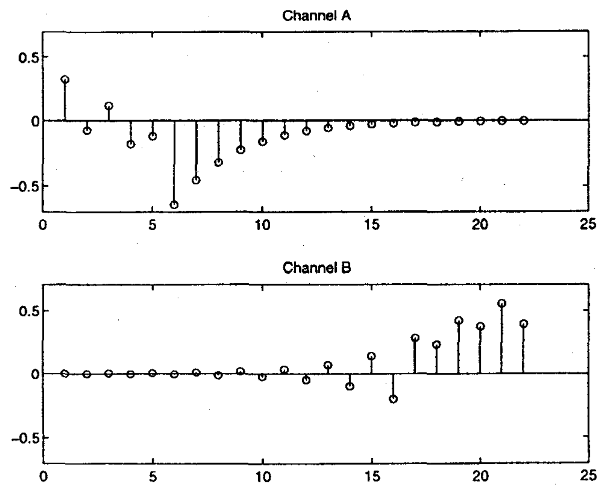

Fig. 1. Channel A and B impulse responses.

TABLE I
CHANNEL-EQUALIZER COMBINATIONS

Chan.	Eq.	ISI*	Comments
A	A	1.96×10^{-10}	Perfectly Equalizable
A	B	2.32	Closed eye
B	B	3.37×10^{-10}	Perfectly Equalizable
B	A	1.66	Closed Eye

*ISI is computed as $ISI = \frac{\sum_i |h_i| - \max_i |h_i|}{\max_i |h_i|}$.

II. Example

Consider an example with two length-22, ℓ_2 unit-norm, FIR $T/2$-spaced channels A and B, both of which are such that their T-spaced subchannels contain no roots that are within a distance of 0.1 to any other root in that subchannel. (Fig. 3 shows the zero locations of the $T/2$-spaced channels.) This nonzero proximity implies that the channels' true roots are correctly extracted from the intersection of zeros in application of (1) since the identifiability conditions of [1] are well satisfied. With $\sim$ denoting approximately equals, the fact that $\Gamma^0(z)$ and $\Gamma^1(z)$ have to be estimated from data and, more generally, good engineering practice should demand the following property of the algorithm of [1].

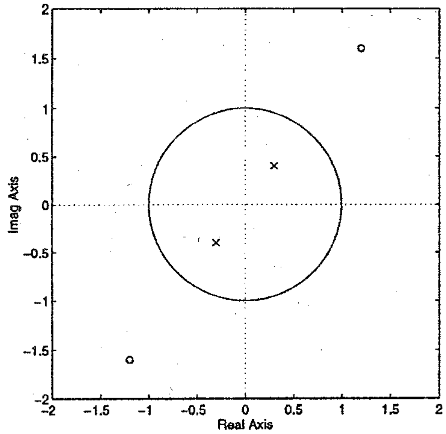

Fig. 2. Specialized allpass example.

Conjecture 1:

$$\Gamma_A^0(z) \sim \Gamma_B^0(z) \quad \text{and} \quad \Gamma_A^1(z) \sim \Gamma_B^1(z) \Rightarrow A(z) \sim B(z). \tag{2}$$

Specifically, forming $\Gamma^0(z)$ and $\Gamma^1(z)$ as described in (1) for channels A and B yields the ℓ_2 unit-norm polynomials, whose coefficients are such that

$$||\Gamma_A^0(z) - \Gamma_B^0(z)|| < 8 \times 10^{-3} \tag{3}$$
$$||\Gamma_A^1(z) - \Gamma_B^1(z)|| < 1 \times 10^{-12}. \tag{4}$$

Conjecture 1 therefore implies that these two channels are similar when, in fact, the channels are grossly different, as suggested by impulse responses in Fig. 1.

Length-22 MMSE $T/2$-spaced equalizers, in fact, result in channel-equalizer combinations that are closed eye when the equalizer is applied to the incorrect channel (see Table I). Thus, Conjecture 1 appears to be false, even though FIR identifiability conditions are well satisfied.

III. IIR Insertion

The premise of identifiability of no reflected roots in $T/2$-spaced FIR channels indicates that there is no FIR cascade with a core channel that leaves the Γ's (and subsequent equalizers) unaltered. Consider, however, the cascade of an IIR structure with a core channel and its effects on the Γ functions.

Theorem 1: Let $K(z)$ be an IIR transfer function, and let $H(z)$ be the transfer function for a core channel. Then, $H(z)$ and $H(z)K(z)$ have the same $\Gamma^0(z), \Gamma^1(z)$ pair *if and only if* $K(z)$ is allpass with $K(z) = K(-z)$.

Proof: Suppose the two channels have the same $\Gamma^0(z), \Gamma^1(z)$ pair. Using (1), it follows that

$$K(z)K(z^{-1}) = 1 \quad \text{and} \quad K(z)K(-z^{-1}) = 1 \tag{5}$$

which implies $K(z) = K(-z)$.

The other direction[1] of the proof follows directly from construction. □

[1]Note that the "if" direction of this theorem is detailed in [4] for arbitrary integer oversampling rates. That is, the cyclic allpass class is shown to yield identical Γ sets. The above theorem shows the cyclic allpass class to be the only such class of IIR cascades yielding equivalent Γ sets.

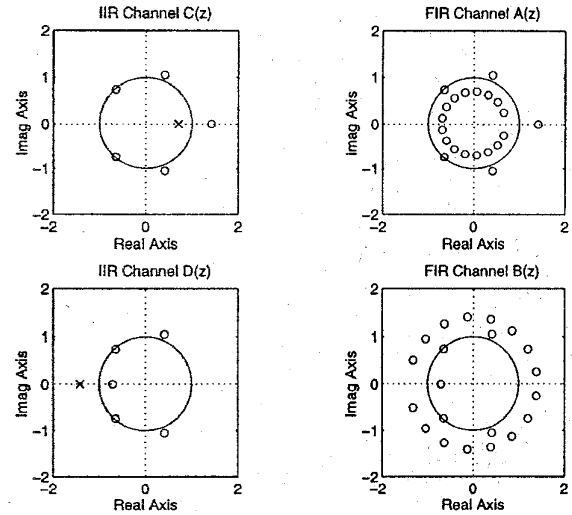

Fig. 3. Rational and FIR channel model roots.

The class of $K(z)$ thus consists of allpass channels with pole patterns symmetric with respect to reflection through the origin, which are referred to as cyclic allpass structures in [4], since they are obtained by allpass rotations. See, for example, Fig. 2. Although the $\Gamma^0(z)$ and $\Gamma^1(z)$, and equalizer computed from them, will be unaltered with the cascade insertion of this cyclic all-pass structure, these allpass insertions ruin perfect equalizability and can even close the eye of the total channel-equalizer combination.

IV. Example Revisited

The example in Section II and its construction can be understood in light of the cyclic allpass effect on the second-order-statistics indicators. Consider first the all-pass belonging to the class described in Theorem 1 with zeros at ± 0.7071 and poles reflected about the origin at $\pm 1/0.7071$. A core channel is described by

$$C(z) = (1 + 0.47z^{-1} + 1.19z^{-2} + 0.86z^{-3} + 1.24z^{-4}) \times \frac{(z - \frac{1}{0.7071})}{(z - 0.7071)} \tag{6}$$

and the cascade of the allpass and $C(z)$ is

$$D(z) = (1 + 0.47z^{-1} + 1.19z^{-2} + 0.86z^{-3} + 1.24z^{-4}) \times \frac{(z - \frac{1}{0.7071})}{(z - 0.7071)} \frac{(z + 0.7071)}{(z + \frac{1}{0.7071})} \frac{(z - 0.7071)}{(z - \frac{1}{0.7071})} \tag{7}$$

See Fig. 3 for pole-zero plots of $C(z)$ and $D(z)$.

Channel A is formed from an FIR approximation of $C(z)$. A length-17 FIR approximation to the pole at 0.7071 results in a ring of 16 zeros evenly spaced around a circle of radius 0.7071 with an absent zero in place of the pole. *Channel B* is formed from an FIR approximation of $D(z)$, which is the cascade of the allpass and core channel. The positive pole and zero of the allpass are canceled by the core channel, and a length-17 FIR approximation to the remaining allpass pole results in a ring of 16 zeros evenly spaced around a circle of radius 1/0.7071 with an absent zero at $z = -1/0.7071$. The root locations for the rational and resulting FIR approximations are described in Fig. 3. Despite the ring of zeros, none are reflections of any other across the origin. (Recall that no two T-spaced subchannels' zeros are closer than 0.1 to any other subchannel zero). Thus, the identifiability condition of [1] for this $T/2$-spaced example is well satisfied. Note that both FIR channels A and B have 21 zeros. These two FIR channels are the ones used in the example of Section II.

This example thus demonstrates how identifiable FIR approximations to IIR channel pairs described in Theorem 1 yield a counterexample to Conjecture 1. (In fact, arbitrarily close $\Gamma^0(z)$ and $\Gamma^1(z)$ approximation is possible with longer FIR's, and only in the limit of an infinite length approximation does the example collapse into the special case of reflected roots, given odd-length approximations). The result of the allpass addition is to yield virtually equal Γ sets for the two channels that produce grossly different channel estimates.

V. Conclusion

This letter demonstrates by a simple example how identifiable FIR approximations to nonidentifiable IIR structures result in robustness concerns for popular second-order-statistics algorithms. This example suggests that the exclusion of just those channels with reflected zeros is insufficient in application of algorithms related to that of [1]. The excluded channels should also include those with zero patterns approximating the cyclic allpass structure. The cyclic allpass class is shown to be the only class of IIR cascades, which results in equivalent Γ indicators. This fact does not imply, however, that the aforementioned identifiable FIR approximations are the only class of channels that prove to be counterexamples to conjecture 1. In fact, our example motivates the search for other root patterns that can result in excessive sensitivity.

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