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On Some Key Issues in the Windsurfer Approach to Adaptive Robust Control

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In the context of iterative identification and control, where the control objective is to maximize the bandwidth of the closed-loop system while achieving step tracking, we provide a methodology that achieves this goal for stable systems.

Key Words—Adaptive control; robust control; internal model control; identification.

Abstract—We examine a number of crucial questions that arise in the windsurfer approach to adaptive robust control. Considerations are limited to the case where the plant is stable and has no zeros on the imaginary axis. The key conclusion is that, given a strictly proper stable model of a strictly proper stable plant, we can improve the performance robustness of the closed-loop system through the windsurfer approach if the plant and the existing model have no unstable zeros within the designed closed-loop bandwidth and if the deterioration in performance robustness caused by increasing the closed-loop bandwidth results in a sufficiently high signal-to-noise ratio for a certain closed-loop output error. Situations that may cause the iterative identification and control design process to terminate prematurely are identified. A simulation example is used to illustrate the results discussed.

1. INTRODUCTION

1.1. *Background and objectives*

A new adaptive control paradigm known as the *windsurfer approach* was first introduced in Anderson and Kosut (1991). The objective of this approach is to increase the bandwidth of a closed-loop system, if possible to a specified value through an *iterative identification and control design procedure*, given that the initial model of the plant may involve significant error in the high-frequency region. Furthermore, as the closed-loop bandwidth is being increased, the closed-loop frequency response is to be kept approximately flat in the passband so that the closed-loop transient response is not too oscillatory or having excessive peak overshoot.

Iterative identification and control design is a topic of growing interest (see e.g. Anderson and Kosut, 1991; Zang *et al.*, 1991; Schrama, 1992; Schrama and Van den Hof, 1992; Lee *et al.*, 1993; Partanen and Bitmead, 1993). Although these schemes are different in detail and have been proposed for achieving different control objectives, they have all originated from the awareness that, in any model-based control design task, the model serves no other purpose than that of designing a controller. It is therefore not surprising that the identification criterion adopted in each of these schemes is determined by the respective control performance criterion. Furthermore, the controllers employed in each of these schemes are designed on the basis of models (except possibly the first one) identified from data obtained under closed-loop conditions. An historical perspective on as well as a tutorial introduction to the *joint design of identification and control* may be found in Gevers (1993).

A scheme for the windsurfer approach was presented in Lee *et al.* (1993). It was demonstrated by simulations that the bandwidth of a closed-loop system can be increased by the iterative applications of the internal model control (IMC) method (see Morari and Zafiriou, 1989) and a closed-loop system identification procedure pioneered by Hansen (1989).

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In this paper we examine a number of crucial questions that arise in this approach. When can one redesign the controller and expand the closed-loop bandwidth, without re-identifying; When should one re-identify? What does one want to identify in the re-identification process? What can one identify in the re-identification process? How can an identified model be verified against the desired purpose? Will re-identification always lead to improved closed-loop performance? Attention is restricted to strictly proper stable plants with no finite zeros on the imaginary axis. Extensions to more general situations are currently under investigation (see Campi *et al.*, 1994).

1.2. *Structure of the paper*

In Section 2 we describe the IMC design state of the windsurfer approach and show that it is safe to increase the designed closed-loop bandwidth gradually if the plant is stabilized by the existing controller. Section 2 also introduces some of the key concepts and notation used in the paper. Properties of good models for the windsurfer approach are established in Section 3. The control-relevant system identification method employed by the windsurfer approach is described in Section 4. Conditions necessary for identifying a good model and methods for verifying experimentally that an identified model is suitable for the desired purpose (or otherwise) will be given. In Section 5 we study mechanisms that may influence the iterative identification and control design of the windsurfer approach. Situations that may lead to the premature termination of the iterative process will be indicated. In Section 6 two methods for model validation are described. A procedure for the identification of a better model while avoiding the potential danger of causing instability in the actual closed-loop system will then be suggested. Conditions under which the performance robustness of a closed-loop system can be improved through the windsurfer approach are discussed in Section 7. A simulation example is presented in Section 8. We conclude the paper in Section 9.

2. PRELIMINARIES

In this section we describe a closed-loop system where, on the basis of a strictly proper stable model, a sequence of controllers is designed for a strictly proper stable plant. This description also introduces some of the key concepts and notation used in this paper. In particular, Section 2.1 outlines the IMC method in the manner that it is applied in the control design step of the windsurfer approach. The concepts of nominal performance and robust stability are then introduced. In Section 2.2 it is shown that we can increase the designed closed-loop bandwidth of the system, while maintaining the stability of the actual closed-loop system, if the increment is sufficiently small. We conclude this section with a definition of performance robustness relevant to the windsurfer approach.

2.1. *Controller design in the windsurfer approach*

The IMC method is applied in the control design step of the windsurfer approach where the reference input is a step function. Although the IMC method is generally applicable to the case where the plant and the models are not necessarily stable, we restrict ourselves to the case where the plant and the models are strictly proper and stable. In this situation the designed closed-loop bandwidth of the system is determined by a single design parameter.

Consider a closed-loop system as shown in Fig. 1, where G is the transfer function of a strictly proper stable plant. A sequence of such models (identified from data obtained in closed-loop) eventuates in the windsurfer approach. We use G_i to denote the ith member in the sequence of strictly proper stable models $\{G_0, G_1, G_2, \ldots\}$. On the basis of G_i, a finite sequence of controllers $\{K_i^0, K_i^1, \ldots, K_i^f\}$ is designed such that, while keeping the closed-loop frequency responses approximately flat within the pass bands, the corresponding closed-loop bandwidths form an increasing sequence $\{\lambda_i^0, \lambda_i^1, \ldots, \lambda_0^f\}$. Note that we shall in general use K_i to denote one of the controllers in the sequence $\{K_i^0, K_i^1, \ldots, K_i^f\}$ when it is immaterial to the discussion which particular controller is involved. Figure 1 shows that

$$K_i = \frac{Q_i}{1 - G_i Q_i}, \qquad (1)$$

with Q_i defined in the IMC method by

$$Q_i = [G_i]_m^{-1} F_i, \qquad (2)$$

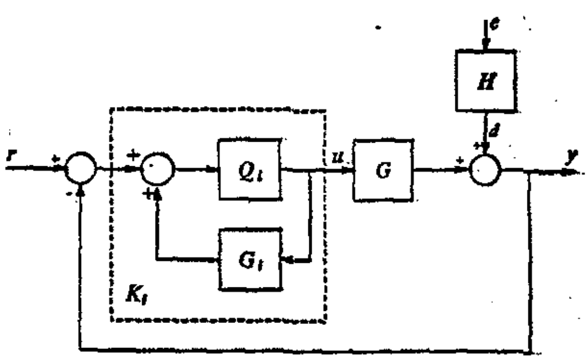

Fig. 1. Internal model control structure.

where $[G_i]_m$ is the *minimum-phase factor* of G_i, and

$$F_i = \left(\frac{\lambda_i}{s+\lambda_i}\right)^n, \quad \lambda_i > 0, \tag{3}$$

is a suitable IMC filter when the model is stable and when the reference input is a step function. The integer n of the IMC filter is chosen such that Q_i is proper. The design parameter λ_i must be chosen such that the *actual closed-loop transfer function*

$$T_i = \frac{TK_i}{1+GK_i} \tag{4}$$

is stable.

It can be shown that the *designed closed-loop transfer function*

$$\bar{T}_i = \frac{G_iK_i}{1+G_iK_i}$$

can be written as

$$\bar{T}_i = G_iQ_i \tag{5}$$

or

$$\bar{T}_i = F_i[G_i]_a, \tag{6}$$

where $[G_i]_a$ is the all-pass factor associated with G_i.

Remarks.

- Since $[G_i]_a(j\omega)$ does not affect the magnitude of $\bar{T}_i(j\omega)$, it is clear that $|\bar{T}_i(j\omega)|$ is flat in its passband and λ_i is the *designed closed-loop bandwidth* with an attenuation of $-3n$ dB.
- Note that the system becomes open-loop when λ_i approaches zero. Since G is stable, it is always possible to make T_i stable by choosing a sufficiently small λ_i.

Although the designed closed-loop transfer function $\bar{T}_i$ is always well behaved, the actual closed-loop transfer function T_i may become unstable when λ_i is too large. We introduce the following definitions.

Definition 1. The designed closed-loop system involving K_i has *robust stability* if the stability of $\bar{T}_i$ implies the stability of T_i. K_i is said to *robustly stabilize* G_i.

Definition 2. For any two closed-loop systems designed by the method described in this subsection, we say that the one with a larger value of λ_i has a better *nominal performance.*

2.2. *Improving nominal performance while maintaining stability*

In Section 2.1 we have indicated that a closed-loop system may not have robust stability when λ_i becomes too large. Since the objective of the windsurfer approach is to increase the closed-loop bandwidth to a specificed value, the following question appears naturally:

- When can the closed-loop bandwidth be increased *with safety*; that is, *without losing robust stability,* while retaining the use of the model G_i?

To answer the above question, we recall from Lee *et al.* (1993) that if T_i (corresponding to λ_i) is stable then there exists a strictly proper transfer function R_i such that

$$G = G_i + \frac{R_i}{1-Q_iR_i}.$$

It can then be shown that

$$\tilde{T}_i = Q_i(1-\tilde{T}_i)R_i, \tag{7}$$

where $\tilde{T}_i = T_i - \bar{T}_i$ is the error in the closed-loop transfer function induced by the error in the model G_i when the designed closed-loop bandwidth is λ_i.

Suppose that the designed closed-loop bandwidth is increased to $\lambda_i' > \lambda_i$; then, corresponding to λ_i', we can write

$$\tilde{T}_i' = Q_i'(1-\tilde{T}_i')R_i',$$

where

$$R_i' = \frac{R_i}{1+[G_i]_m^{-1}(F_i'-F_i)R_i}.$$

Since Q_i' and $\bar{T}_i'$ are stable by design, $\tilde{T}_i'$ and T_i' are stable if and only if R_i' is stable. However, R_i' is stable if $\lambda_i' - \lambda_i > 0$ is *sufficiently small.* Hence we have the following conclusion.

Conclusion 1. We can increase the designed closed-loop bandwidth *cautiously* if the existing closed-loop system has robust stability.

Remark. Note that even when T_i is stable, its response to the reference input could be significantly different from that of $\bar{T}_i$, if, relative to the frequencies where G_i has significant errors, λ_i is not sufficiently small. To address this issue, we now introduce the concept of robust performance that is relevant to the windsurfer approach.

Definition 3. With respect to the given reference input r and a specified finite (usually suitably small) $\sigma > 0$, the closed-loop system is said to have *robust performance* with designed closed-loop bandwidth λ_i if and only if

$$J_i \stackrel{\text{def}}{=} \|v_i\|_2^2 \le \sigma,$$

where $v_i = \tilde{T}_i r$ is the *tracking error.*

Remarks.

- Robust stability is necessary for robust performance, but it is not sufficient.
- It is important to note that a closed-loop system may have high nominal performance (large λ_i) but poor robust performance ($J_i > \sigma$), and vice versa.
- For a model with significant modelling errors in the high-frequency region, the closed-loop system can be designed to have good robust performance if the designed closed-loop bandwidth is sufficiently small.
- While λ_i is being increased, a stage can be reached (before the occurrence of instability) where, because of the modelling errors associated with G_i making a significant contribution to J_i, the performance robustness has deteriorated beyond an acceptable level. At this stage the designed closed-loop bandwidth is $\lambda_i = \lambda_i^f$, and we decide to obtain a more accurate model G_{i+1} before continuing to open up the bandwidth.

3. PROPERTIES OF GOOD MODELS

In Section 2.2 we have concluded that when performance robustness of the closed-loop system has deteriorated beyond an acceptable level, *it is necessary to identify* a model better than the existing one before the designed closed-loop bandwidth can be increased further.

It is clear from Section 2.2 that we can increase the designed closed-loop bandwidth as long as the closed-loop system has robust performance. Therefore it is natural that, when the closed-loop system looses robust performance, we attept to seek a new model that will allow robust performance of the closed-loop system to be restored through controller redesign (while the designed closed-loop bandwidth remains unchanged). This prompts us to ask the following question.

- What *would we like* to identify, in order that, with the new model, robust performance of the closed-loop system can be improved through controller redesign?

Before we proceed to answer the last question, we should observe that each stage of the windsurfer approach involves an existing model G_i and an updated model G_{i+1}. Since every stage of the iteration proceeds in a similar fashion, it suffices to discuss only the stage where $i = 0$. Therefore we shall denote the existing model by G_0 and the updated model by G_1. This system of notation will carry over to all transfer functions and signals involved in the following discussions.

Suppose that G_1 is identified when λ_0 has reached λ_0^f. A new controller K_1^0 will then be designed on the basis of G_1 such that λ_1^0 has the same value as λ_0^f. Obviously, we should like $J_1^0 = \|\tilde{T}_1^0 r\|_2^2$ to be small. By using (1)–(5), with appropriate adjustments made to the notation, we can write $\tilde{T}_1^0 = T_1^0 - \bar{T}_1^0$ as

$$\tilde{T}_1^0 = \frac{\dfrac{G - G_1}{G_1}\bar{T}_1^0}{1 + \dfrac{G - G_1}{G_1}\bar{T}_1^0}(1 - \bar{T}_1^0). \tag{8}$$

Clearly, it is necessary that $\tilde{T}_1^0$ be stable. Since $G - G_1$ is unknown, we conclude the following.

Conclusion 2. We *would like* to identify G via a G_1 of sufficient accuracy such that the model G_1 satisfies the sufficient condition of robust stability

$$\left\| \frac{G - G_1}{G_1}\bar{T}_1^0 \right\|_\infty < 1.$$

Furthermore, we observe that the magnitude of the designed sensitivity function $1 - \bar{T}_1^0$ in the right hand side of (8) could approach a magnitude significantly greater than one if G_1 has unstable zeros within the passband of $\bar{T}_1^0 = F_1^0[G_1]_a$. In order that $\tilde{T}_1^0$ have a small magnitude, we require in addition to the above robust stability condition the following.

Conclusion 3. We *would like* to identify G via a G_1 of sufficient accuracy such that

$$\left| \frac{G(j\omega) - G_1(j\omega)}{G_1(j\omega)} \bar{T}_1^0(j\omega) \right|$$

is sufficiently small for all frequencies above the lesser of the passband of $\bar{T}_1^0$ and the smallest critical frequency corresponding to the unstable zeros of $\bar{T}_1^0$.

Remarks.

- Observe that the unstable zeros of $\bar{T}_1^0$ are those of G_1, which, in a situation with good identification, will be those of the plant G.
- If G_1 has unstable zeros located within the passband of $\bar{T}_1^0$, it is likely that there is a range of frequency within the passband of $\bar{T}_1^0$ where the magnitude of the designed sensitivity

function $1-\bar{T}_1^0$ is significantly greater than one. This has the following consequences:

(1) there is a range of frequency within the passband of $\bar{T}_1^0$ where the designed system has poor disturbance rejection and the measurement noise is not well attenuated:

(2) since the magnitude of the designed sensitivity function is the inverse of the distance of the open-loop frequency response curve from the critical point of stability at $-1+j0$, the designed system may have poor stability margins and transient response if the magnitude of the designed sensitivity function is excessively large near the edge of the system passband.

For these reasons, we may not want to increase the designed closed-loop bandwidth λ_1 beyond λ_0^f if G_1 is found to have unstable zeros with critical frequencies within the passband of $\bar{T}_0^f$.

4. SYSTEM IDENTIFICATION IN THE WINDSURFER APPROACH

Notwithstanding the fact that we have established in the last section properties of a good model for the windsurfer approach, it is important to ask the following question.

- What *can we* identify by using the system identfication procedure embedded in the windsurfer approach?

In this section we answer this question in three steps. In Section 4.1 we show that consideration of the control objective leads to a *(closed-loop) control-relevant system identification problem* that can be transformed into an open-loop system identification problem. This transformation is achieved by employing an identification framework pioneered by Hansen (1989), where, instead of the plant itself, a strictly proper stable transfer function (to be denoted by R_0^f) that parametrizes the plant is identified. In Section 4.2 we shall show that it is possible to identify R_0^f accurately only if the signal-to-noise ratio of a certain closed-loop output error resulting from the existing controller is high. Furthermore, by recognizing the relation between the signal component of the closed-loop output error and deterioration in robust performance, we can restate the conditions necessary for obtaining an accurate estimate of R_0^f in terms of the level of deterioration in robust performance against the effect of noise disturbance. In Section 4.3 we show how to verify indirectly that an estimate of R_0^f is unbiased.

4.1. *Control-relevant system identification*

It was indicated at the end of Section 2.2 that when the designed closed-loop bandwidth has reached a certain value denoted by λ_0^f, the robust performance measure

$$J_0^f = \|v_0^f\|_2^2$$

associated with the closed-loop system designed on the basis of G_0 would become excessively large. It was shown in Section 3 that at this stage, we like to identify a new model G_1 such that

$$\left|\frac{G(j\omega)-G_1(j\omega)}{G_1(j\omega)}\bar{T}_1^0(j\omega)\right|$$

is sufficiently small in an appropriate frequency range. Unfortunately it is not clear how to process input–output measurements to determine G_1 so that this condition is naturally or automatically satisfied. To overcome this difficulty, we use input–output measurements and possibly the reference input of the stable closed-loop system as shown in Fig. 2 to identify G_1 such that

$$\left\|\left(\frac{GK_0^f}{1+GK_0^f}-\frac{G_1K_0^f}{1+G_1K_0^f}\right)r\right\|_2^2$$

is minimized. This closed-loop identification problem can be transformed into an open-loop identification problem by employing Hansen's (1989) framework of identification. We state this result in the following theorem. It is a special case of Theorem 2 in Lee *et al.* (1993) when the plant G and the model G_0 are stable.

Theorem 1. Let $K_0^f=(1-G_0Q_0^f)^{-1}Q_0^f$ stabilize G and G_0, where Q_0^f is a proper stable transfer function, so that G can be parametrized by a strictly proper stable transfer function R_0^f via

$$G = G_0+\frac{R_0^f}{1-R_0^fQ_0^f}.$$

Let

$$G_1 = G_0+\frac{\hat{R}_0^f}{1-\hat{R}_0^fQ_0^f} \qquad (9)$$

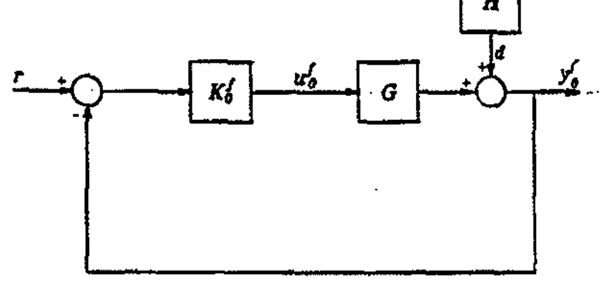

Fig. 2. Closed-loop system just before identification.

be another model stabilized by K_0^f, where $\hat{R}_0^f$ is a strictly proper stable estimate of R_0. Also define

$$\xi_1 = (1 - \bar{T}_0^f)(\beta - \hat{R}_0^f\alpha), \qquad (10)$$

where $\alpha = Q_0^f r$, $\beta = y_0^f - G_0 u_0^f$, and u_0^f and y_0^f are respectively the input and output of the plant resulting from the application of K_0^f. Then ξ_1 can be expressed as

$$\xi_1 = \left(\frac{GK_0^f}{1+GK_0^f} - \frac{G_1K_0^f}{1+G_1K_0^f}\right)r + w_0^f, \qquad (11)$$

where

$$w_0^f = (1 - T_0^f)He \qquad (12)$$

is the effect of the noise disturbance e on the actual closed-loop output.

Remarks.

- If we define $H = (1 - R_0^f Q_0^f)^{-1} S_0^f$, where S_0^f is a proper stable and inversely stable transfer function, then the actual closed-loop system has Hansen's open-loop representation

$$\beta = R_0^f\alpha + S_0^f e. \qquad (13)$$

- From Theorem 1, it is clear that minimizing

$$\left\|\left(\frac{GK_0^f}{1+GK_0^F} - \frac{G_1K_0^f}{1+G_1K_0^f}\right)r + w_0^f\right\|_2^2$$

  with respect to G_1 is equivalent to minimizing

$$\|(1 - G_0Q_0^f)(\beta - \hat{R}_0^f\alpha)\|_2^2$$

  with respect to $\hat{R}_0^f$, provided that G_1 is updated according to

$$G_1 = G_0 + \frac{\hat{R}_0^f}{1 - \hat{R}_0^f Q_0^f}.$$

  Note that the appropriate signal model (which has also taken the data filters L into consideration) for this system identification problem is

$$\bar{\beta} = R_0^f\bar{\alpha} + \Psi_0^f e,$$

  where

$$\bar{\alpha} = L(1 - \bar{T}_0^f)\alpha,$$
$$\bar{\beta} = L(1 - \bar{T}_0^f)\beta,$$
$$\Psi_0^f = L(1 - \bar{T}_0^f)S_0^f.$$

- Since the 'input' α in (13) (and hence $\bar{\alpha}$) is independent of the noise disturbance e, identifying R_0^f and S_0^f (or equivalently Ψ_0^f) is an *open-loop* identification problem.
- We identify R_0^f and Ψ_0^f using a prediction error method (see Ljung, 1987) as shown in Fig. 3. Data filters L (typically low-pass) are usually employed to shape the bias-distribution of the estimates (which is due to under-modelling) such that the model error is small in the appropriate frequency range.

We can summarise the above discussions as follows.

Conclusion 4. We can transform the closed loop identification of G into an open-loop identification problem for

$$R_0^f = \frac{G - G_0}{1 + Q_0^f(G - G_0)}.$$

4.2. *Accurate identification of* R_0^f

In the following we show that the problem of identifying R_0^f accurately can be solved effectively (using finitely many input–output measurements) if the signal-to-noise ratio of a certain closed-loop output error (to be defined immediately) is high. In particular, the normalized variance for an unbiased estimate of R_0^f is small if the signal-to-noise ratio associated with the closed-loop output error is sufficiently high.

From Fig. 4, we observe that the *closed-loop*

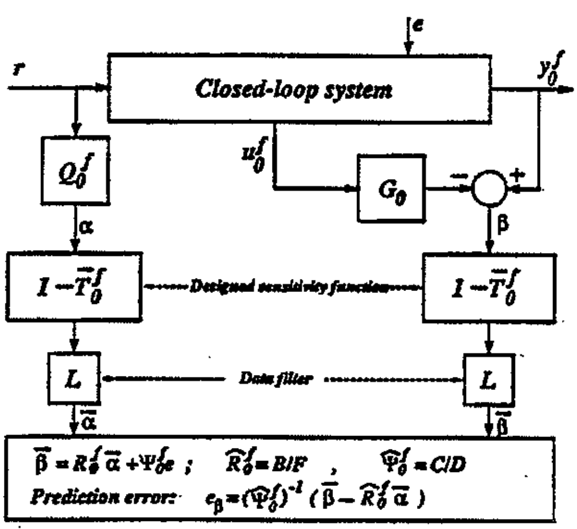

Fig. 3. Identification of R_0^f and Ψ_0^f.

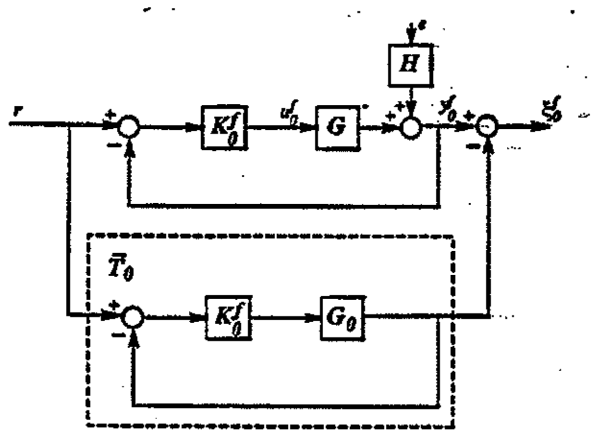

Fig. 4. Closed-loop output error.

output error ξ_0^f is defined as $\xi_0^f = y_0^f - \bar{T}_0^f r$. By substituting the expressions for α and β into (10) and noting that

$$u_0^f = \frac{Q_0^f}{1 - G_0 Q_0^f}(r - y_0^f),$$

we can obtain

$$\hat{R}_0^f = \arg\min_\rho \|\xi_0^f - \rho Q_0^f(1 - \bar{T}_0^f)r\|_2^2. \quad (14)$$

Now we can use the fact that $y_0^f = T_0^f r + (1 - T_0^f)He$ to write

$$\xi_0^f = v_0^f + w_0^f, \quad (15)$$

where

$$v_0^f = \tilde{T}_0^f r. \quad (16)$$

Remark. Note that the tracking error v_0^f cannot be measured directly. It can only be estimated from the closed-loop output error ξ_0^f.

It is apparent that v_0^f is the signal component in ξ_0^f that carries the useful information about the existing modelling errors under closed-loop condition, and w_0^f is the noise component in ξ_0^f which obstructs the determination of $\hat{R}_0^f$. Therefore we can draw an immediate conclusion.

Conclusion 5. We can identify R_0^f successfully if the signal-to-noise ratio associated with the closed-loop output error resulting from the existing controller K_0^f is high.

We next show that the normalized variance for an unbiased estimate of R_0^f is small in the frequency range where the signal-to-noise ratio associated with the closed-loop output error is sufficiently high.

By substituting (15) and (16) into (14) and noting from (7) that $\tilde{T}_0^f = Q_0^f(1 - \bar{T}_0^f)R_0^f$, we can write

$$\hat{R}_0^f = \arg\min_\rho \|Q_0^f(1 - \bar{T}_0^f)(R_0^f - \rho)r + w_0^f\|_2^2.$$

In practice, we use sampled input–output data to estimate a discrete-time model for $\hat{R}_0^f$ before converting it to a continuous-time transfer function. We assume that the errors involved in this conversion are negligible. Following Ljung (1987), we write the variance of an unbiased estimate of R_0^f approximately as

$$\mathbf{E}(|\hat{R}_0^f(j\omega) - R_0^f(j\omega)|^2) \sim \frac{m}{M} \frac{\Phi_{w_0^f}(\omega)}{|Q_0^f(j\omega)[1 - \bar{T}_0^f(j\omega)]|^2 \Phi_r(\omega)},$$

where $\Phi_{w_0^f}(\omega)$ is the power spectral density of w_0^f, under the condition that the order of the discrete-time model for $\hat{R}_0^f$ (denoted by m) and the number of data (denoted by M) are large and the ratio m/M is small. Since

$$\Phi_{v_0^f}(\omega) = |Q_0^f(j\omega)[1 - \bar{T}_0^f(j\omega)]R_0^f(j\omega)|^2 \Phi_r(\omega)$$

is the power spectral density of v_0^f, we can write the normalized variance of $\hat{R}_0^f$ as

$$\mathbf{E}\left(\left\|\frac{\hat{R}_0^f(j\omega) - R_0^f(j\omega)}{R_0^f(j\omega)}\right\|^2\right) \sim \frac{m}{M}\frac{\Phi_{w_0^f}(\omega)}{\Phi_{v_0^f}(\omega)}$$

for the frequencies where $R_0^f(j\omega) \neq 0$.

Remark. For a finite number of data, the normalized variance of $\hat{R}_0^f$ can be small only in the frequency range where the signal-to-noise ratio associated with the closed-loop output error is sufficiently high.

We now summarise the above discussion as follows.

Conclusion 6. We can obtain an unbiased estimate of R_0^f with a small normalized variance in a certain frequency range $\omega_1 \leq \omega \leq \omega_2$ if

(i) the model set used in the estimation of R_0^f is sufficiently general;

(ii) for some sufficiently large $\mu > 0$, the following condition holds:

$$\frac{\Phi_{v_0^f}(\omega)}{\Phi_{w_0^f}(\omega)} \geq \mu \quad \text{for } \omega_1 \leq \omega \leq \omega_2.$$

It is clear that nothing comes for free, and it is prudent to ask the following question.

- What is the price that we have to pay, in terms of system performance, before a sufficiently high signal-to-noise ratio of the closed-loop output error can be achieved?

We next show that it is necessary to have a certain level of deterioration in robust performance (relative to the effect of noise disturbance) before the closed-loop output error can achieve a sufficiently high signal-to-noise ratio.

By using (12) and (16), we deduce that

$$\frac{\Phi_{v_0^f}(j\omega)}{\Phi_{w_0^f}(j\omega)} \geq \mu \quad \text{for } \omega_1 \leq \omega \leq \omega_2, \quad \omega_1 \geq 0,$$

if and only if

$$|\tilde{T}_0^f(j\omega)|^2 > \mu\, |[1 - T_0^f(j\omega)]H(j\omega)|^2 \Phi_e(\omega) \quad \text{for } \omega_1 \leq \omega \leq \omega_2 \quad (17)$$

where $\Phi_e(\omega)$ is the power spectral density of the noise disturbance e. Upon integration, we get

$$\frac{1}{\pi}\int_{\omega_1}^{\omega_2} |\tilde{T}_0^f(j\omega)|^2\,\Phi_r(\omega)\,d\omega$$

$$> \frac{\mu}{\pi}\int_{\omega_1}^{\omega_2} |[1-T_0^f(j\omega)]H(j\omega)|^2\,\Phi_e(\omega)\,d\omega.$$

By Parseval's theorem, and noting that $\omega_1 \geq 0$, we can write

$$J_0^f = \frac{1}{2\pi}\int_{-\infty}^{\infty} |\tilde{T}_0^f(j\omega)|^2\,\Phi_r(\omega)\,d\omega.$$

$$J_0^f > \frac{1}{\pi}\int_{\omega_1}^{\omega_2} |\tilde{T}_0^f(j\omega)|^2\,\Phi_r(\omega)\,d\omega.$$

Therefore

$$J_0^f > \frac{\mu}{\pi}\int_{\omega_1}^{\omega_2} |[1-T_0^f(j\omega)]H(j\omega)|^2\,\Phi_e(\omega)\,d\omega.$$

We can now restate the conditions necessary for the estimation of R_0^f as follows.

Conclusion 7. We can obtain an unbiased estimate of R_0^f with a small asymptotic normalized variance in a certain frequency range of interest ($\omega_1 \leq \omega \leq \omega_2$; $\omega_1 \geq 0$) if

(i) the model set used in the estimate of R_0^f is sufficiently general;

(ii) there is robust performance deterioration bounded below by

$$\frac{\mu}{\pi}\int_{\omega_1}^{\omega_2} |[1-T_0^f(j\omega)]H(j\omega)|^2\,\Phi_e(\omega)\,d\omega$$

for a sufficiently large $\mu > 0$.

Remark. It is obvious that a problem is ill-posed if the value of σ that specifies the tolerable level of deterioration in robust performance (see Definition 3) does not satisfy the inequality

$$\frac{1}{\pi}\int_{\omega_1}^{\omega_2} |[1-T_0^f(j\omega)]H(j\omega)|^2\,\Phi_e(\omega)\,d\omega < \frac{\sigma}{\mu}$$

for a sufficiently large $\mu > 0$.

4.3. *Practically unbiased estimation of R_0^f*

In Section 4.2 we have shown that the normalized variance of $\hat{R}_0^f$ can be small if the signal-to-noise ratio associated with the closed-loop output error is sufficiently high. However normalized variance can be used as a measure of the quality of an estimate only if the estimate is unbiased. It is therefore necessary to verify that $\hat{R}_0^f$ is a practically unbiased estimate (or an unfalsified model as discussed in Ljung *et al.*, 1991) of R_0^f. In this subsection we show how to infer that $\hat{R}_0^f$ is a practically unbiased estimate of R_0^f by verifying that $(1+G_1K_0^f)^{-1}G_1K_0^f$ is a practically unbiased estimate of $(1+GK_0^f)^{-1}GK_0^f$.

We begin by considering

$$\xi_1 = \left(\frac{GK_0^f}{1+GK_0^f} - \frac{G_1K_0^f}{1+G_1K_0^f}\right)r + \frac{1}{1+GK_0^f}He.$$

Clearly, if $(1+G_1K_0^f)^{-1}G_1K_0^f$ is a practically unbiased estimate of $(1+GK_0^f)^{-1}GK_0^f$ then the power spectral density of ξ_1 should reflect the effects of the noise disturbance only. We can perform this verification experimentally after G_1 is obtained (as we describe in Section 6). Now recall that, if it is necessary to update the model G_0, the magnitude of

$$\frac{GK_0^f}{1+GK_0^f} - \frac{G_0K_0^f}{1+G_0K_0^f} = Q_0^f(1-\tilde{T}_0^f)R_0^f$$

must be significant in a certain frequency range $[\omega_1, \omega_2]$. Therefore, before the model G_0 is updated, both the magnitude of the frequency weighting $Q_0^f(1-\tilde{T}_0^f)$ and the magnitude of R_0^f cannot be small in $[\omega_1, \omega_2]$. Since we can write

$$\frac{GK_0^f}{1+GK_0^f} - \frac{G_1K_0^f}{1+G_1K_0^f} = Q_0^f(1-\tilde{T}_0^f)(R_0^f - \hat{R}_0^f),$$

it can easily be deduced that if $(1+G_1K_0^f)^{-1}G_1K_0^f$ is a practically unbiased estimate of $(1+GK_0^f)^{-1}GK_0^f$ in $[\omega_1, \omega_2]$ then $\hat{R}_0^f$ is a practically unbiased estimate of R_0^f in $[\omega_1, \omega_2]$. We can therefore conclude the following.

Conclusion 8. We can verify that $\hat{R}_0^f$ is a practically unbiased estimate of R_0^f in $[\omega_1, \omega_2]$ by verifying experimentally that $(1+G_1K_0^f)^{-1}G_1K_0^f$ is a practically unbiased estimate of $(1+GK_0^f)^{-1}GK_0^f$ in $[\omega_1, \omega_2]$.

5. MECHANISMS THAT INFLUENCE PERFORMANCE ROBUSTNESS AND IDENTIFICATION

In this section we study mechanisms that influence performance robustness of systems designed by the IMC method. We show that there are three mechanisms that may lead to deterioration in robust performance. However, only one of them contributes to the high signal-to-noise ratio needed for a successful estimation of R_0^f. These observations allow us to deduce situations where the iterative identification and control design process may continue or may terminate prematurely.

Recall that in Section 4.2 we have shown that a certain level of deterioration in robust performance is necessary before we can attempt to find a good estimate of R_0^f. However, we should ask the following question.

- Does it mean that, irrespective of the causes, deterioration in robust performance is always helpful to the identification of R_0^f?

The answer is *obviously no*.

With appropriate substitutions in (16) and (12) respectively, we can obtain

$$v_0^f = \frac{\frac{G-G_0}{G_0}\bar{T}_0^f}{1+\frac{G-G_0}{G_0}\bar{T}_0^f}(1-\bar{T}_0^f)r, \qquad (18)$$

$$w_0^f = \frac{1}{1+\frac{G-G_0}{G_0}\bar{T}_0^f}(1-\bar{T}_0^f)He. \qquad (19)$$

Since $J_0^f = \|v_0^f\|_2^2$, we observe that, disregarding changes in disturbance suppression ability, deterioration in robust performances is governed by the value of

$$J_0^f = \frac{1}{2\pi}\int_{-\infty}^{\infty}\Phi_{v_0^f}(\omega)\,d\omega$$

We therefore conclude from the right-hand side of (18) that for a given reference input, there are three factors that contribute to J_0^f through $\Phi_{v_0^f}(\omega)$.

1. The effect of the term $[(G-G_0)/G_0]\bar{T}_0^f$ in the numerator is independent of the phase angle of $[(G-G_0)/G_0]\bar{T}_0^f$. We call this the *phase-insensitive factor*.

2. The effect of the term $1+[(G-G_0)/G_0]\bar{T}_0^f$ in the denominator depends on the gain and phase margins of $[(G-G_0)/G_0]\bar{T}_0^f$. We call this the *stability margin factor*.

3. The effect of the term $1-\bar{T}_0^f$ depends on the existence of unstable zeros of G_0 within the passband of $\bar{T}_0^f = F_0^f[G_0]_a$. We call this the *unstable-zeros-dependent factor*.

By using (18) and (19), we can write the signal-to-noise ratio associated with the closed-loop output error as

$$\frac{\Phi_{v_0^f}(\omega)}{\Phi_{w_0^f}(\omega)} = \frac{\left|\frac{G(j\omega)-G_0(j\omega)}{G_0(j\omega)}\bar{T}_0^f(j\omega)\right|^2}{|H(j\omega)|^2}\frac{\Phi_r(\omega)}{\Phi_e(\omega)}.$$

This equation indicates that for a given reference input and noise disturbance scenario, only an increase in the magnitude of the phase insensitive factor can increase the signal-to-noise ratio of the closed-loop output error. We now summarise the above discussions as follows.

Conclusion 9.

1. There are three factors that can cause the performance robustness to deteriorate: the phase-insensitive factor, the stability margin factor and the unstable-zeros-dependent factor. Among these, only the phase-insensitive factor can contribute to improving the signal-to-noise ratio associated with the closed-loop output error.

2. When the unstable-zeros-dependent factor or the stability margin factor are the main causes of deterioration in robust performance, the signal-to-noise ratio associated with the closed-loop output error may be poor, and it may be difficult to obtain a practically unbiased estimate of R_0^f with a small asymptotic normalized variance. This may cause subsequent difficulties in continuing the iterative identfication and control design process.

Remarks.

- From (18), it is clear that $\Phi_{v_0^f}(\omega)$ cannot be large in the frequency range where the designed sensitivity function has small magnitude. This implies that the frequency range $[\omega_1, \omega_2]$ emphasized in Section 4 cannot be well below λ_0^f.
- From the definitions of the phase-insensitive factor and the stability margin factor, we can deduce that it is possible to estimate R_0^f accurately only in the frequency range where the designed complementary-sensitivity function weighted multiplicative modelling error has large magnitude and small phase lag. This implies that the frequency range $[\omega_1, \omega_2]$ cannot be well above λ_0^f (where $\bar{T}_0^f$ has small magnitude and large phase lag).
- When the stability margin factor or the unstable-zeros-dependent factor are the main causes of deterioration in robust performance, it may be difficult to obtain an accurate estimate of R_0^f. This may lead to premature termination of the iterative identification and control design process. In particular, when the existing model G_0 has unstable zeros within the passband of the designed closed-loop transfer function T_0^f, the designed sensitivity (unstable-zeros-dependent factor) may have large magnitude in a certain frequency region. This fundamental limit in control performance (discussed in Freudenberg and Looze, 1985) causes a deterioration in designed and robust

performances with no improvement in the signal-to-noise ratio associated with the closed-loop output error.

6. IDENTIFICATION AND VALIDATION OF NEW MODELS

In Section 4.2 we have shown that under noisy conditions, the accuracy of the identified model can be improved by increasing the signal-to-noise ratio associated with the closed-loop output error. It was also shown that this is equivalent to having a certain level of deterioration in robust performance relative to the effect of noise disturbance. It is clearly undesirable from the control point of view for robust performance to deteriorate too seriously, while on the other hand it is necessary to have a sufficiently high signal-to-noise ratio in the closed-loop output error before identification can successfully be carried out. Furthermore, it is important to ensure that a model with the right properties is identified. We should therefore like to ask the following practical questions.

1. When should we try to identify a better model?
2. Have we actually identified a good model for our purpose?

Before we can answer these questions, we need methods for validating an identified model. In Section 6.1 we describe a frequency-domain method for model validation. In Section 6.2 we give a time-domain method for model validation. In Section 6.3 we draw on the results of Lee *et al.* (1994), which compared the two methods of model validation and suggest a procedure for identifying a better model.

6.1. *A frequency-domain method for model validation*

In the following we present a model validation method in the frequency domain. It should be emphasized that the model validation procedure is designed with the closed-loop control objective in mind.

Recall that, given the existing model G_0, it is necessary to identify an improved model G_1 when $J_0^f = \|v_0^f\|_2^2$ is excessively large. Evidently ξ_0^f could be large (implying undesirable performance) with one or both of v_0^f and w_0^f large. If the former is larger, there is a potential to reduce it by improved model identification. But this only works (in a particular frequency band $[\omega_1, \omega_2]$) if the signal-to-noise ratio is sufficiently high. Specifically, when only finitely many input–output measurements are available for identifying R_0^f (which parametrizes G), it was shown in Section 4.2 that the normalized variance of $\hat{R}_0^f$ is small only if the signal-to-noise ratio $\Phi_{v_0^f}(\omega)/\Phi_{w_0^f}(\omega)$ associated with $\xi_0^f = v_0^f + w_0^f$ is sufficiently high. Obviously, then one needs to estimate power spectra for w_0 and v_0 (or more precisely ξ_0). We now proceed as follows.

From $v_0 = (T_0 - \bar{T}_0)r$ and $\xi_0 = v_0 + w_0$, we observe that when $r = 0$, the sole contributor to ξ_0 is w_0. Therefore we can compute $\Phi_{w_0}(\omega)$ after measuring ξ_0 with $r = 0$. When $r \neq 0$, we have $\xi_0 = v_0 + w_0$. Assuming that v_0 and w_0 are uncorrelated (which follows if r and e are uncorrelated—a typical situation), $\Phi_{\xi_0}(\omega) = \Phi_{v_0}(\omega) + \Phi_{w_0}(\omega)$. By visual comparison of $\Phi_{\xi_0}(\omega)$ with $\Phi_{w_0}(\omega)$, we evaluate the significance of $\Phi_{v_0}(\omega)$ with respect to $\Phi_{w_0}(\omega)$. If $\Phi_{\xi_0^f}(\omega)$ is significantly larger than $\Phi_{w_0^f}(\omega)$ in a frequency band spanning one decade and centred around λ_0^f (when the designed closed-loop bandwidth is λ_0^f), the model G_0 is invalidated for the design of closed-loop systems with bandwidths larger than or equal to λ_0^f.

The method just described can also be used to validate G_1 after it has been identified (both before and after model reduction is performed). We simply replace G_0 by G_1, while retaining K_0^f, in the simulation of the designed closed-loop response to the reference input. This allows us to compute ξ_1 and its power spectrum $\Phi_{\xi_1}(\omega)$. By visually comparing $\Phi_{\xi_1}(\omega)$ with $\Phi_{w_0^f}(\omega)$, we have good confidence that G_1 is a reliable model of G (when the designed closed-loop bandwidth is λ_0^f) if $\Phi_{\xi_1}(\omega)$ is comparable to $\Phi_{w_0^f}(\omega)$ up to λ_0^f.

6.2. *A time-domain method for model validation*

We now describe a time-domain model validation method. This is useful both for establishing that G_0 should be rejected (that is, as a flag for re-identification) as well as for validating a new model, G_1, replacing G_0.

Referring to Fig. 3 and (10), we notice that $e_\beta = L\xi_1$ when $\hat{\Psi}_0^f = 1$, where e_β is the prediction error (also known as the residual). We also observe from (9) that $G_1 = G_0$ when $\hat{R}_0^f = 0$, and from (11) that $\xi_1 = \xi_0^f$ when $G_1 = G_0$. Therefore we have $e_\beta = L\xi_0^f$ when $\hat{\Psi}_0^f = 1$ and $\hat{R}_0^f = 0$. This suggests that G_0 should be rejected if the cross-correlation of the prediction error e_β with the future values of 'input' $\bar{\alpha}$ exceed its (3σ) confidence limits when $\hat{\Psi}_0^f = 1$ and $\hat{R}_0^f = 0$. This reasoning is independent of the true Ψ_0^f. See Ljung (1987) for more details of model validation by correlation techniques. (Actually it is also easy to apply the same method to validate a pair of newly identified $\hat{R}_0^f$ and $\hat{\Psi}_0^f$

before $\hat{R}_0^f$ is used to calculate G_1. We simply check that the correlation of e_B with the future values of $\bar{\alpha}$ are within their respective confidence intervals.)

6.3. *Identification of a better model*

The methods of model validation described in Sections 6.1 and 6.2 were compared critically in Lee *et al.* (1994). The key conclusions of that study are summarized in the following.

- Correlation function estimates and power spectrum estimates are both useful for model validation where the goodness of fit is based on a closed-loop control criterion.
- Correlation function estimates are more sensitive than power spectrum estimates in the sense that the former tend to invalidate a model before identifying a better model is necessary and possible. This *does not imply* that the correlation method is useless. On the contrary, it suggests that the correlation method is useful for detecting incipient modelling errors.
- Power spectrum estimates not only suggest when a model becomes inadequate, but they also indicate the frequency range in which the signal-to-noise ratio is high for identification.
- There is a limit on the achievable accuracy for performing identification on closed-loop systems if existing controllers are designed on the basis of models with unstable zeros. (Recall the effect of the unstable-zeros-dependent factor remarked upon at the end of Section 5.) Therefore prior knowledge of unstable zeros in the existing model (say G_0) is important. Specifically, let ω_z be the minimum critical frequency corresponding to the unstable zeros of G_0; simulation experience confirmed that it is very difficult, if not impossible, to identify a model better than G_0 if $\lambda_0^f \geq \frac{1}{2}\omega_z$. It should be remarked that this is reminiscent of design tradeoffs discussed in Freudenberg and Looze (1985), as opposed to an ill-posed problem.
- In general, we update G_0 if
  - (i) both methods of model validation suggest so; and
  - (ii) $\lambda_0^f < \frac{1}{2}\omega_z$.

We now suggest a procedure for identifying a better model. Notice that in the frequency range where the current model G_0 has significant modelling errors, the signal-to-noise ratio of the closed-loop output error can be increased by increasing the magnitude of the reference input or by increasing the designed closed-loop bandwidth. If practical operation constraints do not allow the magnitude of the reference input to be increased then the signal-to-noise ratio of the closed-loop output error can only be increased by increasing the designed closed-loop bandwidth. This, however, has the potential danger of causing instability in the actual closed-loop system if the designed closed-loop bandwidth is increased excessively. To avoid this danger, we proceed as follows.

1. Reduce the rate of increasing the designed closed-loop bandwidth λ_0 once the correlation method for model validation has invalidated G_0.
2. Attempt to identify R_0^f (when $\lambda_0 = \lambda_0^f$) as soon as the power spectrum method for model validation suggests that ξ_0^f has a sufficiently high signal-to-noise ratio, provided that $\lambda_0^f < \frac{1}{2}\omega_z$.
   - (a) Use the collected data to identify a set of models by experimenting with the likely model structures. Perform model verification on each of these models.
   - (b) If an identified model is found to be sufficiently accurate, accept it for the next stage of control design. Otherwise, increase the designed closed-loop bandwidth slightly, collect a new set of measurements and repeat the procedures of model estimation and verification.
   - (c) Repeat the last two steps until a sufficiently accurate model is obtained and verified.
3. Terminate the iterative identification and control design procedure if $\lambda_0^f \geq \frac{1}{2}\omega_z$ and ξ_0^f, although unacceptably large, does not facilitate the identfication of a better model.

7. ROBUST PERFORMANCE IMPROVEMENT

Now we know what can be identified and how an identified model can be validated. We have also indicated in Section 3 what we would like to identify. It is therefore logical to ask the following question.

- How does the object which we *can* identify relate to the object which we *would like* to identify?

The answer is that the objects are virtually the same, although it is not obvious. What we can identify is couched in terms of R_0^f, and what we would like to identify is couched in terms of G_1. We need to connect these characterizations. In this section we show that, provided that certain conditions are satisfied, the controller designed on the basis of the model G_1 updated through an estimate of $\hat{R}_0^f$ can improve the performance robustness of the system.

Recall from (17) that, just before we attempt to update the model G_0 through identifying R_0^f, it is necessary that

$$|\tilde{T}_0^f(j\omega)|^2\Phi_r(\omega)$$
$$> \mu |[1 - T_0^f(j\omega)]H(j\omega)|^2\Phi_e(\omega) \quad \text{for } \omega_1 \le \omega \le \omega_2$$

for a sufficiently large $\mu > 0$. Furthermore, it is also necessary that $|\tilde{T}_0^f(j\omega)|$ in the above inequality be mainly contributed by the phase-insensitive factor before an accurate estimate of R_0^f can be obtained. This implies that in order to improve the robust performance through identification and redesign, it is necessary that the phase-insensitive factor (which is also the complementary sensitivity weighted multiplicative factor (which is also the complementary sensitivity weighted multiplicative modelling error) associated with the updated model G_1 and the redesigned controller K_1^0 (while keeping $\lambda_1^0 = \lambda_0^f$) be small in the frequency range $[\omega_1, \omega_2]$. Hence it is relevant to consider the magnitude of the ratio

$$\left(\frac{G - G_1}{G_1}\tilde{T}_1^0\right)\Big/\left(\frac{G - G_0}{G_0}\tilde{T}_0^f\right)$$

in the frequency range $[\omega_1, \omega_2]$.

Before the main results are presented in Theorems 3 and 4, we state a theorem (which follows directly from Theorem 4 of Lee *et al.*, 1993) that is relevant to the choice of the relative degree of $\hat{R}_0^f$, and establish two lemmas that we use in the proof of Theorem 3.

Theorem 2. Let the controller K_0^f and the proper stable transfer function Q_0^f designed by the IMC method described in Section 2.1. Then the relative degree of

$$R_0^f = \frac{G - G_0}{1 + Q_0^f(G - G_0)}$$

is given by

$$\text{rel deg}\{R_0^f\} = \min(\text{rel deg}\{G\}, \text{rel deg}\{G_0\}).$$

Remark. The relative degree of the strictly proper plant G is usually unknown. It is therefore necessary to allow, in the identification of R_0^f, the relative degree of $\hat{R}_0^f$ to take the smallest possible value of one.

Lemma 1. Suppose that G_0 has a relative degree of $n \ge 1$, $Q_0^f = [G_0]_m^{-1}F_0^f$ is proper, and $\hat{R}_0^f$ has a relative degree of $q \ge 1$. If G_1 is updated according to

$$G_1 = G_0 + \frac{\hat{R}_0^f}{1 - Q_0^f\hat{R}_0^f}$$

then

(i) G_1 has a relative degree k, where

$$k \ge n \qquad \text{if } q = n,$$
$$k = \min(n, q) \quad \text{otherwise;}$$

(ii) there exists a strictly proper IMC filter

$$F_1^0 = F_0^f\left(\frac{\lambda_0^f}{s + \lambda_0^f}\right)^i, \quad i \text{ is an integer}$$

(where F_0^f has a relative degree such that $Q_0^f = [G_0]_m^{-1}F_0^f$ is proper) such that $Q_1^0 = [G_1]_m^{-1}F_1^0$ has at least the relative degree of Q_0^f;

(iii) if G_1 has no zeros along the imaginary axis then Q_1^0/Q_0^f is bounded along the imaginary axis; in particular, there exists a finite δ such that

$$\sup_{\omega_1 \le \omega \le \omega_2}\left|\frac{Q_1^0(j\omega)}{Q_f^0(j\omega)}\right| = \delta.$$

Proof. (i) There are three cases to be considered, namely $q = n$; $q < n$ and $q > n$.

For the case where $q = n$, consider

$$\lim_{s\to\infty} s^q G_1 = \lim_{s\to\infty} s^n G_0 + \lim_{s\to\infty}\frac{s^q\hat{R}_0^f}{1 + q_0^f\hat{R}_0^f}.$$

Since

$$\lim_{s\to\infty} s^n G_0 = c_1, \quad c_1 \ne 0,$$

and

$$\lim_{s\to\infty}\frac{s^q\hat{R}_0^f}{1 + Q_0^f\hat{R}_0^f} = c_2, \quad c_2 \ne 0,$$

we have

$$\lim_{s\to\infty} s^q G_1 = 0 \quad \text{if } c_2 = -c_1,$$

or

$$\lim_{s\to\infty} s^q G_1 \ne 0 \quad \text{if } c_2 \ne -c_1.$$

Hence the relative degree of G_1 is $k \ge q = n$.

Next consider

$$\lim_{s\to\infty} s^q G_1 = \lim_{s\to\infty} s^q G_0 + \lim_{s\to\infty}\frac{s^q\hat{R}_0^f}{1 + Q_0^f\hat{R}_0^f}$$

for $q < n$. Clearly,

$$\lim_{s\to\infty} s^q G_1 = \lim_{s\to\infty} s^q\hat{R}_0^f \ne 0,$$

since Q_0^f is proper and G_0 has a relative degree larger than q. Therefore the relative degree of G_1 is $k = q < n$.

Similar considerations applying to

$$\lim_{s\to\infty} s^n G_1 = \lim_{s\to\infty} s^n G_0 + \lim_{s\to\infty} \frac{s^n \hat{R}_0^f}{1 + Q_0^f \hat{R}_0^f}$$

for $q > n$ will lead to the conclusion that the relative degree of G_1 is $k = n < q$.

(ii) It follows immediately from the above that $Q_1^0 = [G_1]_m^{-1} F_1^0$ has at least the relative degree of $Q_0^f = [G_0]_m^{-1} F_0^f$ if

$$F_1^0 = F_0^f \left(\frac{\lambda_0^f}{s + \lambda_0^f}\right)^i, \quad i \geq k - n.$$

Obviously, since $k \geq 1$ and the relative degree of F_0^f is at least n, F_1^0 is strictly proper.

(iii) Now it is easy to conclude that for the above choice of F_1^0, Q_1^0/Q_0^f is proper. Therefore if G_1 has no zeros along the imaginary axis and G_0 has no poles along the imaginary axis then Q_1^0/Q_0^f is bounded along the imaginary axis and there exists a finite δ such that

$$\sup_{\omega_1 \leq \omega \leq \omega_2} \left| \frac{Q_1^0(j\omega)}{Q_f^0(j\omega)} \right| = \delta.$$

□

Remarks.

- It will be clear from Theorem 3 that it is undesirable for δ to become excessively large.
- Since all poles of G_0 that are also poles of G are always retained by a well-identified G_1, it is clear that poles of G_0 that are also poles of G, even if they are near the imaginary axis, will not cause δ to assume an excessively large value.
- Zeros of G_1 near the imaginary axis for $\omega_1 \leq \omega \leq \omega_2$ may not appear as zeros of G_0. However, these zeros would be zeros of the plant G if G_1 is a well-identified model of G for $\omega_1 \leq \omega \leq \omega_2$. This would happen only if we increased the closed-loop bandwidth to the frequency range where the plant has (stable or unstable) zeros near the imaginary axis, and the controller has *excessively large* gain. Therefore we can prevent δ from being excessively large by observing well-known design guidelines.
- If G_0 has poles near to the imaginary axis for $\omega_1 \leq \omega \leq \omega_2$ that are not poles of the plant G then a well-identified model G_1 for G either will have no poles at these locations or will have approximate pole zero cancellations at these locations. In these situations δ may become excessively large. It is therefore *important to verify* that an identified model (such as G_0) has no unnecessary poles near the imaginary axis.

Lemma 2. If G_0 and G_1 are strictly proper stable models of the plant G, and $\bar{T}_0^f = G_0 Q_0^f$ is the closed-loop transfer function, where Q_0^f is designed by the IMC method, then there exists a finite η such that

$$\sup_{\omega_1 \leq \omega \leq \omega_2} \left| \frac{G_1(j\omega) - G_0(j\omega)}{G_0(j\omega)} \bar{T}_0^f(j\omega) \right| = \eta.$$

Proof. Clearly the transfer function $G_1 - G_0$ is stable. Also, from the facts that $\bar{T}_0^f = G_0 Q_0^f$ and that the Q_0^f designed by the IMC method is proper and stable, it is easy to conclude that

$$\frac{G_1 - G_0}{G_0} \bar{T}_0^f = (G_1 - G_0) Q_0^f$$

is proper and stable. □

Remarks.

- It will be clear from Theorem 3 that it is undesirable for η to become excessively large.
- η may become excessively large if G_1 has poles near the imaginary axis for $\omega_1 \leq \omega \leq \omega_2$ that are not poles of G_0, and if λ_0^f is very near to the critical frequencies of these poles. However, this is impossible because if G_1 were a well-identified model of the plant G then G would have poles near to $\pm j\lambda_0^f$ that are not poles of G_0. Under these conditions, the actual closed-loop system T_0^f would be unstable or almost unstable. Furthermore, λ_0^f cannot be close to the zeros of G_0 near the imaginary axis for $\omega_1 \leq \omega \leq \omega_2$, because this will result in a controller with an excessively large gain in that frequency range. Hence, by ensuring that the actual closed-loop system T_0^f is far from instability (recall the guidelines given at the end of Section 6.3) and by observing well-known controller design guidelines, we automatically prevent η from taking excessively large values.

Theorem 3. Let G_0 be a stable strictly proper model of the plant G. Suppose that G is stabilized by the controller K_0^f designed according to the IMC method described in Section 2.1, and hence has the description

$$G = G_0 + \frac{R_0^f}{1 - Q_0^f R_0^f}, \tag{20}$$

where

$$Q_0^f = [G_0]_m^{-1} F_0^f, \tag{21}$$

with $[G_0]_m$ the minimum-phase factor of G_0. Let

G_1 be a stable strictly proper model of G updated according to

$$G_1 = G_0 + \frac{\hat{R}_0^f}{1 - Q_0^f \hat{R}_0^f}, \qquad (22)$$

where $\hat{R}_0^f$ is an estimate of R_0^f.

Suppose that

(i) K_1^0 is designed according to the IMC method with $\lambda_1^0 = \lambda_0^f$;

(ii) the conditions set out in Lemma 1 are satisfied.

Then, for each ω in $[\omega_1, \omega_2]$ such that $R_0^f(j\omega) \neq 0$,

$$\frac{\left|\frac{G(j\omega) - G_1(j\omega)}{G_1(j\omega)} \bar{T}_1^0(j\omega)\right|^2}{\left|\frac{G(j\omega) - G_0(j\omega)}{G_0(j\omega)} \bar{T}_0^f(j\omega)\right|^2} < \delta^2(1+\eta)^2 \left|\frac{\hat{R}_0^f(j\omega) - R_0^f(j\omega)}{R_0^f(j\omega)}\right|^2,$$

where

$$\delta = \sup_{\omega_1 \le \omega \le \omega_2} \left|\frac{Q_1^0(j\omega)}{Q_0^f(j\omega)}\right|.$$

Proof. By direct substitution, we can write

$$\frac{\frac{G - G_1}{G_1} \bar{T}_1^0}{\frac{G - G_0}{G_0} \bar{T}_0^f} = \frac{Q_1^0}{Q_0^f} \frac{G - G_1}{G - G_0}. \qquad (23)$$

Now (20)–(22) allow us to write

$$\frac{R_0^f}{[G_0]_m} = \frac{G - G_0}{[G_0]_m + (G - G_0)F_0^f},$$

$$\frac{\hat{R}_0^f}{[G_0]_m} = \frac{G_1 - G_0}{[G_0]_m + (G_1 - G_0)F_0^f}.$$

Direct calculation with the last two equations gives

$$\frac{\hat{R}_0^f - R_0^f}{R_0^f} = -\frac{(G - G_1)[G_0]_m}{(G - G_0)\{[G_0]_m + (G_1 - G_0)F_0^f\}}. \qquad (24)$$

Recall that $G_0 = [G_0]_m[G_0]_a$, and $\bar{T}_0^f = F_0^f[G_0]_a$; we can therefore rewrite (24) as

$$-\frac{G - G_1}{G - G_0} = \left(1 + \frac{G_1 - G_0}{G_0}\bar{T}_0^f\right) \frac{\hat{R}_0^f - R_0^f}{R_0^f}. \qquad (25)$$

Substituting (25) into (23) allows us to write

$$\frac{\left|\frac{G - G_1}{G_1} \bar{T}_1^0\right|}{\left|\frac{G - G_0}{G_0} \bar{T}_0^f\right|} = \left|\frac{Q_1^0}{Q_0^f}\right| \left|1 + \frac{G_1 - G_0}{G_0}\bar{T}_0^f\right| \left|\frac{\hat{R}_0^f - R_0^f}{R_0^f}\right|.$$

Since G_1 and G_0 are strictly proper stable transfer functions, and $\bar{T}_0^f = G_0 Q_0^f$, with Q_0^f proper and stable, it follows from Lemma 2 that

$$\sup_{\omega_1 \le \omega \le \omega_2} \left|\frac{G_1(j\omega) - G_0(j\omega)}{G_0(j\omega)} \bar{T}_0^f(j\omega)\right| = \eta,$$

$$\sup_{\omega_1 \le \omega \le \omega_2} \left|1 + \frac{G_1(j\omega) - G_0(j\omega)}{G_0(j\omega)} \bar{T}_0^f(j\omega)\right| \le 1 + \eta.$$

Furthermore, from Lemma 1, we have

$$\sup_{\omega_1 \le \omega \le \omega_2} \left|\frac{Q_1^0(j\omega)}{Q_0^f(j\omega)}\right| = \delta.$$

Therefore

$$\frac{\left|\frac{G(j\omega) - G_1(j\omega)}{G_1(j\omega)} \bar{T}_1^0(j\omega)\right|^2}{\left|\frac{G(j\omega) - G_0(j\omega)}{G_0(j\omega)} \bar{T}_0^f(j\omega)\right|^2} \le \delta^2(1+\eta)^2 \left|\frac{\hat{R}_0^f(j\omega) - R_0^f(j\omega)}{R_0^f(j\omega)}\right|^2$$

for $\omega_1 \le \omega \le \omega_2$. □

We noted immediately from Theorem 3 that if

$$\left|\frac{\hat{R}_0^f - R_0^f}{R_0^f}\right|^2 \ll \frac{1}{\delta^2(1+\eta)^2}$$

in the frequency range $[\omega_1, \omega_2]$ then the phase-insensitive factor associated with G_1 and $\bar{T}_1^0$ will be much smaller in magnitude than that associated with G_0 and $\bar{T}_0^f$ in the same frequency range. We now prove a stronger result.

Theorem 4. Suppose that

$$\left|\frac{\hat{R}_0^f(j\omega) - R_0^f(j\omega)}{R_0^f(j\omega)}\right|^2 \ll \frac{1}{\delta^2(1+\eta)^2} \left|\left[\frac{G(j\omega) - G_0(j\omega)}{G_0(j\omega)}\right] \bar{T}_0^f(j\omega)\right|^{-2}$$

for $\omega_1 \le \omega \le \omega_2$. Then the tracking error v_1^0 resulting from $\bar{T}_1^0$ (with $\lambda_1^0 = \lambda_0^f$) designed on the basis of

$$G_1 = G_0 + \frac{\hat{R}_0^f}{1 - Q_0^f \hat{R}_0^f}$$

has a power spectrum approximated as

$$\Phi_{v_1^0}(\omega) \approx \left| \frac{G(j\omega) - G_1(j\omega)}{G_1(j\omega)} \bar{T}_1^0(j\omega) \right|^2 \times |1 - \bar{T}_1^0(j\omega)|^2 \Phi_r(\omega)$$

for $\omega_1 \le \omega \le \omega_2$.

Proof. It can be shown that the tracking error induced by the model error associated with G_1 and when the designed closed-loop transfer function is $\bar{T}_1^0$ is given by

$$v_1^0 = \frac{\frac{G - G_1}{G_1} \bar{T}_1^0}{1 + \frac{G - G_1}{G_1} \bar{T}_1^0} (1 - \bar{T}_1^0) r.$$

Therefore

$$\Phi_{v_1^0}(\omega) = \frac{\left| \frac{G(j\omega) - G_1(j\omega)}{G_1(j\omega)} \bar{T}_1^0(j\omega) \right|^2}{\left| 1 + \frac{G(j\omega) - G_1(j\omega)}{G_1(j\omega)} \bar{T}_1^0(j\omega) \right|^2} \times |1 - \bar{T}_1^0(j\omega)|^2 \Phi_r(\omega).$$

Since

$$\left| \frac{\hat{R}_0^f(j\omega) - R_0^f(j\omega)}{R_0^f(j\omega)} \right|^2 \ll \frac{1}{\delta^2(1+\eta)^2} \left| \frac{G(j\omega) - G_0(j\omega)}{G_0(j\omega)} \bar{T}_0^f(j\omega) \right|^{-2}$$

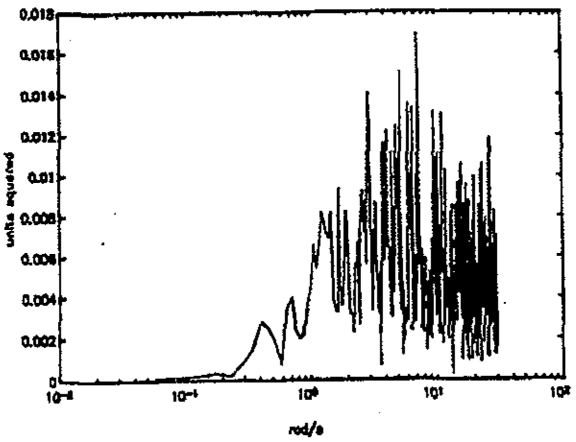

Fig. 7. $\Phi_w(\omega)$ when $\lambda_0 = 1.5$ rad s^{-1}.

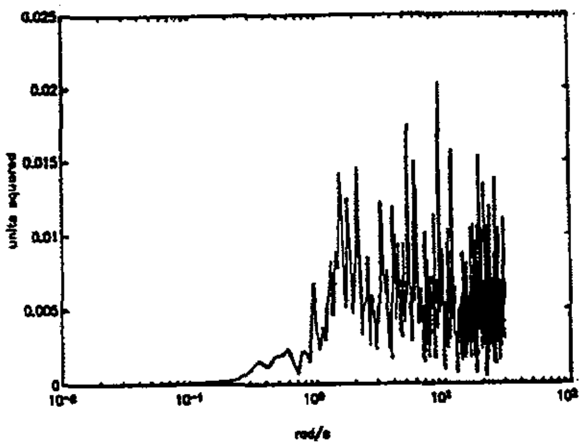

Fig. 8. $\Phi_\xi(\omega)$ when $\lambda_0 = 1.5$ rad s^{-1}.

for $\omega_1 \le \omega \le \omega_2$, we have, from Theorem 3,

$$\left| \frac{G(j\omega) - G_1(j\omega)}{G_1(j\omega)} \bar{T}_1^0(j\omega) \right| \ll 1$$

for $\omega_1 \le \omega \le \omega_2$. It follows that

$$\Phi_{v_1^0}(\omega) \approx \left| \frac{G(j\omega) - G_1(j\omega)}{G_1(j\omega)} \bar{T}_1^0(j\omega) \right|^2 \times |1 - \bar{T}_1^0(j\omega)|^2 \Phi_r(\omega)$$

for $\omega_1 \le \omega \le \omega_2$. □

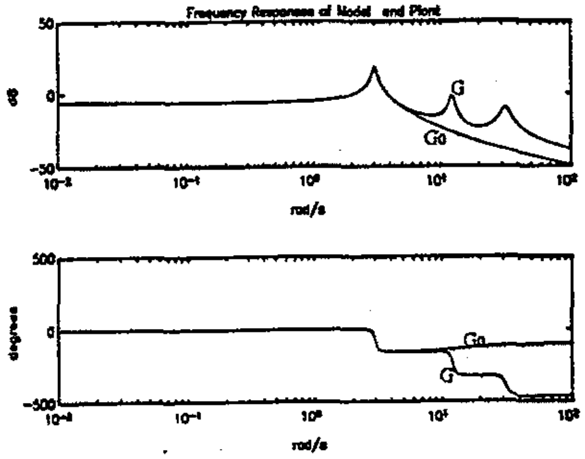

Fig. 5. Frequency response of model G_0.

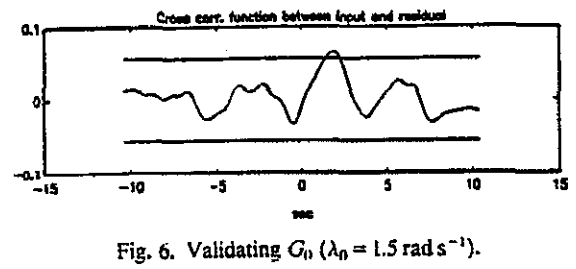

Fig. 6. Validating G_0 ($\lambda_0 = 1.5$ rad s^{-1}).

Theorems 3 and 4 together show that if the phase-insensitive factor associated with G_0^f and $\bar{T}_0^f$ was the main reason for poor robust performance then the pair G_1 and $\bar{T}_1^0$, with a much smaller magnitude of phase-in-sensitive factor, attains a much better robust performance if G_1 has no unstable zeros in the pass band of $\bar{T}_1^0$. We therefore have the following conclusion, which should be read in conjunction with Conclusions 7 and 9.

Conclusion 10. If a practically unbiased estimate of R_0^f with a sufficiently small normalized

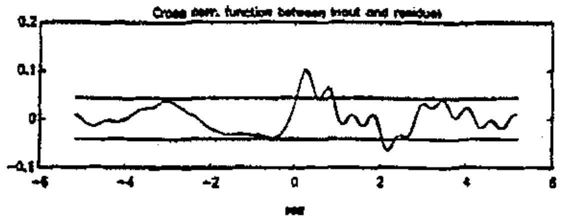

Fig. 9. Validating G_0 ($\lambda_0^f = 3\ \mathrm{rad\,s^{-1}}$).

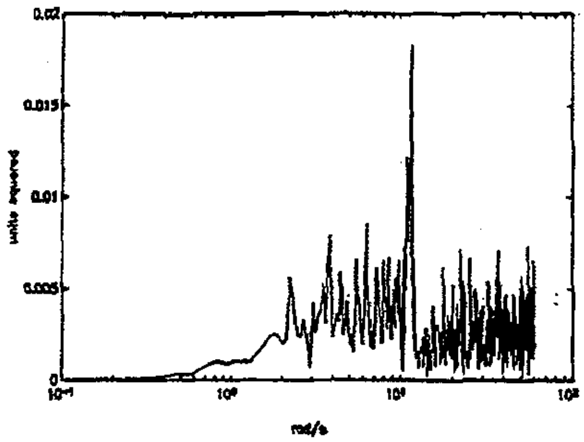

Fig. 10. $\Phi_w(\omega)$ when $\lambda_0^f = 3\ \mathrm{rad\,s^{-1}}$.

variance can be obtained over the frequency range $\omega_1 \le \omega \le \omega_2$, with $\hat{R}_0^f$ and G_1 satisfying the constraints stated in Lemmas 1 and 2, then it is possible to achieve robust performance improvement through controller redesign if the unstable zeros of G_1 are outside the designed closed-loop passband.

8. SIMULATION RESULTS

In this section we show by a simulation example that, through the method outlined above, it is possible to increase the bandwidth of a closed-loop system to its fundamental limit

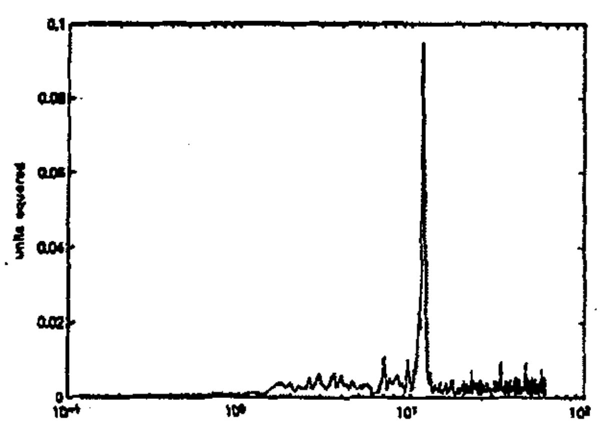

Fig. 11. $\Phi_\xi(\omega)$ when $\lambda_0^f = 3\ \mathrm{rad\,s^{-1}}$.

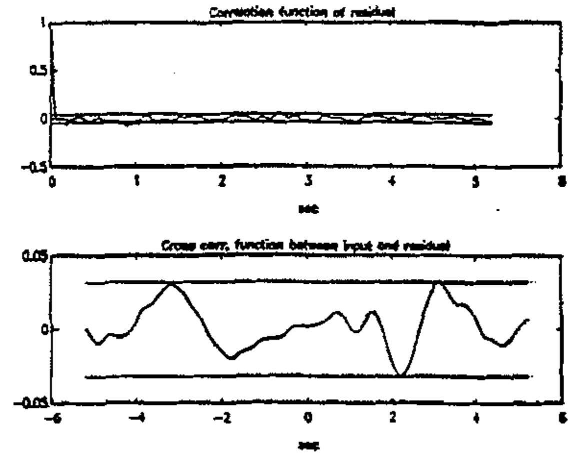

Fig. 12. Validating $\hat{R}_0^f$ and Ψ_0^f ($\lambda_0^f = 3\ \mathrm{rad\,s^{-1}}$).

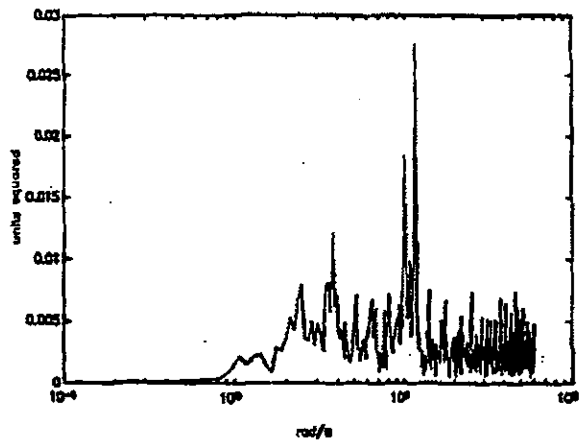

Fig. 13. $\Phi_{\xi_1}(\omega)$ when $\lambda_0^f = 3\ \mathrm{rad\,s^{-1}}$.

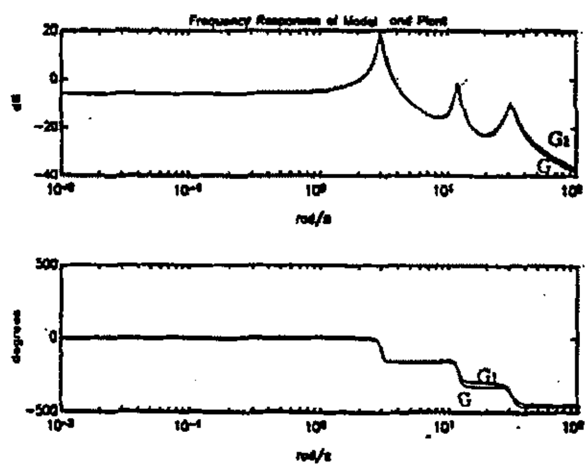

Fig. 14. Frequency response of model G_1.

imposed by the unstable zeros of the plant, given that the initial model has significant modelling errors in the high-frequency region.

In the following the plant involved is a simulated flexible-link robot arm whose transfer function G has poles at $s = -0.0996 \pm j3.0017$, $-0.3339 \pm j12.131$ and $-1.845 \pm j31.481$, zeros at

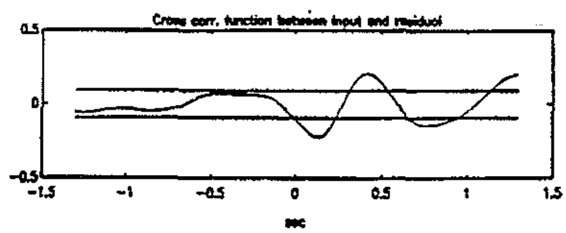

Fig. 15. Validating G_1($\lambda_1^f = 12\,\mathrm{rad\,s^{-1}}$).

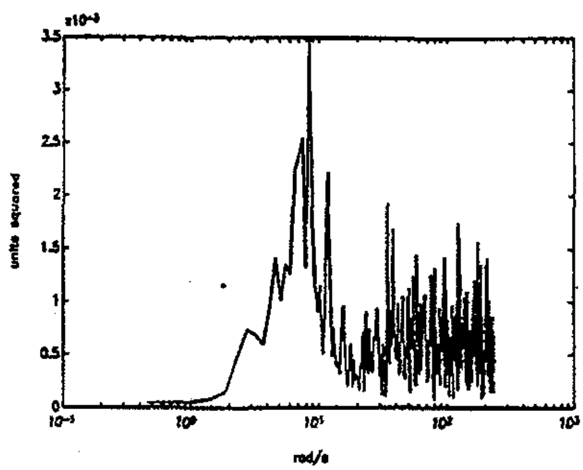

Fig. 16. $\Phi_w(\omega)$ when $\lambda_1^f = 12\,\mathrm{rad\,s^{-1}}$.

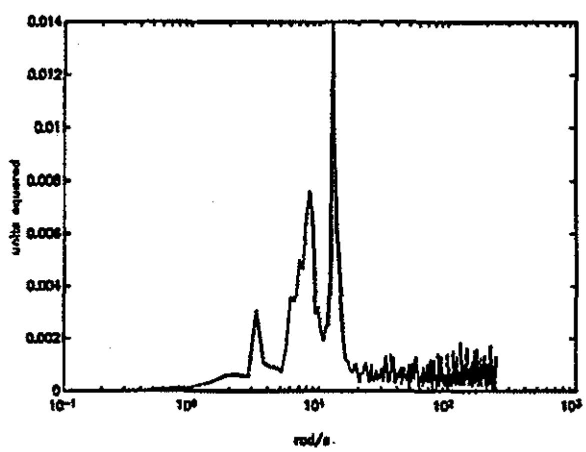

Fig. 17. $\Phi_\varepsilon(\omega)$ when $\lambda_1^f = 12\,\mathrm{rad\,s^{-1}}$.

$s = -13.162$, $-10.646 \pm j12.27$ and $7.169 \pm j11.54$, and $G(0) = 0.5196$.

The initial model G_0 is an open-loop description of G up to and including its first resonant frequency (see Fig. 5). G_0 has a pair of poles at $s = -0.0903 \pm j3.0027$, a zero at $s = -13.31$, and $G_0(0) = 0.5188$.

In this example all identification experiments are performed in closed loop. The input consisted of four periods of a zero-mean square wave of amplitude 1 and period π/λ, where λ is the designed closed-loop bandwidth. The plant output was corrupted by a 'white' noise process generated via Matlab using a normal distribution with zero mean and standard deviation 0.05. During the verification step, one also uses a zero reference input response.

When the designed closed-loop bandwidth is increased to $1.5\,\mathrm{rad\,s^{-1}}$, the method of correlations (see Fig. 6) shows that G_0 is not a good model of G, whereas the method of power spectra (compare Figs 7 and 8) shows that the tracking error is still insignificant. In fact, we are unable to identify a better model than G_0 at this stage. However, as was suggested at the end of Section 6.3, the rate of increasing the designed closed-loop bandwidth is reduced at this stage.

When the designed closed-loop bandwidth has reached $3\,\mathrm{rad\,s^{-1}}$, the method of correlations (see Fig. 9) and the method of power spectra (compare Figs 10 and 11) both indicate that G_0 is not a good model. In particular, comparison of Figs 10 and 11 indicates that the closed-loop output error has a high signal-to-noise ratio at around $12\,\mathrm{rad\,s^{-1}}$. The identified $\hat{R}_0^f$ and $\hat{\Psi}_0^c$ are validated by the method of correlations (see Fig. 12) before $\hat{R}_0^f$ is used to calculate G_1. The resulting G_1 is validated by the method of power spectra (compare Figs 13 and 10). Figure 14 shows the frequency response of G_1. Note that G_1 has poles at $s = -0.0895 \pm j3.0026$, $-0.4834 \pm j12.03$ and $-2.475 \pm j31.502$, zeros at $s = -12.967$, $-7.336 \pm j11.05$ and $9.098 \pm j12.07$, and $G_1(0) = 0.5189$. Using G_1, it is possible to increase the designed closed-loop bandwidth to $12\,\mathrm{rad\,s^{-1}}$ before it is necessary to identify a better model. At this stage, the method of correlations (see Fig. 15) and the method of power spectra (compare Figs 16 and 17) both indicate that G_1 is not a good model. At this stage, the iterative identification and control design process has to be terminated, because we are unable to identify a better model despite considerable efforts and numerous attempts. Notice that the designed closed-loop bandwidth ($12\,\mathrm{rad\,s^{-1}}$) is close to the critical frequency corresponding to the *unstable zeros* of G_1 (at $s = 9.098 \pm j12.07$) and G (at $s = 7.169 \pm j11.54$). From the view point of control design, it follows from Freudenberg and Looze (1985) that, owing to the presence of the unstable zeros of G_1 and G, the system has reached its fundamental performance limitation.

9. CONCLUSIONS

We have examined a number of crucial questions that arise in the windsurfer approach for the case of stable plants with no zeros on the imaginary axis. Among the issues that we have clarified are the following.

1. When can one redesign the controller and expand the closed-loop bandwidth, without re-identifying?
2. When should one re-identify?
3. What does one want to identify in the re-identification procedure?
4. What can one identify in the re-identification procedure?

In order to check if an identified model is actually good for our purpose, we have presented two methods for validating an identified model experimentally before it is employed in controller redesign.

The main conclusion of this paper is that, given a strictly proper stable model of a strictly proper stable plant, it is possible to improve the robust performance of a closed-loop system through the windsurfer approach if

(i) the deterioration in performance robustness caused by increasing the closed-loop bandwidth is mainly contributed by the phase-insensitive factor;

(ii) the deterioration in performance robustness caused by increasing the closed-loop bandwidth resulted in a sufficiently high signal-to-noise ratio associated with the closed-loop output error;

(iii) the designed closed-loop bandwidth has not approached the minimum critical frequency corresponding to the unstable zeros of the plant or the existing model.

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