For left matrix fraction realizations, step MF1 gives the solution rows $t_1 = a_7$ and $t_2 = a_6$. The external change rows for t_1 are $u_{1,1} = a_3$, $u_{1,2} = a_4$ and for t_2 are $u_{2,1} = a_4$. The only change to ensure (2.9) in step MF2 is $t_1 \to t_1 + (1/2)t_2$. From MF3, all the minimal parametrized left matrix fraction realizations $T(z,\alpha)^{-1}Q(z,\alpha)$ are given by:

$$T(z,\alpha) = \begin{pmatrix} z^3 - 3z^2/2 + z/4 - 7/4 & -3z/4 + 9/8 \\ -z^2 + z/2 + 1/2 & z^2 - 3z/2 + 1/4 \end{pmatrix} + \begin{pmatrix} \alpha_{11} & \alpha_{12} \\ 0 & \alpha_{21} \end{pmatrix}\begin{pmatrix} z-1 & -1/2 \\ -z+1 & z-3/2 \end{pmatrix}$$

$$Q(z,\alpha) = \begin{pmatrix} z^2 + z/2 - 1/4 & z^2 + z/2 + 7/4 & z^2 - z/2 + 7/4 \\ z - 1/2 & z - 1/2 & -z - 1/2 \end{pmatrix} + \begin{pmatrix} \alpha_{11} & \alpha_{12} \\ 0 & \alpha_{21} \end{pmatrix}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \end{pmatrix}.$$

Although numerators are expressed, for brevity, by the closed formula in MF3, a simple procedure can be incorporated to give an explicit updating of them in the matricial frame. The right matrix fraction realizations can be obtained in a similar dual way from the solution columns b_4, b_6, b_{11} of the final B^N.

V. Conclusions

In this note we propose a recursive method to obtain all parametrized minimal realizations of finite data. The main advantage is that the method gives two models and that the recursiveness question of state-space realizations is solved. In contrast to the recursive solutions of [11], all solutions are described in a nice recursive way by means of some parameters.

The algorithm brings a new common frame to deal with state-space and matrix fraction realizations, each model obtained not from the other, but directly from data, and the explicit link between both recursive models is shown. In relation to computer memory requirements, the size of the matrices actually used increase with n (minimal dimension), as the remaining rows and columns are set to zero and structural information is retained in indices. Exploitation of the null entries pattern gives a low computational cost of the updating. For state-space realizations, the parameters fit simply into certain entries of the state matrix.

An open question is how to extend the method to give a link with continued fraction models. It seems that this would require to introduce changes based on auxiliary vectors as in [2]. In this case, the algorithm would give three independent models: continued fraction, matrix fraction, and state-space realizations.

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Comments on "Stabilizability and Detectability of Discrete-Time Time-Varying Systems"

Brian D. O. Anderson and John B. Moore

Abstract—**An alleged counterexample in the above paper[1] to a main result of [1] is shown not to be a counterexample in actuality.**

Introduction

A recent paper[1] presented a purported counterexample to one of the main results of [1]. In [1], a definition of uniform detectability is presented for a linear finite-dimensional time-varying system, and it is then shown that bounded output-injection feedback can stabilize a system which is uniformly detectable according to the definition.

The contribution of the paper[1] is to define a linear system, assert that it is uniformly detectable in terms of the definition of [1], and then to establish that there is no bounded output-injection feedback to stabilize the system. If correct, this chain of reasoning would invalidate the claim of [1].

Below, we review the definition of uniform detectability of [1] and recall the system defined in the paper[1]. We show this system is not uniformly detectable, in contrast to the assertion in the paper[1].

Accordingly, the purported counterexample is not really a counterexample to a main result of [1].

Analysis of Alleged Counterexample

Consider the unforced linear finite-dimensional time-varying system

$$x_{k+1} = F_k x_k \quad y_k = H_k' x_k \tag{1}$$

and define the state transition matrix $\phi_{k,l}$ for $k \ge l$ by $\phi_{k,k} = I$, $\phi_{k+1,k} = F_k$ and $\phi_{k,l} = \phi_{k,k-1}\phi_{k-1,l}$. In [1], one finds:

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The authors are with the Department of Systems Engineering, Research School of Physical Sciences, Canberra, ACT 2601, Australia.

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Definition: The pair $[F_k, H_k]$ is uniformly detectable if there exist integers s, $t \geq 0$ and constants d, b with $0 \leq d < 1$, $0 < b < \infty$ such as whenever

$$\|\phi_{k+t,k}\xi\| \geq d\|\xi\| \tag{2}$$

for some ξ and k, then

$$\xi' M_{k+s,k}\xi \geq b\xi'\xi \tag{3}$$

where

$$M_{k+s,k} = \sum_{i=k}^{k+s} \phi'_{i,k} H_i H'_i \phi_{i,k}. \tag{4}$$

Now consider the following system

$$F_{2j} = \begin{pmatrix} 0 & 0 \\ 4 & 0 \end{pmatrix} \; F_{2j+1} = \begin{pmatrix} 0 & \frac{1}{2} \\ 0 & 0 \end{pmatrix} \tag{5}$$

$$H'_{2j} = [0 \;\; 1] \;\; H'_{2j+1} = [0 \;\; 0]. \tag{6}$$

In the paper,[1] it is asserted that the requirements of the definition are fulfilled with $s = 0$, $t = 1$, $d = 3/4$, and $b = 3/64$.

Choose $k = 2j$, $\xi = [1 \;\; 0]'$. Then with $t = 1$

$$\left\|\phi_{k+1,k}\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right\| = \left\|\begin{bmatrix} 0 & 0 \\ 4 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right\| \geq \frac{3}{4}\left\|\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right\|. \tag{7}$$

Also, with $k = 2j$ and $s = 0$

$$M_{k+s,k} = H_{2j}H'_{2j} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

and then with $\xi = [1 \;\; 0]'$, evidently $\xi' M_{k+s,k}, \xi = 0$, and so there cannot hold

$$\xi' M_{k+s,k}\xi \geq b\xi'\xi \tag{8}$$

with $b = 3/64$. Yet uniform detectability requires, since (7) holds, that (8) must hold.

Hence, the system defined by (5) and (6) is not uniformly detectable for the reason asserted in the paper,[1] i.e., by virtue of the satisfaction of the definition for $s = 0$, $t = 1$, $d = 3/4$, and $b = 3/64$. This still leaves open the question of whether other choices of s, t, d, and b could render it uniformly detectable.

Simple calculations reveal that

$$M_{2j+s,2j} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \qquad \forall s \geq 0.$$

Now for $k = 2j$, we have

$$\phi_{k+1,k} = \begin{bmatrix} 0 & 0 \\ 4 & 0 \end{bmatrix}$$

$$\phi_{k+2,k} = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\phi_{k+3,k} = \begin{bmatrix} 0 & 0 \\ 8 & 0 \end{bmatrix}$$

$$\phi_{k+4,k} = \begin{bmatrix} 4 & 0 \\ 0 & 0 \end{bmatrix}.$$

Suppose, to obtain a contradiction, that particular values $\bar{t}$, $\bar{s}$, $\bar{d} < 1$, and $\bar{b} > 0$ will work in the uniform detectability definition. Choose $\xi = [1 \;\; 0]'$. Evidently, no matter what $\bar{t}$ is, there holds with k even

$$\|\phi_{k+\bar{t},k}\xi\| > \|\xi\| > \bar{d}\|\xi\|.$$

But also $\xi' M_{k+\bar{s},k}\xi = 0$ for any $\bar{s}$, which means that $\xi' M_{k+s,k}\xi \geq \bar{b}\xi'\xi$ fails. Hence it is impossible, through variation of the defining constraints s, t, d, and b away from those chosen in the paper[1] to secure uniform detectability.

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A New Adaptive Estimator for Linear Systems

Elon Rimon and K. S. Narendra

***Abstract*—In this note, we propose a new adaptive estimator for the linear time-invariant plant $\dot{x}_p = A_p x_p + B_p u$, whose parameters A_p and B_p are unknown. The proposed estimator is of the form**

$$\dot{\hat{x}}_p = \hat{A}_p \hat{x}_p + \hat{B}_p u$$

$$\dot{\hat{A}}_p = -[e\hat{x}_p^T]$$

$$\dot{\hat{B}}_p = -[eu^T]$$

and is (globally) uniformly asymptotically stable, provided that u is persistently exciting. In particular, there is no need to solve the Lyapunov equation $A_p^T P + PA_p = -Q < 0$ for a positive definite matrix P. This result also implies that in many other adaptive situations it is not necessary to explicitly solve the Lyapunov equation.

I. Introduction

Consider the following linear estimation problem. Given a multiple-input linear time-invariant plant with unknown parameters, of nth-order, whose states are accessible

$$\dot{x}_p = A_p x_p + B_p u \tag{1}$$

where A_p, B_p are unknown, $A_p \in \mathbb{R}^{n\times n}$ and $B_p \in \mathbb{R}^{n \times p}$, A_p is asymptotically stable, and the control input u is bounded. Let $\hat{A}_p$ and $\hat{B}_p$ denote the estimated A_p and B_p. The problem is to dynamically adjust $\hat{A}_p$ and $\hat{B}_p$ as a function of the observed input u and the state x_p, such that $\hat{A}_p$ and $\hat{B}_p$ converge to A_p and B_p. The estimator is to be obtained without the explicit use of the derivatives of the accessible signals.

In a recent book [3], the currently known solution to this problem is described. The exposition begins with the scalar plant

$$\dot{x}_p = a_p x_p + b_p u \qquad a_p < 0$$

in which a_p and b_p are unknown parameters. Two adaptive models are presented, and they include the adaptive laws for adjusting $\hat{x}_p(t)$, $\hat{a}_p(t)$, and $\hat{b}_p(t)$, which are the estimates of $x_p(t)$, a_p, and b_p, respectively. These are given by (2) and (3)

$$\begin{aligned} \dot{\hat{x}}_p &= \hat{a}_p \hat{x}_p + \hat{b}_p u \\ \dot{\hat{a}}_p &= -e\hat{x}_p \\ \dot{\hat{b}}_p &= -eu \qquad \text{(Model 1)} \end{aligned} \tag{2}$$

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The authors are with the Department of Electrical Engineering, Center for Systems Science, Yale University, Yale Station, New Haven, CT 06520.

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