A proper rational matrix M that has no poles on the imaginary axis is called (J, J')-lossless if $[M(s)]^* JM(s) \leq J'$ for all s in the right-half plane, with equality holding on the imaginary axis. Here * denotes the complex conjugate transpose. The claims are the following.

Claim 1 (Lemma 2.1 of the Above Paper[1]): A proper rational matrix M that has no poles on the imaginary axis is (J, J')-lossless iff there exists a $P > 0$ such that

$$\begin{cases} D^* JD = J'; \\ D^* JC + B^* P = 0; \\ A^* P + PA + C^* JC = 0. \end{cases} \tag{1}$$

Here $[A, B, C, D]$ is any minimal realization of M.

Claim 2 (Corollary 2.3 of the Above Paper[1] and Lemma 3.1 of [3]): Every (J, J')-lossless matrix M has a J-orthogonal complement N. (That is, such that $[M \;\; N]T$ is square and (J, J)-lossless for some permutation matrix T.)

As stated these claims are not fully correct. Consider the following rational matrix and minimal realization

$$M(s) = \frac{1}{s+1} \begin{bmatrix} s\sqrt{2} \\ \sqrt{2} \\ 1-s \end{bmatrix} \stackrel{s}{=} \left[\begin{array}{c|c} -1 & 1 \\ \hline -\sqrt{2} & \sqrt{2} \\ \sqrt{2} & 0 \\ 2 & -1 \end{array}\right]. \tag{2}$$

It is readily checked that $[M(s)]^* J_{2,1} M(s) = 1$ for all s, hence, that M is (J, J')-lossless with $J := J_{2,1}$ and $J' := 1$. The matrix P, however, satisfying (1) is, in this case, $P = 0$, contradicting the first claim. The M in (2) also serves as a counterexample to the second claim. Suppose, to obtain a contradiction, that M defined in (2) does have a J-orthogonal complement N; that is, such that $[M \;\; N]$ is square and (J, J)-lossless, with $J := J_{2,1}$. Since Claim 1 is correct if $J = J'$ (more of this later), any minimal realization $[A, B, C, D]$ of the square rational matrix $[M \;\; N]$ satisfies (1) for some $P > 0$. Then M obviously has realization $[A, BV, C, DV]$ where $V := [1 \; 0 \; 0]^T$. After some manipulation it follows that

$$[M(s)]^* JM(s) = J' - (s + \bar{s}) V^* B^* (\bar{s}I - A^*)^{-1} \times P(sI - A)^{-1} BV. \tag{3}$$

We know that $[M(s)]^* JM(s) = J' = 1$, so the second term on the right in (3) must be identically zero. This cannot be, however, since $P > 0$ and $(sI - A)^{-1} BV$ is nonzero (because $M(s) = C(sI - A)^{-1} BV + DV$ is nonzero, nonconstant). This completes the counterexample to the second claim.

The problems can be fixed by strengthening the assumptions somewhat. In the counterexample the J and J' have a different number of negative eigenvalues (that is, $r \neq \rho$). In $\mathcal{H}_\infty$ control, the type of J-losslessness that is important is when J and J' have the same number of negative eigenvalues.

Lemma 3: Claim 1 and Claim 2 are correct if J and J' have the same number of negative eigenvalues (that is, if $r = \rho$).

This result is known, although it is not stated explicitly in the form of Claims 1 and 2 (see [4]). In the above paper[1] it is shown that Claim 1 implies Claim 2, so we only need to prove Claim 1 for the case that $r = \rho$. If $P > 0$ satisfies (1) then

$$[M(s)]^* JM(s) = J' - (s + \bar{s}) B^* (\bar{s}I - A^*)^{-1} P(sI - A)^{-1} B$$

and, hence, in that case M is (J, J')-lossless. Conversely, suppose the $(m + r) \times (\mu + \rho)$ rational matrix M is (J, J')-lossless and partition M compatibly as

$$M = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}.$$

Since $r = \rho$ we have that M_{22} is square. It then follows (see [2]) that U defined as

$$U := \begin{bmatrix} 0 & I_r \\ M_{11} & M_{12} \end{bmatrix} \begin{bmatrix} I_\mu & 0 \\ M_{21} & M_{22} \end{bmatrix}^{-1} \tag{4}$$

is inner, hence, in particular that U is stable. As in [1], a minimal realization of M easily gives a minimal realization of U. By minimality and stability of U, the observability gramian of U is positive definite and this gramian, call it P, can be shown to satisfy the three conditions (1). Note that we used here that the M_{22} is square, which is the reason we need that $r = \rho$.

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Sampled-Data Controller Reduction Procedure

Anton G. Madievski and Brian D. O. Anderson

Abstract—**The problem of controller order reduction aimed at preserving the closed-loop performance of a sampled-data closed-loop system is investigated. Fast sampling of the system at a multiple of the sampling frequency followed by lifting allows capturing of the system's intersample behavior and yields a time-invariant single-rate system; this then permits standard order-reduction ideas to be applied. Special weighting functions aimed at preserving the closed-loop transfer function are obtained, and weighted balanced truncation is used to reduce the controller. An example shows that without the use of fast-sampling, an unstable closed loop can result from the reduction.**

I. Introduction

The great importance and usefulness of controller reduction is now widely recognized, and much attention has been paid to the subject over the past years. The main reason is that the linear quadratic Gaussian (LQG) and H_∞ design procedures lead to controllers which have order equal to, or roughly equal to, the order of the plant ([2] for LQG). Often, controllers of a lower order will result in acceptable performance and will be desired for their greater simplicity.

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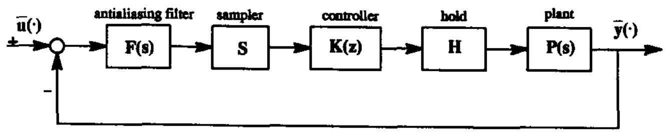

Fig. 1. The closed-loop system.

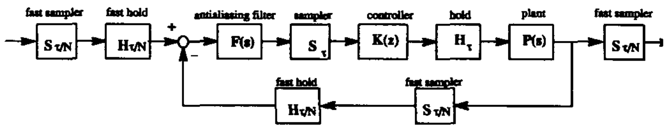

Fig. 2. The fast-sampled closed-loop system.

Model reduction by means of balanced realizations and Hankel-norm approximations has been studied by Moore and Glover [7], [14]. Enns introduced frequency weighting to balanced realizations [5] and applied this approach for maintaining closed-loop stability [6]. Anderson and colleagues have developed a frequency-weighted Hankel-norm technique for controller reduction [1], [13]. None of this earlier work explicitly treated sampled-data systems.

In this paper, our objective is to apply a balanced realization controller order-reduction method to sampled-data closed-loop systems to preserve the closed-loop behavior.

Consider a hybrid closed loop where the plant is continuous time and the controller is discrete time. (Such a configuration represents the usual situation.) This closed loop is drawn in Fig. 1, where P stands for the $p \times m$ continuous-time plant, K for the $m \times p$ discrete controller, F for the strictly proper stable antialiasing filter, S for the sampler with the sampling period τ, and H for the hold element, here assumed to be a zero-order hold. (In the multivariable situation, F, S, and H are diagonal operators.) This is a periodically time-varying sampled-data system. To replace this system by a time-invariant one capturing intersample behavior of the system, one can sample it at a high frequency and then lift the obtained system. (Lifting techniques have been studied in [11].)

There exist frequency-dependent weighting functions on the error between the original and reduced-order controller transfer function matrices with the property that minimizing the weighted error corresponds approximately to minimizing an error between the two closed-loop transfer function matrices. We shall apply a weighted balanced realization technique to reduce the controller.

Unfortunately, reduction based on weighted balanced truncation is limited to (open-loop) stable controllers. One way to handle the problem in the unstable case is to additively decompose the full-order controller transfer function into stable and completely unstable parts with the balanced realization technique applied to the stable part only. The system so obtained is a time-invariant single-rate discrete-time system (with sample interval equal to that of the controller).

An outline of this paper is as follows. In Section II we introduce fast sampling followed by lifting for the closed-loop system in Fig. 1. A time-invariant system results. In Section III we obtain the weighting functions for preserving the closed-loop transfer function and actually reduce the controller by the weighted balanced truncation method. A practical example to confirm the approach is given in Section IV, followed by some concluding remarks in Section V.

II. Fast Sampling and Lifting

The purpose of this section is to introduce the fast sampling and lifting operation for the system in Fig. 1, i.e., to replace the periodically time-varying sampled-data system (with continuous-time input and output) by a time-invariant system.

To do so one should obtain a discrete-time approximation of the system by sampling and then lift the system as has been described in [11]. The sampling interval is τ/N, where τ is the controller sampling time. The sampled system is a multirate N-periodic discrete-time system. Lifting involves passing from an N-periodic linear $p \times m$ discrete-time sampled system to an equivalent $pN \times mN$ discrete-time linear time-invariant system. Observe that the equivalence is an isomorphism of the systems in the sense that both essential algebraic and analytic properties of the systems are preserved. In particular, the lifted system is stable if and only if the N-periodic system is stable, and in this case the operator norms (associated with regarding the system as an operator mapping square-summable input to square-summable output) are equal.

To take into account intersample behavior of the system in Fig. 1, we introduce fast sampling of the system with the sampling time τ/N chosen to be a submultiple N of the controller sampling time τ; see Fig. 2.

Normally, τ/N is chosen to be smaller than the fastest significant time constant in the scheme of Fig. 1, e.g., the inverse of $5\times$ closed-loop bandwidth.

It is intuitively clear that the performance of the setup of Fig. 2 will in some way mimic that of the scheme of Fig. 1. We shall show how a reduced-order controller can be obtained for the scheme of Fig. 2. If it is satisfactory, in the sense of causing very little error in the closed-loop behavior of Fig. 2, it will have this property for the scheme of Fig. 1 also.

To obtain a time-invariant system, so that standard reduction procedures can be applied, we lift the system in Fig. 2. Given state-space realizations of the plant P and antialiasing filter F as

$$P(s) = C_p(sI - A_p)^{-1}B_p + D_p \tag{2.1a}$$

$$F(s) = C_f(sI - A_f)^{-1}B_f \tag{2.1b}$$

the state-space realizations of the mN-input, pN-output lifted plant $\mathcal{P}$ and the pN-input, pN-output filter $\mathcal{F}$ can be written in the form

$$\mathcal{P}(z) = \mathcal{C}_p(zI - \mathcal{A}_p)^{-1}\mathcal{B}_p + \mathcal{D}_p \tag{2.2a}$$

$$\mathcal{F}(z) = \mathcal{C}_f(zI - \mathcal{A}_f)^{-1}\mathcal{B}_f + \mathcal{D}_f \tag{2.2b}$$

where

$$\mathcal{A}_p = a_p^N, \quad \mathcal{B}_p = [a_p^{N-1}b_p \quad \cdots \quad a_p b_p \quad b_p],$$
$$\mathcal{A}_f = a_f^N, \quad \mathcal{B}_f = [a_f^{N-1}b_f \quad \cdots \quad a_f b_f \quad b_f],$$
$$\mathcal{C}_p = [C_p^T \quad a_p^T C_p^T \quad \cdots \quad (a_p^T)^{N-1}C_p^T]^T,$$
$$\mathcal{C}_f = [C_f^T \quad a_f^T C_f^T \quad \cdots \quad (a_f^T)^{N-1}C_f^T]^T,$$
$$\mathcal{D}_p = \begin{pmatrix} D_p & 0 & \cdots & 0 \\ C_p b_p & D_p & \cdots & 0 \\ \cdots & \cdots & \cdots & 0 \\ C_p a_p^{N-2} b_p & C_p a_p^{N-3} b_p & \cdots & D_p \end{pmatrix}$$
$$\mathcal{D}_f = \begin{pmatrix} 0 & 0 & \cdots & 0 \\ C_f b_f & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & 0 \\ C_f a_f^{N-2} b_f & C_f a_f^{N-3} b_f & \cdots & 0 \end{pmatrix}$$
$$a_p = \exp(A_p \tau/N), \quad a_f = \exp(A_f \tau/N),$$
$$b_p = \int_0^{\tau/N} \exp(A_p t)dt\, B_p, \quad b_f = \int_0^{\tau/N} \exp(A_f t)dt\, B_f.$$

(The realizations (2.2) are minimal if (2.1) are minimal, for almost all choices of τ.)

The lifted controller $\mathcal{K}$ can be written as

$$\mathcal{K}(z) = E_1 K(z) E_2$$

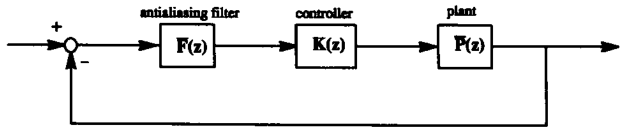

Fig. 3. The lifted closed-loop system.

where

$$E_1 = (I_m \quad I_m \quad \cdots \quad I_m)^T \in \mathbf{R}_{mN\times m},$$
$$E_2 = (I_p \quad 0_p \quad 0_p \quad \cdots \quad 0_p) \in \mathbf{R}_{p\times pN},$$
$$I_n - n\times n \text{ identity matrix}, \quad 0_p - p\times p \text{ zero matrix}.$$

E_2 in the formula corresponds to a slow (every τ seconds) sampler, which passes through only the first element of an input vector and is in the off mode when the following $N-1$ elements of the input vector arrive. E_1 corresponds to a τ-second zero-order hold.

Introducing $\bar{P}$ and $\bar{F}$ as

$$\bar{P} = \mathcal{P}E_1$$

and

$$\bar{F} = E_2\mathcal{F}$$

the periodically time-varying sampled-data system in Fig. 1 can be replaced by a time invariant system in Fig. 3.

Associated with the closed-loop linear periodically time-varying operator T of Fig. 1 is the corresponding linear time invariant operator $\mathcal{T}$ of Fig. 3

$$\mathcal{T} = \bar{P}K\bar{F}(I+\bar{P}K\bar{F})^{-1}. \tag{2.3}$$

Controller reduction will be performed with $\mathcal{T}$ in mind, knowing that good controller reduction for $\mathcal{T}$ provides good controller reduction for T.

III. Controller Reduction

In this section we will reduce the order of the controller K of the setup in Fig. 3 using the approach given in [1] and [8]. The major aim of this reduction is the preservation of the closed-loop transfer function. This means that the error in approximation of the controller K by the reduced-order controller K_r is measured by

$$\|W(z)[K(z)-K_r(z)]V(z)\|_\infty \tag{3.1}$$

where weights W and V are dictated by the requirement to preserve (as far as possible) the closed-loop transfer function. In minimizing the error, they cause the approximation process for K to be more accurate at certain frequencies. We shall now determine these weights. Denote by $\mathcal{T}_r$ the transfer function of the closed loop of Fig. 3 with the reduced-order controller K_r, and consider the difference

$$\mathcal{T}-\mathcal{T}_r = \bar{P}K\bar{F}(I+\bar{P}K\bar{F})^{-1} - \bar{P}K_r\bar{F}(I+\bar{P}K_r\bar{F})^{-1}. \tag{3.2}$$

To a first-order approximation in $K-K_r$, there holds

$$\mathcal{T}-\mathcal{T}_r \approx (I+\bar{P}K\bar{F})^{-1}\bar{P}(K-K_r)\bar{F}(I+\bar{P}K\bar{F})^{-1}. \tag{3.3}$$

This suggests the choice of weighting functions

$$W(z) = (I+\bar{P}K\bar{F})^{-1}\bar{P} \tag{3.4a}$$
$$V(z) = \bar{F}(I+\bar{P}K\bar{F})^{-1} \tag{3.4b}$$

and the minimization problem

$$\min \|W(K-K_r)V\|_\infty. \tag{3.5}$$

To solve this problem, at least in an approximate way, we suggest the frequency-weighted balanced truncation technique [1], [6], [8] be applied to the stable part of K. (The unstable part of K is copied into K_r.) We shall now briefly review this technique. Consider asymptotically stable frequency-weighting functions and associated minimal state-variable realizations

$$W(z) = C_w(zI-A_w)^{-1}B_w + D_w$$

and

$$V(z) = C_v(zI-A_v)^{-1}B_v + D_v$$

(W and V are stable when the closed-loop $\mathcal{T}$ is stable). The basic idea is to change the gramians to reflect the introduction of the frequency weighting. The frequency-weighted transfer function $W(z)\ K(z)\ V(z)$ has a representation with the following state-space matrices

$$\bar{A} = \begin{pmatrix} A_W & B_WC & B_WDC_V \\ 0 & A & BC_V \\ 0 & 0 & A_V \end{pmatrix} \quad \bar{B} = \begin{pmatrix} B_wDD_v \\ BD_V \\ B_V \end{pmatrix}$$
$$\bar{C} = (C_W \quad D_WC \quad D_WDC_V).$$

(Replace $K(z)$ by its stable part in an additive decomposition, if $K(z)$ is not stable).

Let

$$\bar{U} = \begin{pmatrix} U_W & U_{12} & U_{13} \\ U_{12}^T & U & U_{23} \\ U_{13}^T & U_{23}^T & U_V \end{pmatrix}$$

and

$$\bar{Y} = \begin{pmatrix} Y_W & Y_{12} & Y_{13} \\ Y_{12}^T & Y & Y_{23} \\ Y_{13}^T & Y_{23}^T & Y_V \end{pmatrix}$$

be the solutions of the following Lyapunov equations

$$\overline{AUA^T} + \overline{BB^T} = \bar{U} \tag{3.6a}$$
$$\overline{A^TYA} + \overline{C^TC} = \bar{Y}. \tag{3.6b}$$

Now, U and Y can be regarded as the frequency-weighted controllability and observability gramians for the original controller $K(z)$ (or its stable part).

Consider a coordinate basis change to $\{A, B, C\}$ which makes

$$U_{\text{new}} = Y_{\text{new}} = \text{diag}(\mu_1, \mu_2, \cdots, \mu_n), \quad \mu_i \geq \mu_{i+1}, \quad i = 1, 2, \cdots, n-1.$$

This new realization $\{A, B, C\}$ is called a frequency-weighted balanced realization.

Now, the controller reduction is achieved by eliminating the rows and columns of A, B, and C corresponding to smallest $(\mu_{r+1}, \mu_{r+2}, \cdots, \mu_n)$ in $U_{\text{new}} = Y_{\text{new}}$. This yields $K_r(z)$ (or its stable part).

A detailed, computer-oriented description of this weighted balanced truncation algorithm is given in [8].

This frequency-weighted balanced truncation technique allows one to reduce the controller $K(z)$ preserving as much as possible the closed-loop transfer function $\mathcal{T}$. Unlike in the nonweighted or single-side weighted case, a stable $K(z)$ may not yield a stable $K_r(z)$ [15]. A frequency error bound for the frequency-weighted controller order reduction when stability is preserved can be found in [12].

IV. Example

We now present a practical example to confirm the applicability of the approach. This example has been studied in [3] and [4].

The system considered in this example comprises four spinning disks. The disks are connected by a flexible rod, a motor applies torque to the third disk, and the angular displacement of the first disk is the variable of interest. The state-space matrices of the eight state plant are given by

$$A_p = \operatorname{diag}\left\{\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}, \begin{bmatrix}-0.015 & 0.765\\ -0.765 & -0.015\end{bmatrix}, \begin{bmatrix}-0.028 & 1.410\\ -1.410 & -0.028\end{bmatrix}, \begin{bmatrix}-0.04 & 1.85\\ -1.85 & -0.04\end{bmatrix}\right\}$$

$$B_p^T = [0.026 \quad -0.251 \quad 0.033 \quad -0.886 \quad -4.017 \quad 0.145 \quad 3.604 \quad 0.280],$$

$$C_p = [-0.996 \quad -0.105 \quad 0.261 \quad 0.009 \quad -0.001 \quad -0.043 \quad 0.002 \quad -0.026], \quad D_p = 0.$$

The design of the LQG controller was described in [3] and results in the continuous-time controller with the following state-space representation

$$B_c^T = [-0.4105 \quad -0.0868 \quad -0.0004 \quad 0.0036 \quad 0.0081 \quad -0.0085 \quad -0.0004 \quad -0.0132],$$

$$C_c = [-0.0447 \quad -0.6611 \quad -0.0047 \quad -0.3601 \quad -0.1033 \quad 0.0375 \quad 0.0427 \quad -0.0329], \quad D_c = 0$$

and see (x) at the bottom of the page. The controller is open-loop stable.

A discrete-time controller with sampling time $\tau = 0.1$ sec. is obtained by finding the zero-order hold equivalent of the open-loop controller. (This procedure, although it appears satisfactory in this case, can be subject to criticism on the grounds that it does not seek directly to have the continuous-time closed-loop closely approximated by the sampled-data closed loop as in [9] and [10].) The discrete-time controller can be described by the following state-space form (see (y) at the bottom of the page) and

$$B^T = [-0.0406 \quad -0.0084 \quad -0.0000 \quad 0.0007 \quad 0.0021 \quad -0.0010 \quad -0.0014 \quad -0.0013],$$

$$C = C_c, \quad D = 0.$$

The frequency responses of the continuous controller and its sampled version are depicted in Fig. 4.

The antialiasing filter has transfer function $F(s) = 5/(s+5)$.

The frequency responses of the initial continuous and sampled closed loops are shown in Fig. 5. Here, by frequency response of a sampled system we mean the frequency response of the discrete system obtained using zero-order hold discrete-time equivalents of both the plant and the antialiasing filter.

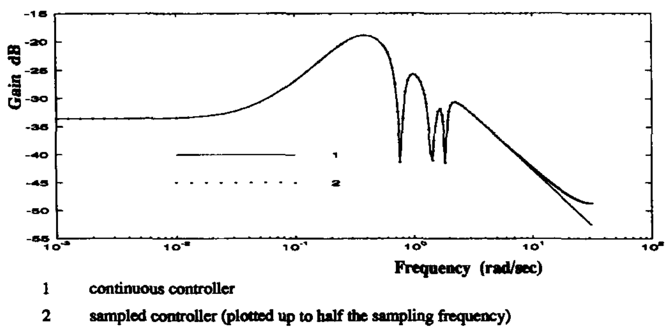

Fig. 4. Frequency responses of continuous and sampled controllers.

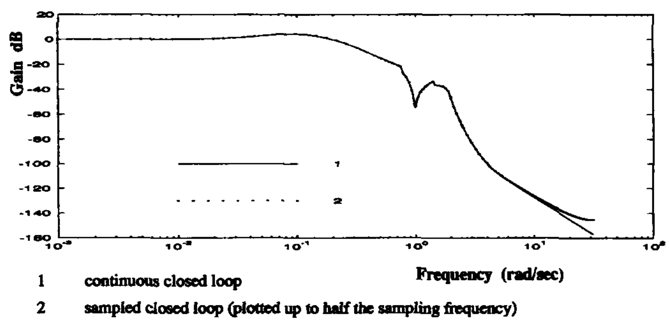

Fig. 5. Frequency responses of continuous and sampled closed loops.

The controller order was reduced using the fast-sampling and lifting technique with $N = 3$ and $N = 10$ (sampling period τ/N). In both cases, frequency weighting was used. The weighted Hankel singular values are (1.5602, 0.4685, 0.0826, 0.0574, 0.0193, 0.0131, 0.0068, 0.0059) for $N = 3$ and (1.5592, 0.4684, 0.0827, 0.0575, 0.0193, 0.0131, 0.0069, 0.0059) for $N = 10$. The controller order was also reduced for the discretized system, which can be considered as fast-sampled and lifted system with $N = 1$. The corresponding weighted Hankel singular values are (1.5539, 0.4660, 0.0817, 0.0568, 0.0191, 0.0130, 0.0068, 0.0059).

Fig. 6 shows the errors of approximation of the full-order continuous closed-loop transfer function by the transfer functions of the closed loops with reduced-order controllers of order 2, obtained in the three above-mentioned ways. The error is $|20\log_{10}(\mathcal{T}/\mathcal{T}_r)|$ where $\mathcal{T}$ and $\mathcal{T}_r$ denote the closed-loop transfer functions with full-order and reduced-order controllers. Clearly, with this measure, it is desirable (though not in general possible) for the error to be zero.

$$A_c = \begin{pmatrix} -0.4077 & 0.9741 & 0.1073 & 0.0131 & 0.0023 & -0.0186 & -0.0003 & -0.0098\\ -0.0977 & -0.1750 & 0.0215 & -0.0896 & -0.0260 & 0.0057 & 0.0109 & -0.0105\\ 0.0011 & 0.0218 & -0.0148 & 0.7769 & 0.0034 & -0.0013 & -0.0014 & 0.0011\\ -0.0361 & -0.5853 & -0.7701 & -0.3341 & -0.0915 & 0.0334 & 0.0378 & -0.0290\\ -0.1716 & -2.6546 & -0.0210 & -1.4467 & -0.4428 & 1.5611 & 0.1715 & -0.1318\\ -0.0020 & 0.0950 & 0.0029 & 0.0523 & -1.3950 & -0.0338 & -0.0062 & 0.0045\\ 0.1607 & 2.3824 & 0.0170 & 1.2979 & 0.3721 & -0.1353 & -0.1938 & 1.9685\\ -0.0006 & 0.1837 & 0.0048 & 0.1010 & 0.0289 & -0.0111 & -1.8619 & -0.0311 \end{pmatrix} \tag{x}$$

$$A = \begin{pmatrix} 0.9596 & 0.0945 & 0.0106 & 0.0012 & 0.0002 & -0.0018 & 0.0001 & -0.0010\\ -0.0094 & 0.9829 & 0.0024 & -0.0084 & -0.0025 & 0.0004 & 0.0011 & -0.0009\\ -0.0000 & -0.0001 & 0.9956 & 0.0763 & -0.0000 & 0.0000 & 0.0000 & -0.0000\\ -0.0031 & -0.0556 & -0.0757 & 0.9653 & -0.0089 & 0.0025 & 0.0038 & -0.0024\\ -0.0148 & -0.2506 & 0.0030 & -0.1361 & 0.9474 & 0.1516 & 0.0172 & -0.0108\\ 0.0008 & 0.0270 & 0.0000 & 0.0147 & -0.1358 & 0.9860 & -0.0018 & 0.0012\\ 0.0139 & 0.2265 & -0.0028 & 0.1229 & 0.0360 & -0.0103 & 0.9635 & 0.1930\\ -0.0015 & -0.0039 & 0.0002 & -0.0020 & -0.0006 & 0.0002 & -0.1830 & 0.9788 \end{pmatrix} \tag{y}$$

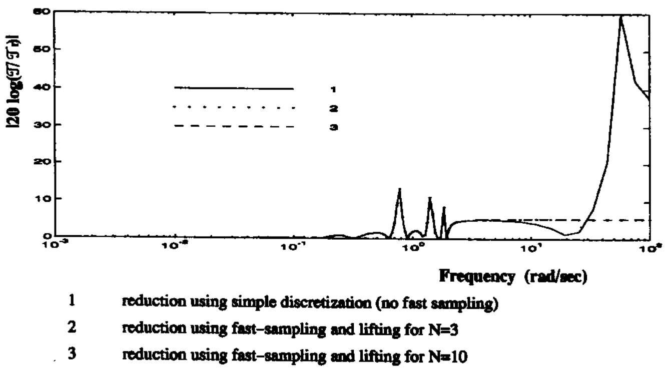

Fig. 6. Errors of approximation of the full-order closed loop by the closed loops with the reduced second order controllers.

The results show that when fast-sampling is used during the reduction process, a superior result is obtained. It is sufficient in this case to use $N = 3$ as the fast sampling rate. This corresponds to an angular frequency of approximately 190 rad/sec; the improvement in matching of $\mathcal{T}$ and $\mathcal{T}_r$ is evident starting at about 10 rad/sec.

Needless to say, whatever the sampling frequency is, it makes sense, especially if there is a problem with stability of the sampled-data closed loop, to use a more sophisticated scheme for obtaining the original (high order) discrete controller transfer function [9], [10].

V. Conclusion

The proposed method allows one to reduce a discrete-time controller which is used in a closed loop with a continuous-time plant, sampler, zero-order hold, and antialiasing filter. This reduction is based on information describing the system's behavior not only at the sampling instants, but in intersample periods as well, and aims to preserve the closed-loop behavior of the sampled-data loop. To get information about the intersample behavior of the system, fast-sampling has been applied, followed by a lifting operation, which gives a time-invariant system. Obviously, the fast sampling procedure incurs an approximation error.

In the whole reduction process, there are actually three different types of error:

i) The error due to replacing a hybrid system by a multirate sampled-data system—this error can be made as small as desired by choosing a fast enough sampling rate for the faster of the two rates.

ii) The error involved in replacing the problem of matching closed-loop transfer functions by the problem of matching (with weights) the open-loop responses of the controller—this error arises from neglecting second-order terms and has the potential to lead to a (mildly) less than optimal result in terms of closed-loop matching.

iii) The error associated with approximating a high-order transfer function by a low-order one—this is obviously unavoidable.

The feasibility, efficiency, and advantage of the proposed method have been confirmed by a practical numerical example.

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Simultaneous Stabilization of Linear Systems Under Stable Additive or Feedback Perturbations

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***Abstract*— In the standard linear, time-invariant, multi-input multi-output unity-feedback system, it is shown that a given plant and one obtained by a known stable additive (or feedback) perturbation of this plant can be simultaneously stabilized by a common controller. The plant is not necessarily stable. No small-gain restrictions are imposed on the stable perturbations. A set of simultaneously stabilizing controllers is explicitly derived for any such pairs of plants. The results extend the standard single connected set of plants description in robust control design methods to two (possibly disjoint) sets of plants.**

I. Introduction

In the standard linear time-invariant (LTI), multi-input multi-output (MIMO) unity-feedback system, we consider simultaneous stabilization of a pair of plants, a nominal plant P, and an additively-perturbed plant $(P + \mathbf{G}_A)$ (similarly, P and a feedback-perturbed

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