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A NEW APPROACH TO ADAPTIVE ROBUST CONTROL

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SUMMARY

A new approach is given for the design of adaptive robust control in the frequency domain. Starting with an initial model of a stable plant and a robust stabilizing controller, the new (windsurfer) approach allows the bandwidth of the closed-loop system to be increased progressively through an iterative control-relevant system identification and control design procedure. The method deals with both undermodelling and measurement noise issues. Encouraging results are obtained in the simulations that illustrate the new idea.

KEY WORDS Adaptive control Robust control Internal model control System identification

1. INTRODUCTION

It has long been understood that a key problem in control system design is to handle the *uncertainties* associated with the plant.[1] Two main techniques for the analysis and design of systems with significant uncertainties are adaptive control[2] and robust control.[3,4]

In the traditional approach to analysis and design of an adaptive control system[2] it is assumed that the unknown plant can be represented by a model in which everything is known except for the values of a finite number of parameters. Once the parameters are estimated (and even during the estimation process), the principle of certainty equivalence is normally invoked to update the controller. Normally the unstructured uncertainties of the model are *ignored* in this approach. Therefore it is not surprising, as pointed out in Reference 5 that these adaptive controllers are often not robust. Further, the extensions of the traditional approach to adaptive control which purportedly cope with unstructured (and other) uncertainties involve conditions which are often hard to apply or to grasp intuitively (see e.g. References 6–8). A further problem with the traditional approach is that extreme transient excursions are possible even when global convergence and asymptotic performance are guaranteed.[9]

To be more specific, we consider an adaptive control system as shown in Figure 1, where G is the unknown transfer function of the plant. The time axis is divided into intervals such that

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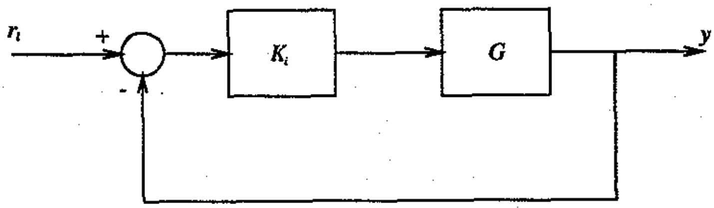

Figure 1. Adaptive control system

during the ith interval the control input applied to the plant is obtained from K_i, where K_i is the transfer function of the controller designed using the model G_{i-1} obtained at the end of the $(i-1)$th time interval.

In an adaptive control problem the ulterior objective for finding G_i, an estimate of G updated from G_{i-1}, is to design a controller K_{i+1} better than K_i such that certain control objectives are improved. For example, if T^{d} represents the desired complementary sensitivity function, then we may like to have

$$\left\| \frac{GK_i}{1+GK_i} - T^{\mathrm{d}} \right\|_\infty \leqslant \left\| \frac{GK_{i-1}}{1+GK_{i-1}} - T^{\mathrm{d}} \right\|_\infty \quad \forall i$$

Implicitly, this means we would like to minimize

$$\left\| \frac{GK_i}{1+GK_i} - T^{\mathrm{d}} \right\|_\infty \quad \forall i$$

Since G, the transfer function of the plant, is unknown, we could only base our design of K_i on G_{i-1} such that

$$K_i = \arg \min_{\gamma} \left\| \frac{G_{i-1}\gamma}{1+G_{i-1}\gamma} - T^{\mathrm{d}} \right\|_\infty \quad \forall i$$

Note that, as usual, we have invoked the principle of certainty equivalence. However, it is important to realize that

$$\left\| \frac{GK_i}{1+GK_i} - T^{\mathrm{d}} \right\|_\infty$$

is not necessarily small even though

$$\left\| \frac{G_{i-1}K_i}{1+G_{i-1}K_i} - T^{\mathrm{d}} \right\|_\infty$$

is minimum. This partly explains why traditional adaptive control systems, which almost invariably invoke the principle of certainty equivalence, often have unsatisfactory robustness properties.

In the robust control approach[3,4] a controller is designed on the basis of a nominal model of the plant with the associated parametric and unstructured model uncertainties explicitly taken into account. Therefore stability robustness is guaranteed and performance robustness is achieved sometimes. The *weakness* of this approach is that it considers only the *a priori* information on the model and *neglects* the fact that some characteristics of the plant could be learnt while it is being controlled. Therefore the robust control approach tends to result in a conservative design in terms of performance. It is likely that *a posteriori* knowledge about the plant could be used to reduce the conservatism in a robust control design.

In this paper we present a new approach for the design of adaptive robust control for a stable plant and the related control-relevant system identification. A preliminary study of the problem under noiseless conditions is reported in Reference 10. The main differences of this paper from Reference 10 are that, instead of updating the models through finding rational function approximations (in the H_∞ sense) of certain plant parametrization and employing proper but non-strictly proper controllers, here we perform system identification using input–output data and employ strictly proper controllers. It was also shown in References 11 and 12 that an iterative approach for model refinement and control robustness enhancement can be developed in the context of an H_2 control problem.

In Section 2 we describe the windsurfer approach[13] to adaptive control. In Section 3 we apply the windsurfer approach to an adaptive model-matching problem and formulate the related *control-relevant system identification criterion*. The relevance of the system identification criterion to the adaptive model-matching problem is explored further in Section 4. In Section 5 we apply Hansen's approach[14–16] to recast the closed-loop system identification problem into an open-loop system identification problem. We also show that with appropriate filtering, the criterion used in the open-loop system identification is highly relevant to the windsurfer approach to adaptive model matching. In Section 6 we develop the relation between the approximate identification and the internal model control (IMC)[4] method of controller design. We present the simulation results in Section 7. In Section 8 we conclude the paper and attempt to give some reasons for the success of the method.

2. THE WINDSURFER APPROACH TO ADAPTIVE CONTROL

By considering how humans learn windsurfing, Anderson and Kosut[13] have made the following observations.

1. The human first learns to control over a limited bandwidth and learning pushes out the bandwidth over which an accurate model of the plant is known.
2. The human first implements a low-gain controller and learning allows the loop to be tightened.

On the basis of these observations, an adaptive robust control design philosophy, *the windsurfer approach*, is proposed in Reference 13. It recognizes that the plant characteristics can differ greatly from the estimated model at any one time, particularly during the initial learning stage. In the new design approach a low-gain controller will first be implemented and the control bandwidth will be small. On the basis of learning a frequency domain description of the plant *in closed loop*, with the learning process progressively increasing the bandwidth over which the plant is accurately known, the controller gain can be increased appropriately over an increasing frequency band. For details see Reference 13. Importantly, in the method suggested, the necessary closed-loop system identification task is transformed into an open-loop system identification problem through the use of coprime fractional representations as discussed in References 14–16.

It was shown recently[17] that the best model for control design cannot be derived from open-loop experiments alone. The controller to be implemented should be taken into account by the system identification experiments. However, this controller is not yet available, since its determination rests on the results of the system identification to be carried out. Hence a general solution to the combination of system identification and control design is necessarily iterative. Although the emphasis of Reference 17 is on the problem of modelling for control purposes, its approach is very similar to that of Reference 13.

In the next section we would like to illustrate the windsurfer approach by considering a model-matching problem in the context of adaptive control.

3. ADAPTIVE MODEL MATCHING

Let G be the unknown transfer function of the plant and let T^{d} represent a desired complementary sensitivity function. We wish to achieve, through iterative system identification and control design, the minimization of the cost function

$$\left\| \frac{GK}{1+GK} - T^{\mathrm{d}} \right\|_{\infty}$$

where K is the transfer function of a controller to be designed.

We begin by designing a controller $K_{1,0}$ to stabilize a known initial model G_0, which may be obtained from an open-loop system identification exercise. If $K_{1,0}$ also stabilizes the unknown transfer function G, then we say that $K_{1,0}$ robustly stabilizes G_0. Notice that we use $K_{j,i}$ to denote the jth controller designed using the ith model which has a transfer function G_i. In general, we attach the subscript j,i to a transfer function to denote that it is either specified or derived on the basis of the ith model for the plant at the jth iteration of control design. Since G_0 may involve significant uncertainties, the resulting controller $K_{1,0}$ may not be able to achieve a small value for

$$\left\| \frac{G_0 K_{1,0}}{1+G_0 K_{1,0}} - T^{\mathrm{d}} \right\|_{\infty}$$

while robustly stabilizing G_0. In general, we need to consider how to handle the question of securing robust stabilization of G_i by $K_{j,i}$. This is bound up with the question of selection of T^{d}. It is in fact to be expected that a sequence of T^{d} will be selected in such a way that the end control objective can be approached in stages. We shall therefore proceed as follows.

In association with each of the models G_i, a sequence of controllers $K_{j,i}$ is designed such that

$$K_{j,i} = \arg \min_{\gamma} \left\| \frac{G_i \gamma}{1+G_i \gamma} - T^{\mathrm{d}}_{j,i} \right\|_{\infty} \quad \forall j \tag{1}$$

where the sequence of functions $T^{\mathrm{d}}_{j,i}$ is specified with $T^{\mathrm{d}}_{j+1,i}$ normally of wider bandwidth than $T^{\mathrm{d}}_{j,i}$. It is also necessary that $T^{\mathrm{d}}_{1,i}$ results in a controller $K_{1,i}$ that robustly stabilizes G_i. A stage will be reached (say when $j = N$) where the bandwidth of the nominal closed-loop transfer function,

$$\bar{T}_{N,i} = \frac{G_i K_{N,i}}{1+G_i K_{N,i}} \tag{2}$$

cannot be increased further without causing the effects of model uncertainties in G_i to be too significant. This occurs when the value of

$$\| T_{N,i} - \bar{T}_{N,i} \|_{\infty}$$

is no longer small, where

$$T_{N,i} = \frac{G K_{N,i}}{1+G K_{N,i}} \tag{3}$$

is the actual closed-loop transfer function of the system.

At this stage it is necessary to improve the accuracy of the model in such a way that is relevant to the control objective. This means that we should try to find an updated model G_{i+1} such that

$$G_{i+1} = \arg\min_{\theta} \left\| \frac{GK_{N,i}}{1+GK_{N,i}} - \frac{\theta K_{N,i}}{1+\theta K_{N,i}} \right\|_{\infty} \tag{4}$$

Once G_{i+1} is found, we can continue to increase the closed-loop bandwidth by repeating the procedure described for G_i previously. However, G_{i+1} should be used instead of G_i and we specify a new sequence of functions $T^{\mathrm{d}}_{j,i+1}$ with $T^{\mathrm{d}}_{1,i+1} = T^{\mathrm{d}}_{N,i}$. The iterative process is continued until the end control objective is achieved or it is prematurely terminated because of one or more of the following constraints:

(1) fundamental performance limitations due to right-half-plane poles and zeros of the plant and/or models[18]
(2) unstable model is obtained (This is a consequence of our simplified control design method. Appropriate extensions of the control design method[4] allow us to deal with this restriction.)
(3) finite control energy
(4) no further improvements in the identified model can be made for a reasonably large set of input–output measurements.

4. CONTROL-RELEVANT SYSTEM IDENTIFICATION

It should be noted that the system identification criterion formulated in Section 3,

$$G_{i+1} = \arg\min_{\theta} \left\| \frac{GK_{N,i}}{1+GK_{N,i}} - \frac{\theta K_{N,i}}{1+\theta K_{N,i}} \right\|_{\infty} \tag{5}$$

would be the formulation of a standard rational function approximation problem provided that G were known. However, as opposed to an approximation problem, we are here dealing with a system identification problem where G is an unknown transfer function and only a finite number of (possibly noisy) input–output measurements are available. Despite this apparent difference, we must emphasize that equation (5) is exactly the dual of the criterion developed by Anderson and Liu[19] in the controller reduction problem based on closed-loop transfer function consideration, where their plant and reduced-order controller are replaced by our controller and estimated model respectively. We can therefore draw a similar conclusion that in our system identification problem there is a reduced weighting placed on the range of frequencies where the loop gain is high. This is very appealing since it agrees with the well-known fact that, for a stable closed-loop system, model errors are more tolerable in the range of frequencies where the loop gain is allowed to be large. More importantly, as we shall explain below, this system identification criterion will enable us to find a new model which allows us to design a closed-loop system with a larger bandwidth than what the original model would allow.

If we rewrite equation (5) in the form

$$G_{i+1} = \arg\min_{\theta} \left\| \left(\frac{1}{1+GK_{N,i}}\right) \left(\frac{\theta K_{N,i}}{1+\theta K_{N,i}}\right) \left(\frac{G-\theta}{\theta}\right) \right\|_{\infty} \tag{6}$$

we see immediately that it is the product of the actual sensitivity function

$$\frac{1}{1+GK_{N,i}}$$

and the nominal complementary-sensitivity-function-weighted multiplicative model error

$$\left(\frac{G_{i+1}K_{N,i}}{1+G_{i+1}K_{N,i}}\right)\left(\frac{G-G_{i+1}}{G_{i+1}}\right)$$

It appears that the frequency-weighting function in equation (6), which involves the unknown actual sensitivity function, cannot be implemented in the system identification procedure. However, we shall show in Section 5 that by recasting the closed-loop system identification into an open-loop system identification problem, we also obtain a system identification criterion which is equivalent to equation (6) but involves only a known frequency-weighting function. Therefore, for the purpose of understanding the effects of the system identification criterion on the identified model, we can treat the frequency-weighting function in equation (6) as a known quantity.

Recall that, as described in Section 3, we were using the model G_i to design a sequence of controllers $K_{j,i}$, with increasing gain over an increasing range of frequencies, such that the closed-loop system has an increasing bandwidth. At the stage where $j = N$, the gain of the controller $K_{N,i}$ has become so large that the high-frequency model uncertainties associated with G_i are no longer insignificant. Any attempt to increase the closed-loop bandwidth further will cause the magnitude of the nominal complementary-sensitivity-function-weighted multiplicative model error

$$\left(\frac{G_iK_{N,i}}{1+G_iK_{N,i}}\right)\left(\frac{G-G_i}{G_i}\right)$$

to become too large at certain frequencies, such that the system may lose performance robustness or even stability robustness. Therefore the gain of the controller $K_{N,i}$ will be limited and the actual sensitivity function

$$\frac{1}{1+GK_{N,i}}$$

will be large beyond the existing limited closed-loop bandwidth. From equation (6) we notice that it is exactly in this range of frequencies, where the actual sensitivity function has large magnitude, that our system identification criterion will penalize the nominal complementary-sensitivity-function-weighted multiplicative model uncertainties of the new model G_{i+1}. We could therefore expect G_{i+1} to have smaller model uncertainties, as compared with G_i, near and beyond the edge of the closed-loop bandwidth that can be achieved with G_i. This will allow us to design controllers $K_{j,i+1}$ that lead to larger closed-loop bandwidth than was possible with G_i. Hence we can say that criterion (6) is control-relevant.

To make concrete the above discussions, we consider an example where the plant has a transfer function

$$G=\frac{9}{s^2+0\cdot06s+9} \tag{7}$$

and an initial model with the transfer function

$$G_0=\frac{1}{s+1} \tag{8}$$

If we employ a controller which has a transfer function of the form

$$K=\frac{\lambda^2(s+1)}{s(s+2\lambda)} \tag{9}$$

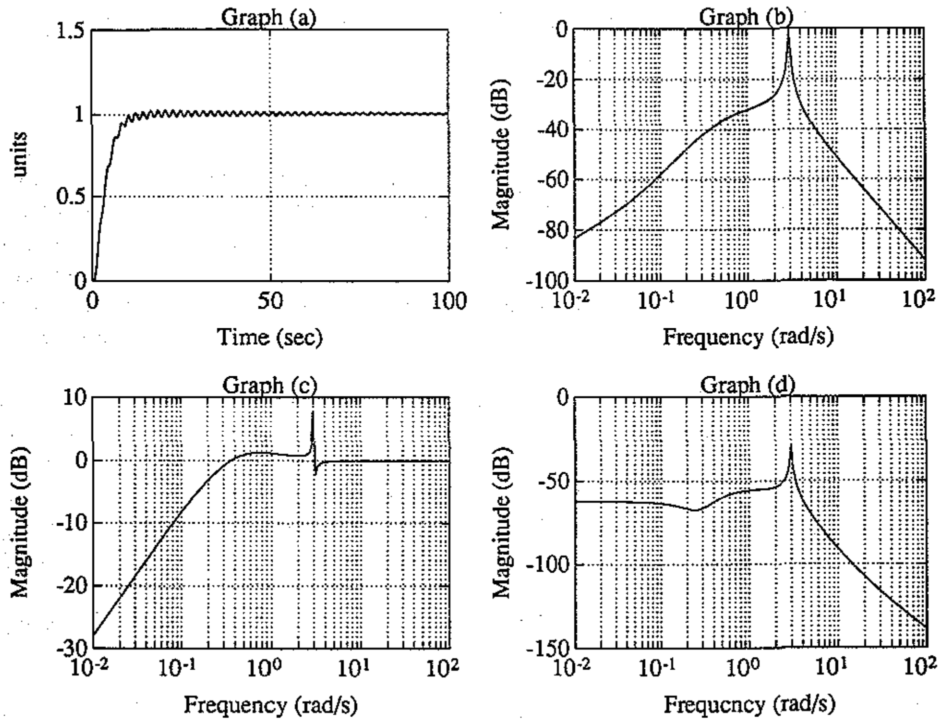

Figure 2. Frequency-weighted multiplicative model errors

then it can be shown that the nominal closed-loop transfer function is given by

$$\bar{T}=\frac{\lambda^2}{(s+\lambda)^2} \tag{10}$$

Therefore λ is the nominal -6 dB bandwidth of the closed-loop system. As λ is increased to $0{\cdot}5\ \mathrm{rad\,s^{-1}}$, the actual closed-loop unit-step response, as shown in graph (a) of Figure 2, has excessive oscillations because the model uncertainties associated with G_0 are no longer insignificant. This is apparent in graph (b) of Figure 2, which shows the magnitude of the nominal complementary-sensitivity-function-weighted multiplicative model error of G_0. We have also shown the actual sensitivity function in graph (c) of Figure 2, which indicates that, if it is incorporated into the system identification criterion (for which the procedure is described later in Sections 5 and 6), the nominal complementary-sensitivity-function-weighted multiplicative error in the new model G_1 will be penalized in the range of frequencies near and beyond the existing closed-loop bandwidth of $0{\cdot}5\ \mathrm{rad\,s^{-1}}$. It can be seen from graph (d) of Figure 2, which shows the nominal complementary-sensitivity-function-weighted multiplicative error of the new model G_1, that this is indeed the case. Therefore the new model G_1 will allow us to increase the closed-loop bandwidth beyond $0{\cdot}5\ \mathrm{rad\,s^{-1}}$.

5. CLOSED-LOOP SYSTEM IDENTIFICATION

We first review a method for closed-loop system identification developed by Hansen and co-workers.[14–16] Subsequently, in Theorem 2, we demonstrate that with appropriate signal filtering, Hansen's method provides a suitable framework to carry out the *control-relevant* system identification formulated in Section 3. For the sake of expository simplicity we shall consider only scalar plants. We begin with the following theorem.[20]

Theorem 1

If $K = X/Y$ is a controller, where X and Y are stable proper transfer functions, and if N and D are stable proper transfer functions that satisfy the Bezout identity

$$NX + DY = 1$$

then the set of all plants stabilized by the controller K is precisely the set of elements in

$$\mathscr{G} = \left\{ \frac{N + RY}{D - RX} \colon R \text{ is a stable proper transfer function} \right\}$$

Consider the feedback system shown in Figure 3, where y and u are the measured output and the control input respectively, e is an unpredictable white disturbance and r_1 and r_2 are user-applied inputs. It is assumed that $K_{j,i}$ is a known stabilizing controller, G is inexactly known and possibly unstable and, as is standard,[21] H is imperfectly known, stable and inversely stable. The system identification problem is to obtain improved estimates of G and H from a finite interval of measured and known data $\{y, u, r_1, r_2 \colon 0 \leqslant t \leqslant T\}$.

Following Hansen,[16] we introduce the stable proper transfer functions $X_{j,i}$, $Y_{j,i}$, $N_{j,i}$ and $D_{j,i}$ which satisfy

$$K_{j,i} = \frac{X_{j,i}}{Y_{j,i}}, \qquad G_i = \frac{N_i}{D_i}, \qquad N_i X_{j,i} + D_i Y_{j,i} = 1$$

The interpretation is that G_i is a known but imperfect model of the plant which is also stabilized by $K_{j,i}$. Applying Theorem 1 as shown in References 15 and 16, there exist stable proper transfer functions $R_{j,i}$ and $S_{j,i}$, with $S_{j,i}$ also inversely stable, such that

$$G = \frac{N_i + R_{j,i} Y_{j,i}}{D_i - R_{j,i} X_{j,i}} \tag{11}$$

$$H = \frac{S_{j,i}}{D_i - R_{j,i} X_{j,i}} \tag{12}$$

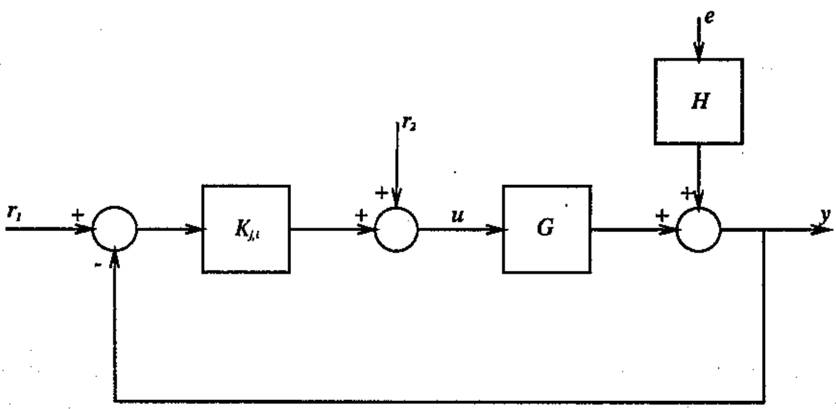

Figure 3. Closed-loop system

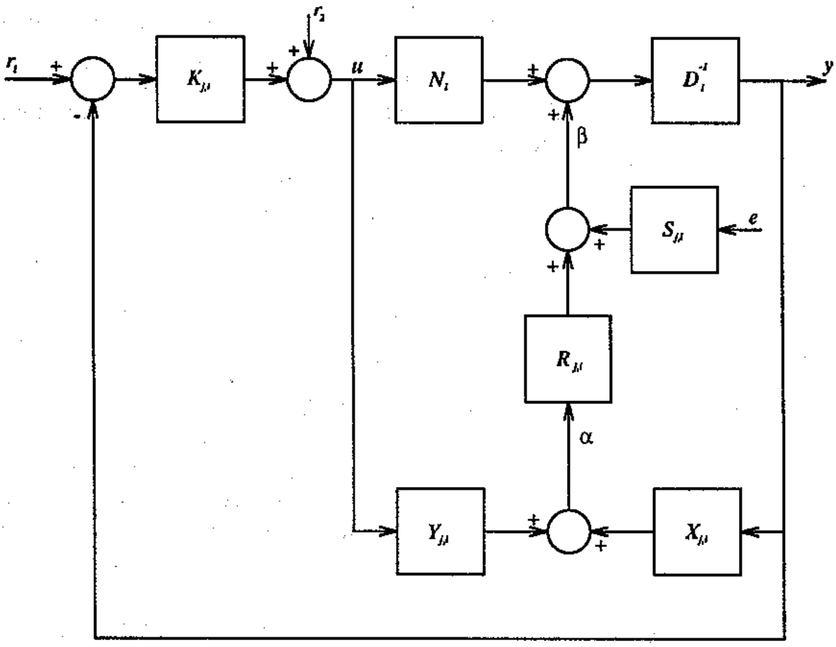

Figure 4. Closed-loop system identification

where $R_{j,i}$ denotes the parametrization of G using the ith model and its associated jth controller $K_{j,i}$.

As a result, system identification of G and H in closed loop is equivalent to system identification of the stable proper transfer functions $R_{j,i}$ and $S_{j,i}$. Using equations (11) and (12), we can represent the feedback system as shown in Figure 4.

From Figure 4 we can write

$$\beta = R_{j,i}\alpha + S_{j,i}e \tag{13}$$

where

$$\alpha = X_{j,i}y + Y_{j,i}u \tag{14}$$

$$\beta = D_i y - N_i u \tag{15}$$

However, since

$$u = K_{j,i}(r_i - y) + r_2$$

and

$$K_{j,i} = \frac{X_{j,i}}{Y_{j,i}}$$

equation (14) can be rewritten as

$$\alpha = X_{j,i}r_1 + Y_{j,i}r_2 \tag{16}$$

It is important to observe from equations (13), (15) and (16) that α depends only on the applied signals r_1 and r_2 operated on by known stable proper transfer functions $X_{j,i}$ and $Y_{j,i}$ respectively, while β depends on the measured signals y and u operated by known stable proper transfer functions D_i and N_i respectively. Moreover, α is independent of the transfer functions G and H and the disturbance e. Hence the system identification of G and H in closed loop has been recast into the system identification of $R_{j,i}$ and $S_{j,i}$ in open loop.

We shall next state a result which is highly relevant to the system identification step of the windsurfer approach to adaptive control.

Theorem 2

Let the controller $K_{j,i}$ stabilize the plant G and the model

$$G_i = \frac{N_i}{D_i}$$

where N_i and D_i are stable proper transfer functions, and let

$$K_{j,i} = \frac{X_{j,i}}{Y_{j,i}}$$

where $X_{j,i}$ and $Y_{j,i}$ are stable proper transfer functions satisfying the Bezout identity

$$N_i X_{j,i} + D_i Y_{j,i} = 1$$

Let G_{i+1} be another model of G, also stabilized by $K_{j,i}$ and therefore having a description

$$G_{i+1} = \frac{N_i + r_{j,i} Y_{j,i}}{D_i - r_{j,i} X_{j,i}} \tag{17}$$

where $r_{j,i}$ is a stable proper transfer function. Also define the filtered output error

$$\xi = Y_{j,i}(\beta - r_{j,i}\alpha) \tag{18}$$

where, with $r_2 = 0$,

$$\alpha = X_{j,i} r_1, \qquad \beta = D_i y - N_i u$$

$$r_1 = \text{reference signal}, \qquad y = \text{plant output}, \qquad u = \text{control input}$$

Thus ξ is an error arising in the (open-loop) identification of $R_{j,i}$ through an estimate $r_{j,i}$. Then the filtered output error can be expressed as

$$\xi = \left(\frac{GK_{j,i}}{1 + GK_{j,i}} - \frac{G_{i+1}K_{j,i}}{1 + G_{i+1}K_{j,i}}\right) r_1 + \frac{1}{1 + GK_{j,i}} He$$

Proof. See Appendix I. □

Remark

Notice that in Theorem 2 it is necessary that $K_{j,i}$ stabilizes G when the system identification procedure is carried out. This can be ensured by increasing the closed-loop bandwidth smoothly and cautiously in the controller design stages (to be described in Section 6). We would always detect a gradual degradation of performance robustness (while stability is still being maintained and the system identification procedure is being carried out) before the closed-loop system lost stability.

Suppose that the value of

$$\left\| \frac{GK_{j,i}}{1+GK_{j,i}} - \frac{G_iK_{j,i}}{1+G_iK_{j,i}} \right\|_\infty \tag{19}$$

has become large. As it was described in Section 3, we want a new identification of G via G_{i+1} for which

$$\left\| \frac{GK_{j,i}}{1+GK_{j,i}} - \frac{G_{i+1}K_{j,i}}{1+G_{i+1}K_{j,i}} \right\|_\infty \tag{20}$$

is small. We are going to use the $r_{j,i}$-parametrization of G_{i+1}. By substituting equations (11) and (17) into expression (20) and noting that

$$K_{j,i} = \frac{X_{j,i}}{Y_{j,i}}$$

we can, after simplification, conclude that

$$\left\| \frac{GK_{j,i}}{1+GK_{j,i}} - \frac{G_{i+1}K_{j,i}}{1+G_{i+1}K_{j,i}} \right\|_\infty = \| Y_{j,i}X_{j,i}(R_{j,i} - r_{j,i}) \|_\infty \tag{21}$$

should be small.

By using equations (13), (18) and (21), we immediately see that (for the system identification procedure relevant to the windsurfer approach to adaptive control) the appropriate signal model is

$$\beta_1 = R_{j,i}\alpha_1 + v \tag{22}$$

where

$$\beta_1 = Y_{j,i}\beta \tag{23}$$

$$\alpha_1 = Y_{j,i}\alpha \tag{24}$$

and v is the term related to the disturbance e.

Remarks

(i) Note that

$$T_{j,i} = \frac{GK_{j,i}}{1+GK_{j,i}}$$

is the actual closed-loop transfer function of the system and

$$\bar{T}_{j,i} = \frac{G_iK_{j,i}}{1+G_iK_{j,i}}$$

is the nominal closed-loop transfer function of the system. Therefore, using similar substitutions to these that resulted in equation (21), we can obtain

$$T_{j,i} - \bar{T}_{j,i} = Y_{j,i}X_{j,i}(R_{j,i} - \bar{R}_{j,i}) \tag{25}$$

However, since

$$\bar{R}_{j,i} \equiv 0 \quad \forall j, \forall i$$

we therefore have

$$T_{j,i} - \bar{T}_{j,i} = Y_{j,i} X_{j,i} R_{j,i} \tag{26}$$

By comparing the argument of the H_∞-norm given in expression (19) with the left-hand side of equation (26), we see immediately that when the value of

$$\left\| \frac{GK_{j,i}}{1 + GK_{j,i}} - \frac{G_i K_{j,i}}{1 + G_i K_{j,i}} \right\|_\infty$$

has become large, i.e. when the closed-loop properties of the actual system ($T_{j,i}$) are significantly different from the closed-loop properties of the nominal system ($\bar{T}_{j,i}$), the value of

$$\| Y_{j,i} X_{j,i} R_{j,i} \|_\infty$$

will be large.

(ii) From the signals defined in Theorem 2, we observed that $R_{j,i}$, the transfer function to be identified, is excited by the signal α, where

$$\alpha = X_{j,i} r_1, \qquad X_{j,i} = \frac{K_{j,i}}{1 + G_i K_{j,i}}$$

Since the nominal closed-loop transfer function of the system is

$$\bar{T}_{j,i} = \frac{G_i K_{j,i}}{1 + G_i K_{j,i}}$$

we can write

$$X_{j,i} = \frac{\bar{T}_{j,i}}{G_i}$$

Therefore $X_{j,i}$ will have large magnitude when we try to push the nominal closed-loop bandwidth beyond the nominal open-loop bandwidth. Since a model usually has its uncertainties become significant for frequencies beyond its bandwidth, from Figure 5 we see that if the spectrum of r_1 is white, we automatically get the right weighting for the input to $R_{j,i}$ for the system identification scheme.

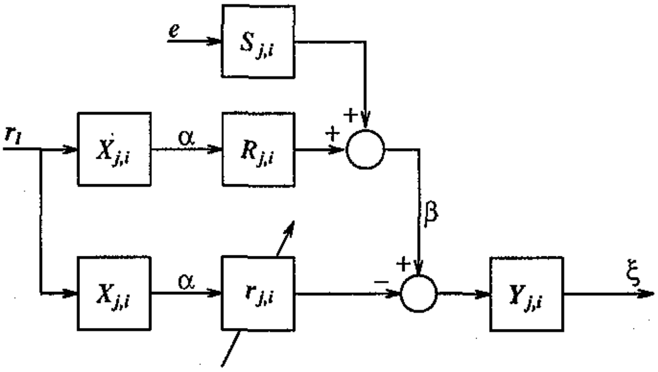

Figure 5. Excitation of $R_{j,i}$

(iii) It is shown in Theorem 2 that the effect of e on ξ is given by

$$\frac{He}{1+GK_{j,i}}$$

Notice that this is the effect of e on y attenuated by the sensitivity function of the actual closed-loop system.

6. APPROXIMATE IDENTIFICATION OF THE $R_{j,i}$ TRANSFER FUNCTION FOR IMC CONTROLLER DESIGN

In Section 5 we have shown that the closed-loop system identification of the plant transfer function G can be reformulated into an open-loop system identification of the stable proper transfer function $R_{j,i}$ that parametrized the transfer function G via the equation

$$G=\frac{N_i+R_{j,i}Y_{j,i}}{D_i-R_{j,i}X_{j,i}}$$

In this and the following sections we shall, for simplicity, study the case where the plant transfer function is stable strictly proper and has no zeros on the imaginary axis of the s-plane and where the IMC method[4] is used to design the controller $K_{j,i}$. We shall also assume that all estimates G_i of the plant are stable strictly proper transfer functions.

If the model

$$G_i=\frac{N_i}{D_i}$$

is stable, we can let $N_i=G_i$ and $D_i=1$ so that

$$G=G_i+\frac{R_{j,i}}{1-R_{j,i}Q_{j,i}} \tag{27}$$

where $Q_{j,i}$ is a stable strictly proper transfer function that parametrized the strictly proper controller

$$K_{j,i}=\frac{X_{j,i}}{Y_{j,i}}$$

and

$$Q_{j,i}\overset{\text{def}}{=}\frac{K_{j,i}}{1+G_iK_{j,i}} \tag{28}$$

The reason for requiring $Q_{j,i}$ and hence $K_{j,i}$ to be strictly proper is that this is a necessary condition for the system to be robust in the presence of high-frequency parasitic or singular perturbation.[22] We also have

$$X_{j,i}=Q_{j,i} \qquad Y_{j,i}=1-Q_{j,i}G_i$$

Since the parametrization of G by $R_{j,i}$ depends intimately on $Q_{j,i}$, we shall briefly explain how $Q_{j,i}$ is obtained in the design of the controller $K_{j,i}$. We will use the notations n_H and d_H to denote the numerator polynomial and the denominator polynomial respectively of a rational transfer function H.

Given a stable model

$$G_i = \frac{n_{G_i}}{d_{G_i}}$$

where d_{G_i} has no zeros in the closed right-half s-plane, if n_{G_i} has no zeros on the imaginary axis of the s-plane, we can write

$$G_i = \frac{\tilde{n}_{G_i} \Pi_i (z_i - s)}{d_{G_i}}$$

where all z_i have positive real parts and $\tilde{n}_{G_i}$ has no zeros in the closed right-half s-plane. By writing G_i as

$$G_i = [G_i]_{\mathrm{m}} [G_i]_{\mathrm{a}}$$

where

$$[G_i]_{\mathrm{m}} = \frac{\tilde{n}_{G_i} \Pi_i (z_i^* + s)}{d_{G_i}}$$

(z_i^* is the complex conjugate of z_i)

$$[G_i]_{\mathrm{a}} = \frac{\Pi_i (z_i - s)}{\Pi_i (z_i^* + s)}$$

we have factored G_i as a product of its minimum phase factor $[G_i]_{\mathrm{m}}$ and the associated all-pass factor $[G_i]_{\mathrm{a}}$. We can find the controller

$$K_{j,i} = \arg \min_{\gamma} \left\| \frac{G_i \gamma}{1 + G_i \gamma} - T^{\mathrm{d}}_{j,i} \right\|_{\infty}$$

with $T^{\mathrm{d}}_{j,i}$ specified as $T^{\mathrm{d}}_{j,i} = F_{j,i}[G_i]_{\mathrm{a}}$, by designing a detuned H_2-optimal controller for a step reference input using the internal model control (IMC) approach.[4] This is achieved by setting

$$Q_{j,i} = [G_i]_{\mathrm{m}}^{-1} F_{j,i} \tag{29}$$

where $F_{j,i}$ is a lowpass filter of the form

$$F_{j,i} = \left(\frac{\lambda_{j,i}}{s + \lambda_{j,i}} \right)^{n+1}$$

n is the relative degree of the model G_i and $\lambda_{j,i}$ is selected small enough so that $K_{j,i}$ robustly stabilizes G_i.

In the ideal situation where $G_i = G$, the nominal and actual closed-loop transfer functions of the system are equal and are given by the transfer function $F_{j,i}[G_i]_{\mathrm{a}}$. Therefore $\lambda_{j,i}$ is both the nominal and actual closed-loop system bandwidth with a $-3(n+1)$ dB attenuation. In general, $G_i \neq G$ and $\lambda_{j,i}$ serves only as an approximate bandwidth of the actual closed-loop system.

With the controller designed using the above procedure, we shall now show that the transfer function to be identified, $R_{j,i}$, is the product of a known stable proper transfer function and an unknown stable strictly proper transfer function. An analysis of the form of the unknown factor in $R_{j,i}$ indicates how it can be sensibly approximated by a low-order transfer function. We shall first rewrite equation (27) as

$$R_{i,j} = \frac{G - G_i}{1 + Q_{j,i}(G - G_i)} \tag{30}$$

Then we can obtain, after substituting equations (28) and (29) into equation (30) and performing some algebraic manipulations,

$$R_{j,i} = \{[G_i]_{\mathrm{m}}(s+\lambda_{j,i})^n\}\left\{\frac{(s+\lambda_{j,i})(d_{G_i}n_G - d_G n_{G_i})}{d_{K_{j,i}}d_G + n_{K_{j,i}}n_G}\right\} \tag{31}$$

Note that equation (31) can also be written as

$$R_{j,i} = \tilde{R}_{j,i}\hat{R}_{j,i} \tag{32}$$

where

$$\tilde{R}_{j,i} = [G_i]_{\mathrm{m}}(s+\lambda_{j,i})^n \tag{33}$$

is a *known* stable proper transfer function and, other than the factor $s+\lambda_{j,i}$ in the numerator,

$$\hat{R}_{j,i} = \frac{(s+\lambda_{j,i})(d_{G_i}n_G - d_G n_{G_i})}{d_{K_{j,i}}d_G + n_{K_{j,i}}n_G} \tag{34}$$

is an *unknown* stable strictly proper transfer function that depends on the unknown transfer function G. Therefore the problem of identifying $R_{j,i}$ has become one of identifying its unknown factor $\hat{R}_{j,i}$. We shall summarize this important result in the following theorems.

Theorem 3

Consider a plant which has an unknown stable strictly proper transfer function G and a model with a known stable strictly proper transfer function G_i. If G and G_i have no zeros along the imaginary axis of the s-plane and

$$G_i = [G_i]_{\mathrm{m}}[G_i]_{\mathrm{a}}$$

where $[G_i]_{\mathrm{m}}$ is the minimum phase factor of G_i and $[G_i]_{\mathrm{a}}$ is the allpass factor of G_i, then with

$$Q_{j,i} = [G_i]_{\mathrm{m}}^{-1}F_{j,i}$$

and

$$F_{j,i} = \left(\frac{\lambda_{j,i}}{s+\lambda_{j,i}}\right)^{n+1}$$

where n is the relative degree of G_i, the controller

$$K_{j,i} = \frac{Q_{j,i}}{1-Q_{j,i}G_i}$$

will robustly stabilize G_i for all sufficiently small values of $\lambda_{j,i} \geqslant 0$.

Proof. See Chaps 4 and 5 of Reference 4. □

Theorem 4

Let the controller be designed according to the conditions stated in Theorem 3; then the unknown stable strictly proper transfer function to be identified,

$$R_{j,i} = \frac{G - G_i}{1 + Q_{j,i}(G - G_i)}$$

can be factorized as

$$R_{j,i} = \tilde{R}_{j,i}\hat{R}_{j,i}$$

where $\hat{R}_{j,i}$ is an unknown stable strictly proper transfer function to be identified and $\tilde{R}_{j,i}$ is a known stable proper transfer function given by

$$\tilde{R}_{i,j} = [G_i]_m(s + \lambda_{j,i})^n$$

where $\lambda_{j,i}$ is the nominal closed-loop system bandwidth with a $-3(n+1)$ dB attenuation.

Furthermore, the order and the relative degree (rel deg) of the transfer function $\hat{R}_{j,i}$ are respectively given by

$$\{\text{order of } \hat{R}_{j,i}\} = \{\text{order of } G\} + \{\text{order of } G_i\} - (M+N) + 1$$

$$\text{rel deg}\{\hat{R}_{j,i}\} = \min(\text{rel deg}\{G\}, \text{ rel deg}\{G_i\})$$

where M is the number of common zeros in G and G_i and N is the number of common poles in G and G_i.

Proof. See Appendix II. □

Remarks

(i) Note that the factorization of $R_{j,i}$ given in Theorem 4 is naturally induced by the IMC[4] controller design procedure that we have adopted.

(ii) The poles of $\hat{R}_{j,i}$ are the poles of $T_{j,i}$, the actual closed-loop transfer function of the system.

(iii) It is important to note that $\hat{R}_{j,i} = 0$ if and only if $G = G_i$.

(iv) The order of $\hat{R}_{j,i}$ depends on the order of G, which is an unknown.

(v) Although Hansen's approach enables us to obtain an unbiased estimate of the transfer function $\hat{R}_{j,i}$, it should be noted that $\hat{R}_{j,i}$ has more parameters to be estimated than G. Furthermore, since the order, hence the number of parameters to be identified in $\hat{R}_{j,i}$, increases while the magnitude of $\hat{R}_{j,i}$ decreases with the stages of iteration, we would expect that under noisy conditions the system identification problem will become harder as the iteration process progresses. There is an obvious analogy in the windsurfing situation. The better is the skill of a windsurfer, the harder it will be for him/her to improve his/her skill further. In fact, it will take a long time under extreme conditions to improve his/her skill. In the system identification problem for $\hat{R}_{j,i}$ the interpretation is that strong probing signals and a long record of measurements are necessary to achieve even a slight improvement if the closed-loop already has good performance and large bandwidth.

Since we do not know the order of $\hat{R}_{j,i}$ *a priori* and since we are going to identify $\hat{R}_{j,i}$ (actually $R_{j,i}$) and update G_i to G_{i+1} when the step response of the actual closed-loop system exhibits unacceptable oscillations and/or overshoots (associated with model uncertainties), we expect $\hat{R}_{j,i}$ to have complex conjugate poles. Therefore the transfer function which serves as an approximation of $\hat{R}_{j,i}$ has to have an order of at least two. Moreover, since the smallest possible relative degree of a strictly proper transfer function is one and the relative degree of G is unknown, we have to assume that the relative degree of $\hat{R}_{j,i}$ could be one. It was shown in equation (21) that the system identification problem is to find

$$r_{j,i} = \arg\min_{\sigma} \| X_{j,i}Y_{j,i}(R_{j,i} - \sigma)\|_\infty \tag{35}$$

If we define

$$r_{j,i} = \tilde{R}_{j,i}\hat{r}_{j,i} \tag{36}$$

where $\hat{r}_{j,i}$ is an unknown second-order stable strictly proper transfer function, then by substituting equations (32) and (36) into equation (35), we can show that the system identification problem becomes one of finding

$$\hat{r}_{j,i} = \arg\min_{\phi} \| X_{j,i}Y_{j,i}\tilde{R}_{j,i}(\hat{R}_{j,i} - \phi)\|_{\infty} \tag{37}$$

Therefore, for the purpose of identifying $\hat{R}_{j,i}$, the signal model can be obtained by appropriately modifying equation (22) and is given by

$$\beta_1 = \hat{R}_{j,i}\alpha_2 + v \tag{38}$$

where

$$\alpha_2 = \tilde{R}_{j,i}\alpha_1 \tag{39}$$

and α_1, β_1 and v have been defined previously. Notice that the signals β_1 and α_2 in the model described by equation (38) can easily be generated, using known filters, from the control input u, the measured output y and the reference input r_1.

Remarks

(i) Since $Y_{j,i}$ is the nominal sensitivity function of the closed-loop system, we immediately see that the frequency shaping in the identification criterion given by equation (37) will force the updated model to have small modelling error at the edge of the closed-loop bandwidth where the nominal sensitivity function cannot be made small by the controller $K_{j,i}$.

(ii) It is important to ensure that the input is sufficiently exciting when we are carrying out a system identification experiment.

(iii) Under noisy conditions the signals to be used in the system identification process should be appropriately lowpass filtered. In a discrete time implementation this can be accomplished by an anti-aliasing filter.

(iv) When updating the model using the equation

$$G_{i+1} = G_i + \frac{r_{j,i}}{1 - r_{j,i}Q_{j,i}}$$

the order of the model may increase. To prevent the model order from increasing indefinitely, we use a frequency-weighted balanced truncation scheme[19] to reduce the order of G_{i+1}. Specifically, we find

$$\hat{G}_{i+1} = \arg\min_{\eta} \left\| \frac{G_{i+1}K_{j,i}}{1 + G_{i+1}K_{j,i}} - \frac{\eta K_{j,i}}{1 + \eta K_{j,i}} \right\|_{\infty}$$

where $\hat{G}_{i+1}$ is the reduced-order model. If the model order is restricted to m, the controller will be at most of order $m + 1$ (see controller design equations given in Theorem 3). In this way the controller complexity will be limited.

7. SIMULATION RESULTS

With reference to Figure 3, we shall present some simulation results of applying the windsurfer approach to the control of a system with

$$G(s) = \frac{9}{(s+1)(s^2+0\cdot 06s+9)}, \qquad H(s) = 1$$

and e as zero-mean disturbance with a constant energy density of 0·0025 from 0 to 100 Hz. We first summarize the procedure in the following algorithm.

Step 1

Set $G_i = G_0$, where G_0 is the transfer function of an initial model of the plant.

Step 2

Factorize G_i as

$$G_i = [G_i]_{\mathrm{m}}[G_i]_{\mathrm{a}}$$

where $[G_i]_{\mathrm{m}}$ is the minimum phase factor of G_i with a relative degree of n and $[G_i]_{\mathrm{a}}$ is the associated allpass factor of G_i.

Step 3

For $j = 1$ find

$$K_{j,i} = \frac{Q_{j,i}}{1 + Q_{j,i}G_i}$$

with

$$Q_{j,i} = [G_i]_{\mathrm{m}}^{-1}F_{j,i}$$

where the parameter $\lambda_{j,i}$ in the transfer function

$$F_{j,i} = \left(\frac{\lambda_{j,i}}{s+\lambda_{j,i}}\right)^{n+1}$$

is chosen such that $K_{j,i}$ robustly stabilizes G_i in the sense that the filtered (noisy) step response of the actual closed-loop system has, at most, few oscillations and/or overshoots. Stop here if such a robust stabilizing controller cannot be found. Also stop here if the robust stabilizing controller results in a closed-loop system which meets the specified bandwidth. Otherwise proceed to the next step.

Step 4

Let $j = j + 1$ and set $\lambda_{j,i} = \lambda_{j-1,i} + \varepsilon$ for small $\varepsilon > 0$ and redesign the controller $K_{j,i}$ using the equations given in Step 3. Stop here if the design produces a robust stabilizing controller with the closed-loop system satisfying the specified bandwidth. Otherwise repeat this step if $K_{j,i}$ robustly stabilizes G_i; else proceed to the next step.

Step 5

Perform control-relevant system identification to obtain $\hat{r}_{j,i}$. For this purpose we apply an algorithm such as least squares to obtain an estimate $\hat{r}_{j,i}$ of $\hat{R}_{j,i}$ which satisfies

$$\beta_1 = \hat{R}_{j,i}\alpha_2 + v$$

This depends on using the signals

$$\beta_1 = Y_{j,i}(y - G_i u), \qquad \alpha_2 = \tilde{R}_{j,i}Y_{j,i}X_{j,i}r_1$$

(We actually used discrete time samples of β_1 and α_2 and an output error algorithm to construct a strictly causal second-order estimate from which a continuous time strictly proper $\hat{r}_{j,i}$ was obtained.) Using $\hat{r}_{j,i}$, the model is updated via the following set of equations:

$$\tilde{R}_{j,i} = [G_i]_m(s + \lambda_{j,i})^n, \qquad r_{j,i} = \tilde{R}_{j,i}\hat{r}_{j,i}, \qquad G_{i+1} = G_i + \frac{r_{j,i}}{1 - r_{j,i}Q_{j,i}}$$

Step 6

If G_{i+1} is stable, find the reduced-order model

$$\hat{G}_{i+1} = \arg\min_{\eta} \left\| \frac{G_{i+1}K_{j,i}}{1 + G_{i+1}K_{j,i}} - \frac{\eta K_{j,i}}{1 + \eta K_{j,i}} \right\|_\infty$$

Otherwise stop here.

Step 7

Set $G_i = \hat{G}_{i+1}$ and return to Step 2.

Remarks

(i) In the algorithm, system identification has to be carried out when

$$\| T_{N,i} - \bar{T}_{N,i} \|_\infty$$

is no longer small. Broadly speaking, this will correspond to a significant difference between the designed nominal performance (depending on G_i and $K_{N,i}$) and the actual performance (depending on G and $K_{N,i}$). In particular, the observed step response may exhibit many more oscillations and/or overshoots than the designed values. This is not of course the same thing as guaranteeing that the H_∞ error above has became large, but neither is it unrelated. To be more precise, we define the peak gain of a system whose transfer function is T by

$$\| T \|_1 = \sup_{\| w \|_\infty \neq 0} \frac{\| Tw \|_\infty}{\| w \|_\infty}$$

This is also equal to the *total variation* of the system's unit-step response[23] (roughly the sum of all consecutive peak-to-valley differences in the unit-step response). It can be shown[24] that if T is a stable strictly proper transfer function, then

$$\| T \|_\infty \leqslant \| T \|_1 \leqslant 2p \| T \|_\infty$$

where p is the order of the transfer function T. Now we consider the peak error

$$\| T_{N,i} - \bar{T}_{N,i} \|_1$$

Since

$$\| T_{N,i} - \bar{T}_{N,i} \|_1 \geqslant \| T_{N,i} \|_1 - \| \bar{T}_{N,i} \|_1$$

then if the observed step response of $T_{N,i}$ exhibits many more oscillations and/or overshoots than the designed step response of $\bar{T}_{N,i}$, we would expect

$$\| T_{N,i} \|_1 \gg \| \bar{T}_{N,i} \|_1$$

and hence

$$\| T_{N,i} - \bar{T}_{N,i} \|_1 \gg \varepsilon, \quad \varepsilon > 0$$

Since the peak gain also provides a *loose* lower bound for the H_∞ gain, it is likely that

$$\| T_{N,i} - \bar{T}_{N,i} \|_\infty$$

becomes large when the observed actual step response exhibits many more oscillations and/or overshoots than the desired one. This explains why, in the simulation, the models are updated whenever the filtered (noisy) actual step response exhibits *unacceptable* oscillations and/or overshoots.

(ii) The algorithm used to obtain an estimate $\hat{r}_{j,i}$ of $\hat{R}_{j,i}$ cannot be expected to give an optimal H_∞ estimate. However, note that efficient algorithms for performing H_∞ system identification are still lacking and the corresponding theory is still not well understood.[25–27]

(iii) Since the stability robustness of the closed-loop system for each $\lambda_{j,i}$ has to be checked by using step response testing, the method is not an on-line procedure. In fact, at this stage of development it is an off-line iterative identification and control design procedure.

The simulation results are presented in Figures 6–8. We start with an initial model which has the transfer function

$$G_0 = \frac{0 \cdot 8}{s + 1 \cdot 2}$$

In all these figures the graphs on the left show the noisy unit-step responses of the actual closed-loop systems and those on the right show the corresponding lowpass-filtered signals. Graphs (a) and (b) of Figure 6 show the responses of the actual closed-loop system with a nominal bandwidth of $0 \cdot 1$ rad s^{-1}. Note that overshoots and oscillations are absent for the response in graph (b). Graphs (c) and (d) of Figure 6 are for a nominal closed-loop bandwidth of $0 \cdot 5$ rad s^{-1}. Note that the response in graph (d) is oscillatory and any attempt to increase the nominal closed-loop bandwidth further is likely to lead to instability. At this stage it is necessary to improve the accuracy of the model if we wish to increase the nominal closed-loop bandwidth further. To ensure that the signals are sufficiently exciting, low-amplitude sinusoids in the relevant frequency range are superimposed on the unit-step input just prior to system identification. The responses are shown in graphs (a) and (b) of Figure 7. The updated model has a transfer function

$$G_1 = \frac{0 \cdot 062528 s^2 - 0 \cdot 33968 s + 10 \cdot 279}{s^3 + 1 \cdot 2801 s^2 + 9 \cdot 1173 s + 10 \cdot 324}$$

The updated model G_1 is used to redesign a nominal closed-loop system with a bandwidth of $0 \cdot 51$ rad s^{-1} and the responses are shown in graphs (c) and (d) of Figure 7. By comparing graph (d) of Figure 7 with that of Figure 6, we observe that the response no longer has oscillations. We also notice that the rise time in graph (d) of Figure 7 is about twice that in

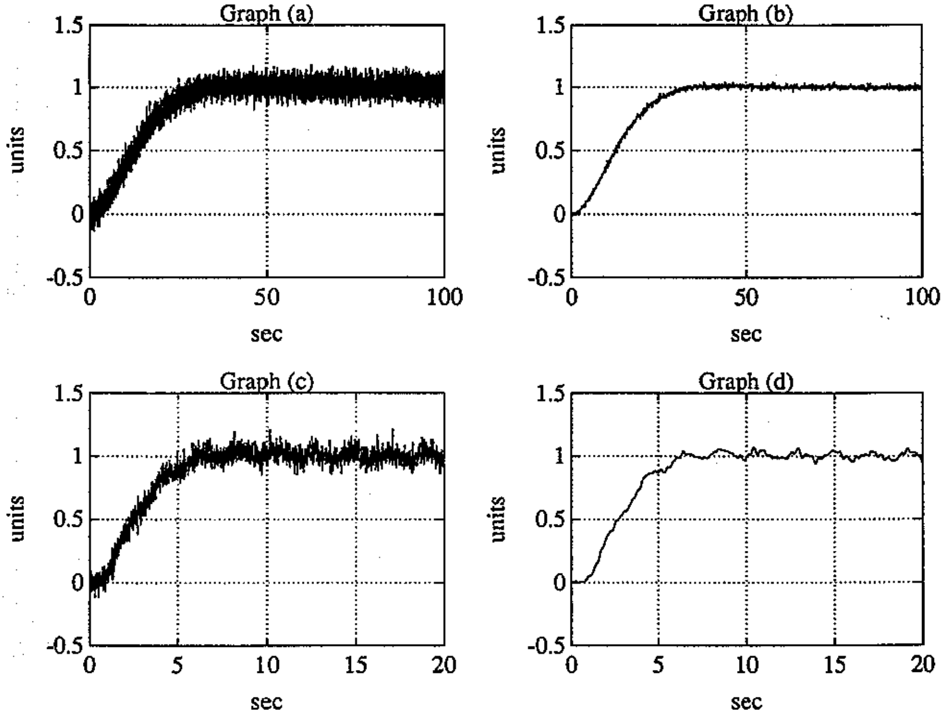

Figure 6. Simulation results 1

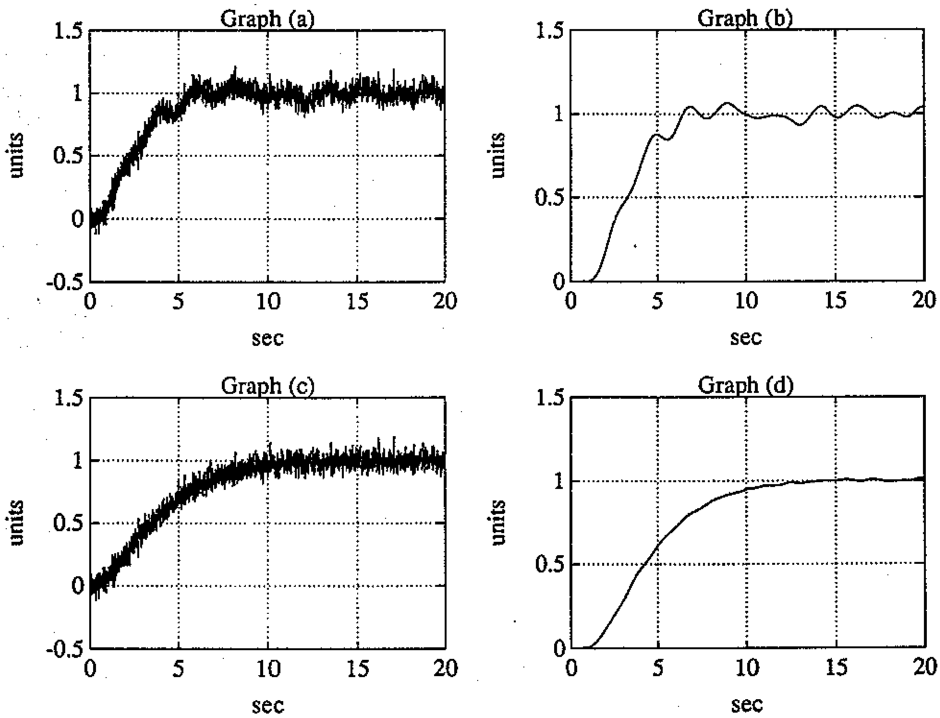

Figure 7. Simulation results 2

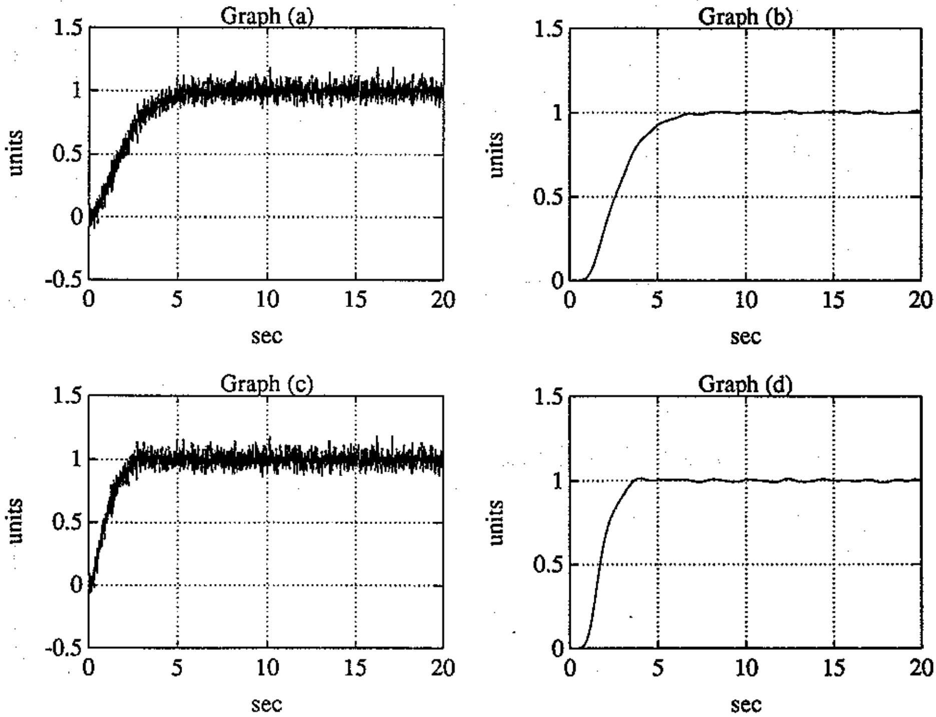

Figure 8. Simulation results 3

graph (d) of Figure 6. Since both G_0 and G_1 have the same relative degree $n = 1$, we would expect graph (d) of Figure 6 and graph (d) of Figure 7 to be similar to the unit-step response of the nominal closed-loop transfer function $[0\cdot5/(s+0\cdot5)]^2$. By comparing with the computed unit-step response of the transfer function $[0\cdot5/(s+0\cdot5)]^2$, we have verified that graph (d) of Figure 7 is very close to the desired one. If we continue to increase the nominal closed-loop bandwidth of the system, we obtain the responses shown in Figure 8, where graphs (a) and (b) are for a bandwidth of 1 rad s^{-1} and graphs (c) and (d) are for a bandwidth of 2 rad s^{-1}.

The frequency responses of G, G_0 and G_1 are presented in Figure 9. Notice that, compared with G_0, the updated model G_1 has effectively captured the effects of the poorly damped resonance of the plant.

For the purpose of comparison we present in Figures 10 and 11 the corresponding results obtained using the procedure described in Reference 10. Recall that, as we have mentioned in Section 1, these are obtained under noiseless conditions using rational function approximations (in the H_∞ sense) of the plant instead of identified models. It is also important to emphasize that, instead of strictly proper controllers, proper but non-strictly proper controllers are used in the procedure described in Reference 10.

To facilitate comparison, we adopt the same initial model

$$G_0 = \frac{0\cdot8}{s+1\cdot2}$$

Graphs (a) and (b) of Figure 10 show the unit-step responses of the actual closed-loop system for nominal closed-loop bandwidths of $0\cdot02$ and $0\cdot04$ rad s^{-1} respectively. Note that graph (b)

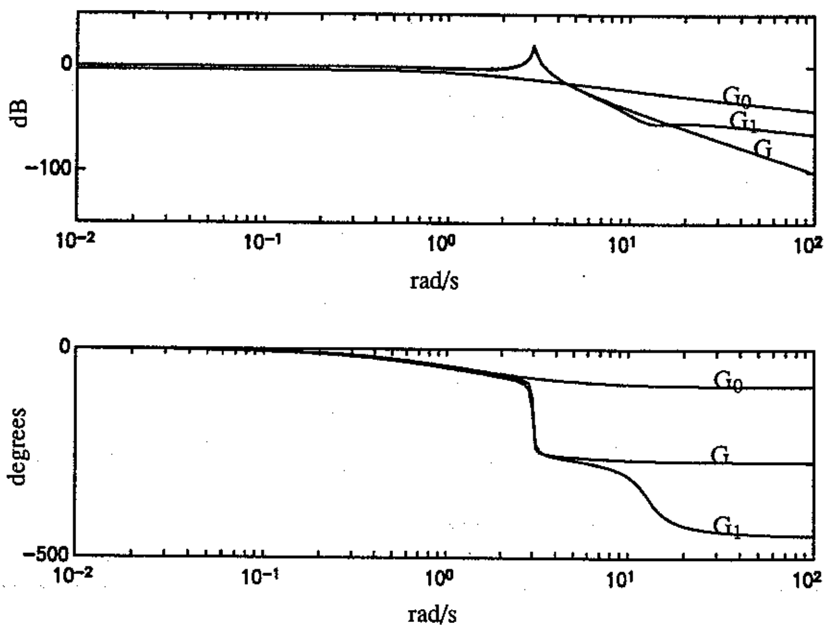

Figure 9. Frequency responses of models and plant

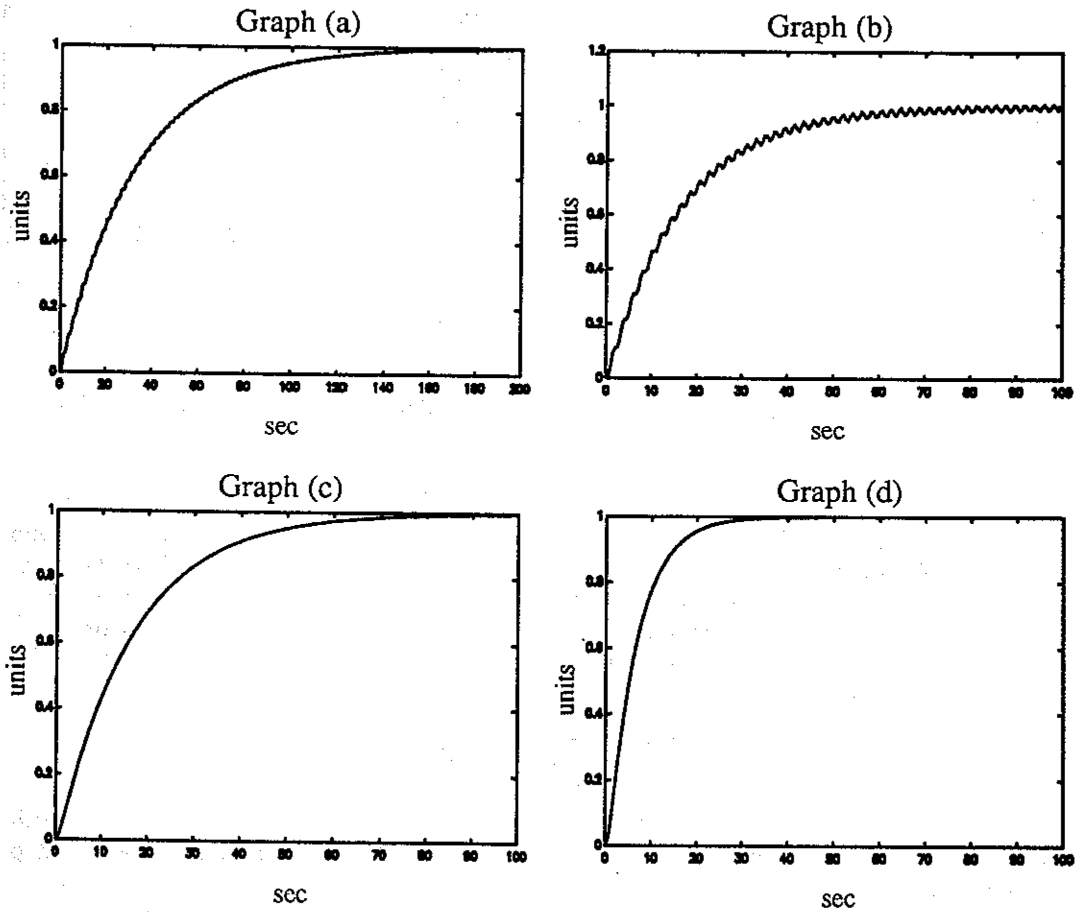

Figure 10. Simulation results using rational approximations of the plant

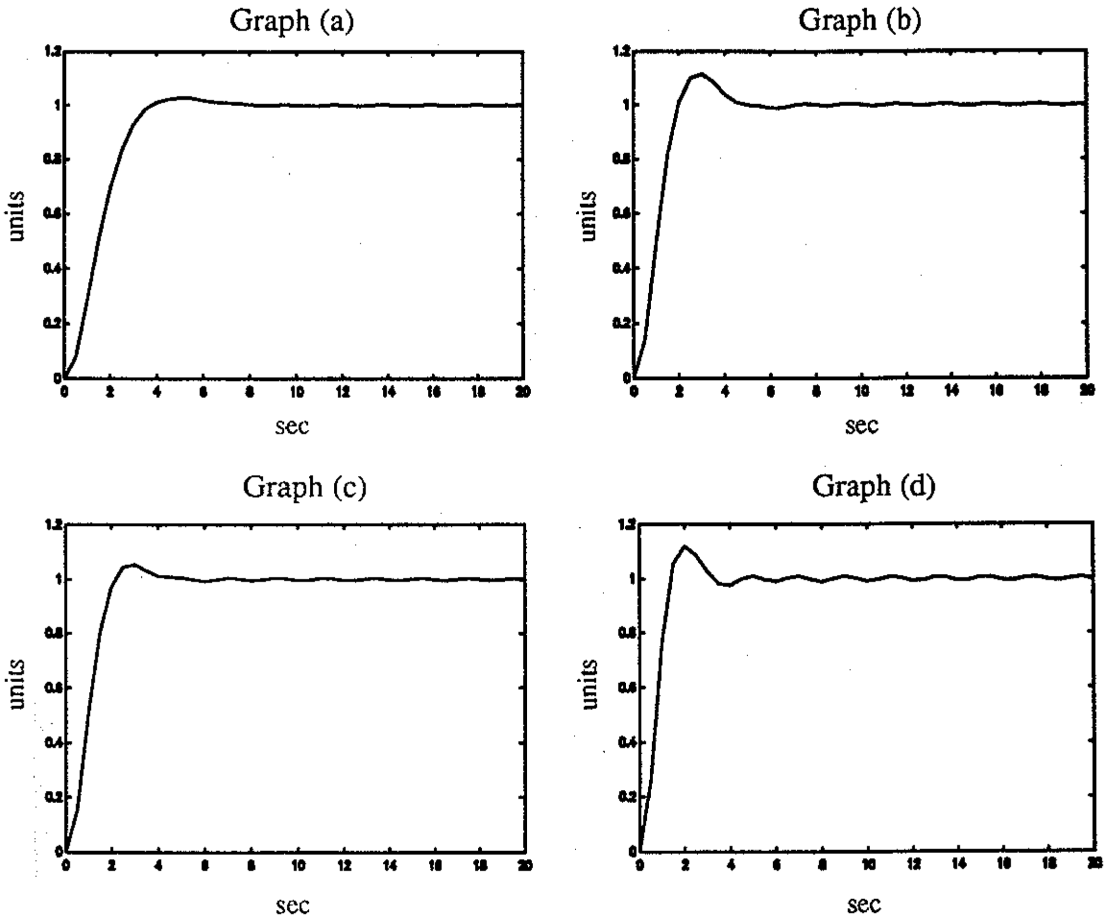

Figure 11. Simulation results using rational approximations of the plant

of Figure 10 shows significant oscillations. After the model is updated to

$$G_1 = \frac{-0\cdot4094s^2 + 2\cdot0572s + 7\cdot175}{s^3 + 1\cdot3027s^2 + 8\cdot9908s + 10\cdot6411}$$

using the procedure described in Reference 10, the unit-step response of the actual closed-loop system for a nominal closed-loop bandwidth of $0\cdot04$ rad s^{-1} is improved and it is shown in graph (c) of Figure 10. Graph (d) of Figure 10 shows the unit-step response when the nominal closed-loop bandwidth is increased to $0\cdot1$ rad s^{-1}. Graphs (a) and (b) of Figure 11 are obtained for nominal closed-loop bandwidths of $0\cdot5$ and 1 rad s^{-1} respectively when the model is G_1. If the model is improved to

$$G_2 = \frac{-0\cdot40612s^2 + 0\cdot80196s + 6\cdot3884}{s^2 + 1\cdot0977s^2 + 8\cdot882s + 9\cdot3027}$$

the unit-step response is given by graph (c) of Figure 11 for a nominal closed-loop bandwidth of 1 rad s^{-1}. When the nominal closed-loop bandwidth is increased to 2 rad s^{-1}, the unit-step response is as shown in graph (d) of Figure 11.

All else being equal, we would expect the noiseless situations to give better results than the noisy conditions. However, by comparing the results given in Figures 6–8 with those given in Figures 10 and 11, we observed that, overall, the results given in Figures 6–8 appear to be better than those given in Figures 10 and 11. Therefore we can conclude that strictly proper controllers are less sensitive to high-frequency model uncertainties and hence require less frequent model updates when we attempt to increase the nominal closed-loop bandwidth of the system. This is important, because, as we have mentioned before, under noisy conditions the system identification process is becoming progressively difficult and it is advantageous to be able to have infrequent but accurate model updates.

8. DISCUSSION AND CONCLUSIONS

We have reviewed in Section 1 the strength and weakness of both the traditional adaptive control and the robust control design methods. These methods should be able to complement each other and there should be natural ways in which they could be blended harmoniously. We proposed that one of the possible ways is by the windsurfer approach which was first mentioned in Reference 13. We have shown, by simulation, that by starting with a (crude) initial model of the plant and a (small-bandwidth) robustly stabilizing controller, the bandwidth of the closed-loop system can be increased progressively through an iterative control-relevant system identification and control design procedure. We shall highlight the following points which we believe are reasons for the success of the approach.

(i) The use of control-relevant frequency weighting in the system identification criterion.

(ii) Updating of the model when the effects of its error are no longer small in the closed-loop response. This will ensure that model uncertainties are emphasized in the correct range of frequencies.

(iii) The controller designed by using the IMC method always has integral action. Therefore it is insensitive to model uncertainties at low frequencies provided that the gain of the model at low frequencies is of the right sign.

(iv) The use of strictly proper controllers to reduce the required number of model updates through identifying the $\hat{R}_{j,i}$ transfer function, a difficult task under noisy conditions.

(iv) The controller designed by using the IMC method induces a natural factorization in the parametrization of the unknown transfer function of the plant. This enables the system identification problem to be solved efficiently.

It is natural in any discussion of adaptive or iterative design to raise the question of convergence. We consider that the work of this paper implicitly established the following practical convergence results, which are confirmed by all the simulations.

1. Assume that a certain closed-loop bandwidth can be achieved for a known stable plant. In the situation where the same plant is imperfectly known, it is possible to adapt the closed-loop system by the method described such that the same closed-loop bandwidth is achieved, assuming that the noise is not so great as to preclude satisfactory identification.
2. For a specified (fixed) closed-loop bandwidth which is achievable with the real plant, the actual closed-loop behaviour approaches very closely the nominal closed-loop behaviour.

In conclusion, we would like to emphasize that only the case of a stable plant with stable models is considered in this preliminary investigation. We hope to address the following problems in the near future.

(i) How to come up with an initial stabilizing controller. Simple cases where the plants have single or double poles at the origin have already been addressed.
(ii) The extension of the method to deal with unstable plants or models.
(iii) Use of orthogonalized exponential in the system identification procedure such that it becomes a convex optimization problem.
(iv) To prove that the algorithm actually converges in some sense.
(v) To study other control design methods in the context of the windsurfer philosophy.

APPENDIX I: PROOF OF THEOREM 2

Since the controller

$$K_{j,i}=\frac{X_{j,i}}{Y_{j,i}} \tag{40}$$

stabilizes the model

$$G_i=\frac{N_i}{D_i}, \qquad N_iX_{j,i}+D_iY_{j,i}=1 \tag{41}$$

then solving equations (40) and (41) simultaneously, we get

$$X_{j,i}=\frac{K_{j,i}}{D_i+N_iK_{j,i}} \tag{42}$$

$$Y_{j,i}=\frac{1}{D_i+N_iK_{j,i}} \quad \text{or} \quad Y_{j,i}=\frac{1-N_iX_{j,i}}{D_i} \tag{43}$$

Substituting $X_{j,i}$ and $Y_{j,i}$ into

$$G_{i+1}=\frac{N_i+r_{j,i}Y_{j,i}}{D_i-r_{j,i}X_{j,i}}$$

will result in

$$G_{i+1}=G_i+\frac{r_{j,i}}{D_i(D_i-r_{j,i}X_{j,i})}$$

Solving for $r_{j,i}$, we get

$$r_{j,i}=\frac{D_i^2(G_{i+1}-G_i)}{1+D_iX_{j,i}(G_{i+1}-G_i)} \tag{44}$$

From Figure 3 in Section 5, with $r_2=0$, we can write for the closed-loop system

$$y=\frac{GK_{j,i}}{1+GK_{j,i}}r_1+\frac{1}{1+GK_{j,i}}He, \qquad u=\frac{K_{j,i}}{1+GK_{j,i}}r_1-\frac{K_{j,i}}{1+GK_{j,i}}He$$

Therefore we can write the equation

$$\beta=D_iy-N_iu$$

as

$$\beta=\frac{D_iK_{j,i}(G-G_i)}{1+GK_{j,i}}r_1+\frac{D_i(1+G_iK_{j,i})}{1+GK_{j,i}}He \tag{45}$$

and the equation

$$\alpha=X_{j,i}r_1$$

as

$$\alpha=\frac{K_{j,i}}{D_i(1+G_iK_{j,i})}r_1 \tag{46}$$

If we form the output error defined by

$$\varepsilon - \beta - r_{j,i}\alpha \tag{47}$$

then by substituting equations (44)–(46) into equation (47) and using the expression for $X_{j,i}$ given by equation (42), we can obtain

$$\varepsilon = \frac{D_i(1+G_iK_{j,i})K_{j,i}(G-G_{i+1})}{(1+GK_{j,i})(1+G_{i+1}K_{j,i})}r_1 + \frac{D_i(1+G_iK_{j,i})}{1+GK_{j,i}}He \tag{48}$$

Since equation (43) can also be written as

$$Y_{j,i} = \frac{1}{D_i(1+G_iK_{j,i})}$$

it is clear that if we define the filtered output error as

$$\xi = Y_{j,i}\varepsilon$$

then

$$\xi = \frac{K_{j,i}(G-G_{i+1})}{(1+GK_{j,i})(1+G_{i+1}K_{j,i})}r_1 + \frac{1}{1+GK_{j,i}}He$$

or

$$\xi = \left(\frac{GK_{j,i}}{1+GK_{j,i}} - \frac{G_{i+1}K_{j,i}}{1+G_{i+1}K_{j,i}}\right)r_1 + \frac{1}{1+GK_{j,i}}He \qquad \square$$

APPENDIX II: PROOF OF THEOREM 4

Using the notations established in Section 6, we have

$$R_{j,i} = \frac{G-G_i}{1+Q_{j,i}(G-G_i)}, \qquad Q_{j,i} = \frac{K_{j,i}}{1+G_iK_{j,i}}$$

Therefore we can write

$$R_{j,i} = \frac{1+G_iK_{j,i}}{1+GK_{j,i}}(G-G_i) \tag{49}$$

We also have

$$G_i = [G_i]_{\mathrm{m}}[G_i]_{\mathrm{a}}$$

where

$$[G_i]_{\mathrm{m}} = \frac{\tilde{n}_{G_i}\Pi_i\,(z_i^*+s)}{d_{G_i}}, \qquad [G_i]_{\mathrm{a}} = \frac{\Pi_i\,(z_i-s)}{\Pi_i\,(z_i^*+s)}$$

Since $Q_{j,i} = [G_i]_{\mathrm{m}}^{-1}F_{j,i}$, we can rewrite the equation

$$K_{j,i} = \frac{Q_{j,i}}{1-Q_{j,i}G_i}$$

as

$$K_{j,i} = \frac{d_{G_i}n_F}{\tilde{n}_{G_i}[\Pi_i\,(z_i^*+s)d_{F_{j,i}} - \Pi_i\,(z_i-s)n_{F_{j,i}}]}$$

Hence we can write

$$1+G_iK_{j,i} = \frac{\Pi_i\,(z_i^*+s)d_{F_{j,i}}}{\Pi_i\,(z_i^*+s)d_{F_{j,i}} - \Pi_i\,(z_i-s)n_{F_{j,i}}} \tag{50}$$

By substituting equation (50) into equation (49) and noting that

$$d_{F_{j,i}} = (s+\lambda_{j,i})^{n+1}, \qquad 1+GK_{j,i} = \frac{d_Gd_{K_{j,i}} + n_Gn_{K_{j,i}}}{d_Gd_{K_{j,i}}}, \qquad G-G_i = \frac{d_{G_i}n_G - d_Gn_{G_i}}{d_Gd_{G_i}}$$

we obtain

$$R_{j,i} = \tilde{R}_{j,i}\hat{R}_{j,i}$$

where

$$\tilde{R}_{j,i} = [G_i]_m(s+\lambda_{j,i})^n$$

is a *known* stable proper transfer function and, other than the factor $s+\lambda_{j,i}$ in the numerator,

$$\hat{R}_{j,i} = \frac{(s+\lambda_{j,i})(d_{G_i}n_G - d_G n_{G_i})}{d_{K_{j,i}}d_G + n_{K_{j,i}}n_G}. \tag{51}$$

is an *unknown* stable strictly proper transfer function.

To obtain the results on the order and relative degree of $\hat{R}_{j,i}$ we shall write

$$[G_i]_m = \frac{\eta_i(s)}{\pi_i(s)}, \qquad [G_i]_a = \frac{\rho_i(-s)}{\rho_i(s)}$$

where each of the polynomials $\eta_i(s)$, $\pi_i(s)$ and $\rho_i(s)$ has degree r, $n+r$ and m respectively. We can then obtain

$$K_{j,i} = \frac{\lambda_{j,i}^{n+1}\pi_i(s)\rho_i(s)}{\eta_i(s)[(s+\lambda_{j,i})^{n+1}\rho_i(s) - \lambda_{j,i}^{n+1}\rho_i(-s)]}$$

If we also write G as

$$G = \frac{\alpha(s)}{\beta(s)}$$

where $\alpha(s)$ has degree p and $\beta(s)$ has degree q, then by substituting all these into equation (51), we get

$$\hat{R}_{j,i} = \frac{(s+\lambda_{j,i})[\alpha(s)\pi_i(s)\rho_i(s) - \eta_i(s)\rho_i(-s)\beta(s)]}{\beta(s)\eta_i(s)[(s+\lambda_{j,i})^{n+1}\rho_i(s) - \lambda_{j,i}^{n+1}\rho_i(-s)] + \lambda_{j,i}^{n+1}\alpha(s)\pi_i(s)\rho_i(s)} \tag{52}$$

By counting the degrees of the resulting numerator and denominator polynomials of $\hat{R}_{j,i}$ given by equation (52), the required results are established immediately. □

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