Frequency Domain Conditions for the Robust Stability of Linear and Nonlinear Dynamical Systems

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Abstract —**This paper presents general frequency domain criteria for the robust stability of systems with parametric uncertainties. These are applied to the robust stability verification of LTI systems having possibly nonrational transfer functions and LTI systems operating under possibly nonlinear time varying passive feedback.**

I. Introduction

A SIGNIFICANT result in the field of robust stability of systems with parametric uncertainty is Kharitonov's theorem, [1], which addresses the problem of Hurwitz invariance of sets of real polynomials defined by

$$f(s) = s^n + \sum_{i=1}^{n} a_i s^{n-i} \in S \tag{1.1}$$

$$a_i^- \leqslant a_i \leqslant a_i^+ \tag{1.2}$$

with a_i^-, a_i^+ known. Kharitonov's theorem states that all members of S are Hurwitz, iff four of its special members are Hurwitz.

Since the publication of [1], a number of related papers have appeared in the literature. These include verification of i) robust Schur stability [2]–[5], ii) stability of polytopes of polynomials [6], iii) stability of a class of differential equations with delays [7], and iv) Hurwitzness of polynomials with independent variations in the even and odd coefficients [8]. Extensions have not been confined to just LTI systems. In fact, results are also available for LTI systems operating under passive, possibly time-varying nonlinear feedback [9], [10], [28], [29]. Such results find application in the design of an important class of adaptive estimators and in adaptive control problems in general [11], [12].

Despite the underlying common theme, the techniques for deriving these results have differed sharply. The main purpose of this paper is to identify a unifying framework within which all these results can be understood. In this regard we mention the work of [13]–[15], [20], and [25], which provide frequency domain simplifications of Kharitonov's original theorem. Motivated by these interpretations, we establish a generalized frequency domain criterion for checking families of polynomials for root confinement in open subsets of the complex plane. These same ideas help check the stability of LTI systems under passive feedback.

The criterion is based on the zero exclusion ideas developed and used in [16], [17], and [24]. Thus to check if the roots of a family of functions lie in a region confined by a closed curve ∂D (i.e., if the family is D-stable) the basic idea is to verify two facts: i) that at least one member of the family is D-stable and ii) that no member, evaluated on ∂D, ever equals zero.

In Section II we show how this criterion reduces to checking certain curves in the complex plane for zero confinement. Moreover, in some special cases, it further reduces to some complex functions having pointwise phase differences that are always less than π in magnitude. We also give conditions under which robust stability can be verified by checking only a finite number of members for D-stability. In Section III we show how the results of [4], [5], [8], and [23] can be obtained by specializing the criteria of Section II. In Section IV some new applications are considered, specifically how to check i) if a controller stabilizes a family of plants and ii) if a family has damping ratios that exceed a certain prespecified value. Also given is a generalized version of the edge theorem of [6]. Unlike in [6], our version applies to general functions and not just to polynomials. In Section V our ideas are extended to the stability of LTI systems operating under passive feedback. The class of systems considered include those in which the transfer function coefficients of the LTI part are multilinear in the parameters.

Barmish in [17] has also derived general frequency domain techniques to reduce robust stability verification of convex sets of polynomials to checking four functions for certain properties. Our results are somewhat more general and cover a wider range of applications including stability of classes of nonlinear time-varying systems.

II. Some Frequency Domain Criteria

In this section we present several frequency domain criteria for checking robust stability. The common thread in these criteria comes from a key result given in Theorem 2.1 below.

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The ideas implicit in this theorem appear in various forms in [16], [17], and [24].

In the sequel, families of scalar functions in $s \in C$ will be characterized by $f(s,A)$, where the n-dimensional real parameter vector $A \in \Gamma$. A zero or root of $f(s,A)$ will refer to s such that $f(s,A)=0$. The following is a standing assumption for all $f(s,A)$ and Γ considered in this paper.

Assumption 2.1: The function $f(s,A)$ with $A \in \Gamma$ is a smooth function of s and A. All roots of $f(s,A)$ vary smoothly with A. Further, any two members of Γ can be joined by smooth curves lying entirely in Γ.

We remark that for polynomial f, the smooth variation of the roots requires that the leading coefficient never crosses zero. The following decomposition of $f(s,A)$ is of interest.

Definition 2.1: Consider a curve ∂D in C. Then a pair of functions $\{h(s,A), g(s,A)\}$ is a *linearly independent decomposition* (LID) of $f(s,A)$ with respect to D if for every $s \in \partial D$,

$$f(s,A) = h(s,A) + g(s,A) \tag{2.1}$$

$$f(s,A) = 0 \text{ iff } h(s,A) = g(s,A) = 0. \tag{2.2}$$

In many cases LID's obey the following restriction.

Assumption 2.2: There exists $c(s)$, nonzero on ∂D and independent of A, such that

$$h(s,A) = c(s)h^*(s,A)$$

$$g(s,A) = jc(s)g^*(s,A)$$

where h^* and g^* take real values on ∂D. Such LID's will be denoted by $\{h^*, g^*\}$ or $\{h, g\}$.

An obvious LID with respect to any parametrization and D is $\{\mathrm{Re}(f(s,A)), j\,\mathrm{Im}(f(s,A))\}$ with $c(s)=1$. This LID will be referred to as the *natural* LID. As shown in Lemma A.1 in the Appendix, for $f(s,A)$ a polynomial affine in A, the following LID with respect to the unit circle obeys Assumption 2.2:

$$h(s,A) = \left[f(s,A) + s^n f(s^{-1},A)\right]/2 \tag{2.3}$$

$$g(s,A) = \left[f(s,A) - s^n f(s^{-1},A)\right]/2. \tag{2.4}$$

Here $c(s) = s^{n/2}$. The curves ∂D will satisfy the following assumption.

Assumption 2.3: The curve ∂D is either

i) closed, smooth, simple and bounds the open simply connected region D, or
ii) smooth, simple, not closed, has imaginary part taking all values from $-\infty$ to ∞ and separates the complex plane into two open simply connected regions.

We also need a formal definition of D-stability.

Definition 2.2: A function $f(s)$ is

i) D-stable if ∂D satisfies i) of Assumption 2.3 and all the roots of $f(s)$ lie in the open region D, and
ii) D_- (resp. D_+) stable if ∂D satisfies ii) of Assumption 2.3 and all the roots of $f(s)$ lie in the open region to the left (resp. right) of ∂D.

In the sequel, results stated for D stability will apply to D_- and D_+ stability as well. We now state Theorem 2.1, whose proof, being similar to corresponding results in [16], [17], and [24], is omitted.

Theorem 2.1: Consider the family of functions $f(s,A)$ and a curve ∂D as above. Suppose the functions $\{h, g\}$ form an LID w.r.t. D. Then all polynomials $f(s,A)$, $A \in \Gamma$, are D-stable iff the following two conditions are met.

i) At least one member of the family is D-stable.
ii) There exists no $s \in \partial D$ and no $A \in \Gamma$, such that $h(s,A) = g(s,A) = 0$.

Notice the LID need not satisfy Assumption 2.2. For the theorem to lead to a graphical criterion, however, this assumption is needed. Suppose it holds. Consider the mapping which for any $s \in \partial D$ takes $A \in \Gamma$ to the real vector $N_D(s,A) = [h^*(s,A), g^*(s,A)]'$. Then ii) of Theorem 2.1 reduces to $N_D(s,A)$ never equalling zero. Plot $g^*(s,A)$ versus $h^*(s,A)$ for all $s \in \partial D$. Call this diagram the Generalized Nyquist diagram of $f(s,A)$ w.r.t. ∂D and the LID $\{h, g\}$. The space of $N_D(s,A)$ will be called the *Generalized Nyquist space* (GNS) of $f(s,A)$. Denote $D_N(s)$ as the image of Γ in GNS. Then we need to check if $D_N(s)$ contains the origin at any $s \in \partial D$.

We thus have a graphical test. For ∂D symmetric about the real axis, the curves need be sketched only for such $s \in \partial D$ as have $\mathrm{Im}[s] \geqslant 0$. The test simplifies if one can find those members of Γ that correspond to the boundaries of $D_N(s)$. Calling the set of these members Γ^* the curves needed are $N_D(s,A)$, $A \in \Gamma^*$. If $N_D(s,A)$ is affine in A, i.e.,

$$N_D(s,A) = N_0(s) + N'(s)A \tag{2.5}$$

$N_0(s)$ and $N'(s)$ independent of A, then Γ^* is a subset of $\partial\Gamma$. For polytopic Γ an even further simplification is possible.

Proposition 2.1: Suppose $N_D(s,A)$ is as in (2.5), with Γ a polytope and Γ_c the set of its corner points. Then at any s, i) $D_N(s)$ is a convex polygon, with each corner having at least one preimage that belongs to Γ_c, ii) $\partial D_N(s)$ is obtained by drawing straight lines joining the corners of $D_N(s)$, and iii) every exposed edge of $D_N(s)$ has at least one preimage that is an exposed edge of Γ.

Proof: From (2.5), for any positive real scalar p and all s,

$$N_D(s, pA_1 + (1-p)A_2) = pN_D(s,A_1) + (1-p)N_D(s,A_2).$$

Thus it follows that any line segment in Γ maps to one in $D_N(s)$, with extreme points mapping to extreme points. Then members of $D_N(s)$ can be expressed as a convex combination of the images of members of Γ_c, whence i) and ii) follow. Since members of each exposed edge in $D_N(s)$ are convex combinations of adjacent corners of $D_N(s)$, and these corners have preimages in Γ_c, iii) also follows. □

Thus only the images of the members of Γ_c are needed. As many of these may have images in the interior of $D_N(s)$, in practice not all the corners of Γ have to be considered. Notice that if $f(s,A)$ satisfies Assumption 2.1 and has the form

$$f(s,A) = f_0(s) + F'(s)A \tag{2.6}$$

with f_0 and F independent of A, then both the LID's mentioned so far result in the satisfaction of (2.5).

For convex Γ, the criterion reduces to checking the phase difference between certain complex scalar functions.

Lemma 2.1: Consider $f(s,A)$ as in (2.6) and ∂D as in Assumption 2.3. Then every convex combination of $f(s,A_1)$ and $f(s,A_2)$ is D-stable iff i) one of them is D-stable and ii) $|\phi(f(s,A_1)) - \phi(f(s,A_2))| < \pi$ for all $s \in \partial D$. Here ϕ denotes the phase and is assumed to vary continuously as s changes along ∂D.

Proof: Since Theorem 2.1 holds we need simply show that ii) of Theorem 2.1 is equivalent to ii) of this lemma. Use the natural LID. Suppose ii) of Theorem 2.1 is violated. Then by (2.6), for some $\lambda \in (0,1)$ and $s \in \partial D$,

$$(1-\lambda)f(s,A_1)+\lambda f(s,A_2)=0. \tag{2.7}$$

Thus $f(s,A_1)/f(s,A_2)$ is a negative real number and ii) of this lemma fails. Conversely, suppose ii) of this lemma fails. Then for some $s \in \partial D$ and real positive p,

$$f(s,A_1)/f(s,A_2)=-p.$$

Defining $\lambda = p/(1+p) \in (0,1)$, (2.7) holds. Thus ii) of Theorem 2.1 fails. □

The following result follows from Lemma 2.1 and Proposition 2.1.

Proposition 2.2: Suppose $f(s,A)$ is as in (2.6) with Γ a polytope. Define γ_i as the corners of Γ and $f(s,\gamma_i)$ as $f_i(s)$. Then the entire family is D-stable iff i) one of the $f_i(s)$ is D-stable, and ii) for all i, j and $s \in \partial D$, $|\phi(f_i(s))-\phi(f_j(s))| < \pi$.

An attraction of Kharitonov's theorem lies in the fact that the stability of an entire set is implied by that of a finite number of its members. Such a finite test is possible only in a limited number of situations. One such is set out in Theorem 2.2 below. We need a further definition.

Definition 2.3: Suppose ∂D, $f(s,A)$, and the LID $\{h(s,A), g(s,A)\}$ satisfy Assumptions 2.3, 2.1, and 2.2, respectively, and for any A, $f(s,A)$ is D-stable only if the following hold.

i) All zeros of $h^*(s,A)$ and $g^*(s,A)$ are on ∂D, are simple, and separate each other.
ii) In traversing ∂D consistently in one direction, at any zero of either $h^*(s,A)$ or $g^*(s,A)$, $\arg\{h^*(s,A)+jg^*(s,A)\}$ either always increases or decreases, the pattern holding regardless of A.

Then f and ∂D are said to be *LPR compatible* w.r.t. $\{h,g\}$.

Examples of LPR compatability are given later. The term LPR abbreviates lossless positive real. Transfer functions g^*/h^* with g^*, h^* obeying i) and ii) are LPR. We now state and prove Theorem 2.2.

Theorem 2.2: Consider the family of functions $f(s,A)$, $A \in \Gamma$, a curve ∂D and an LID $\{h(s,A), g(s,A)\}$. Call the set of $f(s,A)$, S. Suppose

i) f and ∂D are LPR compatible w.r.t. $\{h,g\}$;
ii) $\Gamma = \Gamma_h \times \Gamma_g$ such that $f(s,A)$ is expressible as

$$f(s,A_h,A_g)=h(s,A_h)+g(s,A_g);$$
$$A_h \in \Gamma_h \text{ and } A_g \in \Gamma_g;$$

iii) $\exists S^*$ a subset of S such that for some integer m,

$$S^*=\{h(s,A_{hi})+g(s,A_{gj})|\forall i=1,\cdots,m,\ j=1,\cdots,m,$$
$$A_{hi}\in\Gamma_h \text{ and } A_{gj}\in\Gamma_g\};$$

iv) for every $s \in \partial D$ and $A \in \Gamma$, $f(s,A)$ can be expressed as a convex combination of the members of S^*. Then all members of S are D-stable iff all members of S^* are D-stable.

Proof: We follow the proofs in [15] and [20]. Necessity is obvious. For sufficiency we will show that if all members of S^* are D-stable then at any $s \in \partial D$, $D_N(s)$ is contained in an open half-plane excluding the origin. Theorem 2.1 will then prove the result. Definition 2.3 and iv) above imply that we need only show this half-plane confinement for the images of S^* on GNS.

Denote $h^*(s,A_{hi})$ as $h_i(s)$ and $jg^*(s,A_{gj})$ as $g_j(s)$:

$$f_{ij}(s)=c(s)\{h_i(s)+g_j(s)\}.$$

Q_i refers to the open ith quadrant of the GNS; Q_{ij} are the union of Q_i and Q_j. Notice that Q_{12}, Q_{14}, Q_{23}, and Q_{34} are half-planes that exclude the origin. From Definition 2.3, as the curve ∂D is traversed in any one direction, the image on GNS of each member of S^* either migrates from quadrant to quadrant, strictly in the order Q_1, Q_2, Q_3, Q_4, Q_1, etc., or does so in the opposite order. We consider two cases covering all possibilities.

Case I: For all i in $1,\cdots,m$ and $s \in \partial D$, $g_i(s) \neq 0$. In this case images of all members of S^* lie always in either Q_{12} or Q_{34} and the result holds.

Case II: There exists an $s^* \in \partial D$ and an i for which $g_i(s^*)=0$. We argue that the image of S^* on GNS for this $s^*(D_N^*(s^*))$, must be entirely in one of the two open regions Q_{14} or Q_{23}. Otherwise, because of ii) for some integer p,q, $h_p(s^*)>0$ and $h_q(s^*)<0$. Neither can equal zero as both f_{pi} and f_{qi} are in S^* and are D-stable. Now, this D-stability implies that as we move s^* along ∂D, if $h_p(s^*)+g_i(s^*)$ moves into $Q_1(Q_4)$, $h_q(s^*)+g_i(s^*)$ moves into $Q_3(Q_2)$. Since both have the same g_i, this cannot happen. Thus the entire set $D_N^*(s^*)$ is either in open Q_{14} or Q_{23}. Continuing this argument it follows that at any $s \in \partial D$, $D_N^*(s)$ is contained entirely in one of the four open half-planes described above. The result follows.

Under the hypotheses of the theorem, $D_N(s)$ is a rectangle with sides parallel to the axes of GNS and corners the images of members of S^*. Applications of the results derived so far are given in later sections. We now give examples of LPR compatability beginning with one useful in establishing robust Schurness. The proof of this result is in the Appendix.

Theorem 2.3: Consider $f(s,A)=f_0(s)+[s^n,s^{n-1},\cdots,1]A$, $A \in R^{n+1}$, f_0 a polynomial with degree no greater than n, ∂D the unit circle and the LID of (2.3) and (2.4) with $c(s)=s^{n/2}$. Then f and ∂D are LPR compatible w.r.t. this choice of $\{h,g\}$.

The following result is useful for Hurwitzness and follows from the Hermite–Biehler theorem [18].

Theorem 2.4: Consider f as in Theorem 2.3, ∂D the imaginary axis, and the natural LID with $c(s)=1$. Then f and ∂D are LPR compatible w.r.t. this choice of $\{h,g\}$.

Finally we show that with $f(s,A)$ as in Theorem 2.3 and curves ∂D satisfying the assumption below, the LID of Theorem 2.4 leads to LPR compatability.

Assumption 2.4: The curve ∂D satisfies i) of Assumption 2.3, D is convex, and ∂D is parametrized by a single parameter δ via the continuous mapping

$$\phi_D: [0,2\pi] \to \partial D.$$

From the well-known principle of argument, we have the final theorem of this section.

Theorem 2.5: Suppose $f(s,A)$ and the LID are as in Theorem 2.4 and ∂D satisfies Assumption 2.5. Then the conclusions of Theorem 2.4 hold.

III. Specializations to Known Results

In this section we demonstrate how the results of [1], [4], [5], [8], and [23] can be obtained by specializing Theorem 2.2.

3.1. Kharitonov's Theorem [1]

Consider the set (1.1)–(1.2). Express $f(s)$ as

$$f(s)=p(s^2)+sq(s^2). \tag{3.1}$$

Define

$$p_1(s^2)=a_n^-+a_{n-2}^+s^2+a_{n-4}^-s^4+a_{n-6}^+s^6+\cdots$$
$$p_2(s^2)=a_n^++a_{n-2}^-s^2+a_{n-4}^+s^4+a_{n-6}^-s^6+\cdots$$
$$q_1(s^2)=a_{n-1}^-+a_{n-3}^+s^2+a_{n-5}^-s^4+a_{n-7}^+s^6+\cdots$$
$$q_2(s^2)=a_{n-1}^++a_{n-3}^-s^2+a_{n-5}^+s^4+a_{n-7}^-s^6+\cdots.$$

Notice that with Γ, Γ_p, and Γ_q containing the coefficients of f,p,q, respectively, $\Gamma=\Gamma_p\times\Gamma_q$. Then the following is the statement and our proof of Kharitonov's theorem.

Theorem 3.1: The set S is Hurwitz invariant iff the four polynomials

$$f_{ij}(s)=p_i(s^2)+sq_j(s^2) \tag{3.2}$$

are Hurwitz.

Proof: Define ∂D to be the imaginary axis. We need to establish the D-stability of all members of S. Notice $\mathrm{Re}(f(j\omega))=p(-\omega^2)$ and $\mathrm{Im}(f(j\omega))=\omega q(-\omega^2)$. Thus, from Theorem 2.4, f and ∂D are LPR compatible with the LID $\{p(s), sq(s)\}$. Define S^* to be the set in (3.2). Then i), ii), and iii) of Theorem 2.2 follow from the definition of these polynomials. It remains to be shown that iv) holds. Observe, for all $s=j\omega$, and any p,q corresponding to the members of S,

$$p_1(s^2)\leqslant p(s^2)\leqslant p_2(s^2)$$

and

$$q_1(s^2)\leqslant q(s^2)\leqslant q_2(s^2).$$

Thus the set $D_N(s)$ at any $s=j\omega$ is identical with a rectangle having the generalized Nyquist diagrams of the members of S^* as its corners, whence iv) and hence the result follows.

3.2. Polynomials with Uncoupled Variations in Coefficients of Odd and Even Powers [8]

The work in [8] deals with the Hurwitz invariance of polynomials such as (3.1) where the variations in the coefficients of p and q are uncoupled. Define the sets SP, SQ, SP^*, and SQ^* in the following way. With S the set of all polynomials to be tested,

$$SP=\{p(s^2)|f(s)=p(s^2)+sq(s^2)\in S\}$$
$$SQ=\{q(s^2)|f(s)=p(s^2)+sq(s^2)\in S\}.$$

Further, $SP\supset SP^*$ and $SQ\supset SQ^*$ such that at any $s=j\omega$, $\exists p_i(s^2)$, $p_j(s^2)\in SP^*$ and $q_i(s^2)$, $q_j(s^2)\in SQ^*$ obeying $\forall p(s^2)\in SP$ and $q(s^2)\in SQ$,

$$p_i(s^2)\leqslant p(s^2)\leqslant p_j(s^2) \tag{3.3}$$

and

$$q_i(s^2)\leqslant q(s^2)\leqslant q_j(s^2). \tag{3.4}$$

Further define S^* as

$$S^*=\{p(s^2)+sq(s^2)|p(s^2)\in SP^* \text{ and } q(s^2)\in SQ^*\}.$$

Then the result is as follows.

Theorem 3.2: The set S is Hurwitz invariant iff S^* is the same.

Proof: Using the same ∂D and LID as in Theorem 3.1, all four conditions of Theorem 2.2 follow: i) from Theorem 2.4, ii) from the decoupled nature of the variations in the coefficients of p and q, and iii) and iv) from (3.3)–(3.4) and the definition of S^*. □

It is pertinent to note the results in [25] that view the problems such as those in Theorems 3.1 and 3.2 in terms of behavior of p and q in certain frequency bands. It is not hard to see that this requirement, derived in [25] from the Hermite–Biehler theorem, boils down to the zero exclusion condition at the heart of our results.

3.3. Robust Schur Stability: The Weak Version [4]

The problem considered here and the next subsection is as follows. Consider the set S of real polynomials

$$f(s,A)=f_0(s)+[s^n,s^{n-1},\cdots,1]A$$
$$A=[a_0,a_1,\cdots,a_n]'\in\Gamma \tag{3.6}$$

where the set Γ is as in (3.7)–(3.11). Define for $i=0,1,\cdots,m-1$, $m=[(n+1)/2]$, $[\cdot]$ denoting the integer part of the argument,

$$\alpha_i=a_i+a_{n-i} \tag{3.7}$$
$$\beta_i=a_i-a_{n-i}. \tag{3.8}$$

For even n, $\alpha_{n/2}=a_{n/2}$. For all $i\leqslant n/2,\exists$ constants $\alpha_i^-,\alpha_i^+,\beta_i^-,\beta_i^+$ such that

$$\alpha_i^-\leqslant a_i+a_{n-i}\leqslant\alpha_i^+ \tag{3.9}$$
$$\beta_i^-\leqslant a_i-a_{n-i}\leqslant\beta_i^+. \tag{3.10}$$

For even n,

$$\alpha_{n/2}^-\leqslant a_{n/2}\leqslant\alpha_{n/2}^+. \tag{3.11}$$

We seek conditions under which S is Schur invariant. Define S^* to be the set of all corners of S. Then the solution of [4] is in Theorem 3.3.

Theorem 3.3: All members of S above are Schur iff members of S^* are Schur.

Proof: Use ∂D as the unit circle with the LID of (2.3) and (2.4). Theorem 2.3 ensures i) of Theorem 2.2. Moreover, as a_i+a_{n-i} and a_i-a_{n-i} are allowed independent variations, ii) is also satisfied. Consider (A.1)–(A.3) in the Appendix for $h(s)$ and $g(s)$ with $s=e^{j\omega}$. Notice that by definition of S^*, its members correspond to h,g obtained by substituting all possible combinations of α_i^-, α_i^+, β_i^-, and β_i^+. Thus $D_N(s)$ is an axis parallel rectangle with corners the images of members of S^*. Thus the structure of S,S^* establishes iii) and iv), and hence the result.

3.4. Robust Schur Stability: The Strong Version [5], [23]

The number of members of S^* in the previous subsection grows exponentially with n, the degree of $f(s,A)$. In this subsection we demonstrate the existence of a subset S^{**} of S^*, whose Schur invariance implies the same for S (S and S^*

here are the same as those in Section 3.3). We also show that the number of members of S^{**} grows faster than linearly but no faster than quadratically in n.

It is clear from the proof of Theorem 3.3 that S^{**} should consist of those members of S^* whose images in GNS appear at the corners of $D_N(s)$ at various values of s on ∂D. Thus we need to find those members of S^* that correspond to the extremities of the $h^*(e^{j\omega})$ and $g^*(e^{j\omega})$ defined in the Appendix. From the proof of Lemma A.1, for $n = 2m$,

$$h^*(e^{j\omega}) = [\alpha_0 \cos m\omega + \alpha_1 \cos(m-1)\omega + \cdots + \alpha_{m-1} \cos \omega + \alpha_m] \tag{3.12}$$

$$g^*(e^{j\omega}) = [\beta_0 \sin m\omega + \beta_1 \sin(m-1)\omega + \cdots + \beta_{m-1} \sin \omega]. \tag{3.13}$$

For $n = 2m - 1$,

$$h^*(e^{j\omega}) = [\alpha_0 \cos(m-0.5)\omega + \alpha_1 \cos(m-1.5)\omega + \cdots + \alpha_{m-1} \cos 0.5\omega] \tag{3.14}$$

$$g^*(e^{j\omega}) = [\beta_0 \sin(m-0.5)\omega + \beta_1 \sin(m-1.5)\omega + \cdots + \beta_{m-1} \sin 0.5\omega]. \tag{3.15}$$

The maximum of $h^*(e^{j\omega})$ w.r.t. α_i is then reached at α_i^+ when

$$\cos(n/2-1)\omega > 0 \tag{3.16a}$$

and at α_i^- when

$$\cos(n/2-1)\omega < 0. \tag{3.16b}$$

For the minimum, complementary boundaries are taken. For similar results for g^* w.r.t. β_i, sine terms replace the cosine terms in (3.16). Thus for α_i the sign changes take place at the angles

$$\omega_{ik} = k\pi/(n-2i), \qquad k = 1,3,5,\cdots < n-2i. \tag{3.17}$$

For β_i they occur at

$$\omega_{ik} = k\pi/(n-2i), \qquad k = 2,4,6,\cdots < n-2i. \tag{3.18}$$

If in an ω interval, no sign change takes place, then the corners of D_N are characterized in the entire interval by the same members of S^*. A transferring of the angles of sign change for the α and β coefficients gives us the boundaries of these ω intervals as

$$\omega_{ik} = k\pi/(n-2i),$$
$$k = 1,2,3,\cdots < n-2i,\ i = 0,1,\cdots [n/2]-1.$$

A formula for determining the number of different ω intervals I_n is derived in [5] and [23]. It is, for integer k,

$$I_n = I_{n-2} + 2\phi(n), \qquad n = 2(2k-1)$$
$$= I_{n-2} + \phi(n), \qquad n \neq 2(2k-1)$$

where $\phi(n)$ (the Euler function) is the number of integers in $[1, n-1]$ that are coprime with n. Of course, the number of members of S^{**} equals $4I_n$. A calculation set out in Appendix A.3 shows that I_n increases faster than linearly, but no faster than quadratically in n.

IV. Some New Applications

In this section, the criteria of Section II are applied to address some new situations. In Section 4.1, we give a generalized version of the edge theorem of [6]. Unlike in [6], this result applies to classes of functions ranging beyond polynomials. It is applicable, for example, to delay differential equations. In Section 4.2 we show how the graphical criterion of Section II can be used to verify if families of systems have damping ratios that exceed a given value. In Section 4.3, we consider robust controller design for certain families of LTI plants.

4.1. An Edge Theorem

Consider the set of functions (2.6) where f_0 and F are independent of A and take real values for real s. Suppose Γ is a polytope, Assumptions 2.1 and 2.3 hold, and ∂D crosses the real axis at least once. Then the following holds.

Theorem 4.1 (Edge Theorem): Under the foregoing assumptions, the set of functions (2.6) is D-stable invariant iff the functions corresponding to the exposed edges of Γ are D-stable.

Proof: Necessity is obvious. For sufficiency select the natural LID. From Proposition 2.1, $D_N(s)$ is a convex polygon; we need only show that it excludes the origin. Choose s_0 on ∂D so that it is real. Then $D_N(s_0)$ is a connected line segment on the real axis. From iii) of Proposition 2.1, each of its members must have a preimage in the exposed edges of Γ. Hence D-stability of the exposed edges implies $D_N(s_0)$ excludes the origin. Now if at some $s \in \partial D$, $D_N(s)$ encloses the origin, at some other $s \in \partial D$, at least one of its edges must touch the origin. Then from iii) of Proposition 2.1, the stability of the exposed edges raises a contradiction. □

The theorem in [6] does not explicitly assume that ∂D crosses the real axis. However, it does assume that D is simply connected and $f(s, A)$ is a set of real polynomials. For such a set to be D-stable, ∂D must cross the real axis. We remark that an edge theorem for a more general region D can be found in [26].

For a differential equation with possibly noncommensurate delays, T_i, f_0, and F could be polynomials in $\exp(T_i s)$. Our result applies to families of such systems as long as smooth parameter variations result in smooth root movements.

4.2. Checking for Damping Ratios

Consider the following problem. Suppose the characteristic polynomials of a family of systems lie in S of (1.1)–(1.2); check if the damping ratios exceed a given value, or equivalently check D-stability for $\partial D = j\omega e^{j\theta}$, for some real θ and all real ω varying from $-\infty$ to ∞. We demonstrate the use of ideas in Section II for $n = 3$ and θ such that $\cos 2\theta$, $\cos\theta$, $\sin 2\theta$, and $\sin\theta$ are all positive. Select the natural LID and $c(s) = 1$. Then for $s \in \partial D$,

$$h^*(s) = \omega^3 \sin 3\theta - a_1\omega^2 \cos 2\theta - a_2\omega \sin\theta + a_3 \tag{4.1}$$

$$g^*(s) = \omega^3 \cos 3\theta - a_1\omega^2 \sin 2\theta + a_2\omega \cos\theta. \tag{4.2}$$

From Proposition 2.1, $D_N(s)$ is a convex polygon. Thus from Proposition 2.2 we need to check the phase difference between $N_D(s, A)$ of those A whose images form the corners

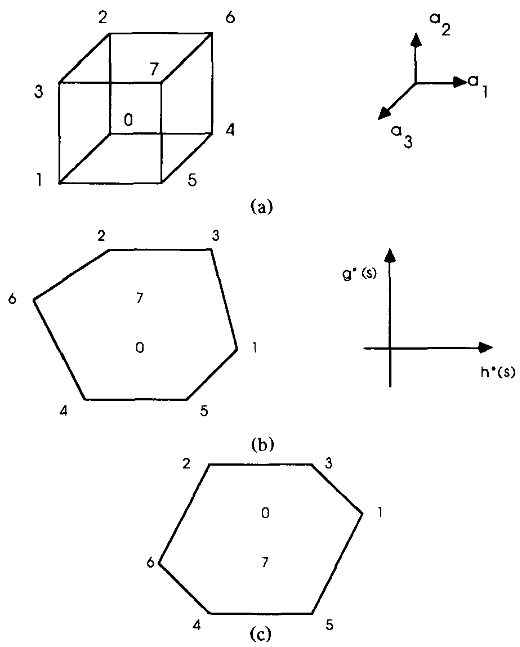

Fig. 1. (a) Ordering of points in parameter space. (b) D_N for low frequencies. (c) D_N for high frequencies.

of D_N. Notice that only $\omega \geqslant 0$ is needed. Fig. 1 gives the general shape and corner locations of D_N. In this figure the cutoff point between "high" and "low" frequencies is $0.5\{\operatorname{cosec}\theta\}(a_2^+ - a_2^-)/(a_1^+ - a_1^-)$. Notice corners 0 and 7 have images always in the interior of $D_N(s)$. To check D-stability, we simply need to apply Proposition 2.2 for $i = 1, \cdots, 6$.

We remark that sufficient but not necessary conditions for such a D-stability appear in [20] and [27]. The approach is to construct a larger set, containing S, having $D_N(s)$, which is the rectangle demanded by Theorem 2.2. Then the preimages of the corners imply D-stability.

4.3. Robust Controller Design

Consider the rational function family $a(s,A)/b(s,A)$, a,b polynomials in s with coefficients affine in the elements of A. Suppose $A \in \Gamma$, Γ a polytope with corners Γ_c. Given a fixed controller $p(s)/q(s)$, we need to check if for all $A \in \Gamma$,

$$1 + p(s)a(s,A)/(q(s)b(s,A))$$

is D-stable. Define

$$\begin{aligned} f(s,A) &= q(s)b(s,A) + p(s)a(s,A) \\ &= f_0(s) + F'(s)A \end{aligned} \tag{4.3}$$

and check if f is D-stable for all $A \in \Gamma$. Since $f_0(s)$ and $F(s)$ obey (2.6), we need to check if for some $A \in \Gamma_c$, (4.3) is D-stable and if $\forall A_i, A_j \in \Gamma_c$ and $s \in \partial D$, $|\phi(f(s,A_i)) - \phi(f(s,A_j))| < \pi$.

V. Systems with Passive Feedback

Consider an LTI plant with rational SISO transfer function $T(s)$, operating under passive, negative feedback (a feedback path is passive if the integral of its input–output product is non-negative). A sufficient condition for closed-loop stability is that $T(s)$ be strictly positive real (SPR), [21], i.e., it be stable, minimum phase, and for all real ω, $\mathrm{Re}[T(j\omega)] > 0$. Such closed loops find application in adaptive systems problems under a variety of guises. In particular, one resulting design problem in adaptive output error identification of a plant $b(s)/a(s)$ is to select a polynomial $p(s)$ with degree equalling that of $a(s)$, such that $p(s)/a(s)$ is SPR [12]. Similar issues also arise under discrete time settings. In this section we address the problem of checking families of functions for SPR-ness. The following definition helps make the problem general.

Definition 5.1: A rational function $T(s)$ is D-SPR if i) its numerator is D-stable, ii) its denominator is D-stable, and iii) $\mathrm{Re}[T(s)] > 0$ for all $s \in \partial D$.

Notice that if $T(s)$ is D-SPR, then so is its inverse. In the sequel, the curve ∂D will satisfy Assumption 2.3. Then we have Lemma 5.1.

Lemma 5.1: Suppose two rational functions $T_i(s) = p_i(s)/q(s)$, $i = 1,2$, are D-SPR. Then so also are all their convex combinations.

Proof: For any convex combination, ii) and iii) follow trivially. Suppose i) is violated. Then as both $p_i(s)$ are D-stable, $\exists \lambda \in (0,1)$ and $s_0 \in \partial D$ such that

$$(1-\lambda)p_1(s_0) + \lambda p_2(s_0) = 0$$

and for a positive real k,

$$p_1(s_0) = -kp_2(s_0).$$

Thus $\mathrm{Re}[T_1(s_0)] = -k\,\mathrm{Re}[T_2(s_0)]$ and both T_i cannot simultaneously satisfy iii). □

We now state the first main result of this section.

Theorem 5.1: Consider the families of scalar polynomials

$$p(s,A) = p_0(s) + P'(s)A, \qquad A \in \Gamma$$

$$q(s,B) = q_0(s) + Q'(s)B, \qquad B \in \Omega$$

where Γ and Ω are polytopes belonging to R^n and R^m, respectively, and $p_0(s)$, $P(s)$, $q_0(s)$, and $Q(s)$ are polynomials in s but independent of A, B. Suppose Γ and Ω have corners A_i and B_j, respectively. Then all members of the family $p(s,A)/q(s,B)$, $A \in \Gamma$ and $B \in \Omega$, are D-SPR iff all functions $p(s,A_i)/q(s,B_j)$ are D-SPR.

Proof: An arbitrary $p(s,A)$ is expressible as a convex combination of the $p(s,A_i)$. Thus for all j and $A \in \Gamma$, by hypothesis and Lemma 5.1, $p(s,A)/q(s,B_j)$ is D-SPR. Thus for all j and $A \in \Gamma$, $q(s,B_j)/p(s,A)$ is D-SPR. The result now follows by reapplying Lemma 5.1. □

Thus if Γ and Ω have m and n corners, then D-SPR invariance verification requires checking mn transfer functions. Further, suppose the sets of $p(s,A)$ and $q(s,B)$ are called S_p and S_q, respectively, and they obey the conditions set out in Theorem 2.2, with S_p^* and S_q^* the respective analogs of the set S^* of that theorem. Then a slight variation of the foregoing argument shows that $p(s,A)/q(s,B)$, $p \in S_p$ and $B \in S_q$, are D-SPR iff all functions $p(s,A)/q(s,B)$ are D-SPR for every $p \in S_p^*$ and $B \in S_q^*$. This last result specializes to the SPR invariance scenarios of [15], [28], and [29] where the sets of polynomials considered are of the form in (1.1)–(1.2).

The next result is motivated by [12]. Consider a plant

$$b(s,K)/a(s,K), \qquad K \in R^n \tag{5.1}$$

where b and a are polynomials in s and have coefficients multilinear in the elements of K. It has been shown in [19] that if the parameters in an LTI system correspond to physical component values then in many cases the plant transfer function has the above structure. In [12], adaptive estimators for such K have been formulated. These require for their convergence the existence of a polynomial $p(s)$ such that $p(s)/a(s,K)$ is SPR. Given the physical significance of elements k_i of K, bounds on them are often available. Consequently, the following is an associated design issue. Suppose $K \in \Gamma$ where Γ is defined by

$$\alpha_i \leqslant k_i \leqslant \beta_i. \tag{5.2}$$

Given a polynomial $p(s)$, verify that for all $K \in \Gamma$

$$T(s,K) = p(s)/a(s,K) \text{ is SPR.} \tag{5.3}$$

Accordingly, the following is of interest.

Theorem 5.2: Suppose $a(s,K)$ is defined as above. Denote Γ_c as the set of corners of Γ, defined by (5.2). Then $T(s,K)$ in (5.3) is D-SPR for all $K \in \Gamma$ iff it is so for all $K \in \Gamma_c$.

Proof: Necessity is obvious. To prove the result it suffices to show that $a(s,K)/p(s)$ D-SPR for all $K \in \Gamma_c$ implies the same for all $K \in \Gamma$. The multilinear dependence of $a(s,K)$ on the k_i ensures that if $a(s,K_1)$ and $a(s,K_2)$ are such that K_1 and K_2 differ in only one element, then both $a(s,K_1)/p(s)$ and $a(s,K_2)/p(s)$ D-SPR imply that for any K that is a convex combination of K_1 and K_2, $a(s,K)/p(s)$ is D-SPR. In the sequel, the zero- and n-dimensional boundaries of Γ will refer to the corners and the entire set, respectively. We prove the result by induction. Notice that any point on an m-dimensional boundary can be expressed as a convex combination of some K_1 and K_2, differing in only one element, and each lying on some $(m-1)$-dimensional boundary. Thus from the foregoing, $a(s,K)/p(s)$ D-SPR for all K on all $(m-1)$-dimensional boundaries implies the same for all K on each m-dimensional boundary. Since $a(s,K)/p(s)$ is D-SPR for all K on the zero-dimensional boundaries, the result follows. □

Finally, the following result obtains from Theorem 5.2 and ideas similar to those in the proof of Theorem 5.1.

Theorem 5.3: Suppose $a(s,K)$ and $b(s,M)$ are polynomials having coefficients multilinear in the elements of K and M, respectively. Suppose (5.2), along with $\gamma_i \leqslant k_i \leqslant \delta_i$, is satisfied. Denote the set of M as Ψ and that of its corners Ψ_c. Then all members of the family $b(s,M)/a(s,K)$ are D-SPR iff $b(s,M)/a(s,K)$ is D-SPR for all $K \in \Gamma_c$ and $M \in \Psi_c$.

VI. Conclusions

We have presented a unifying frequency domain framework, within which most of the currently available results on the robust stability of linear systems with parametric uncertainties can be viewed. The framework encapsulates not just finite dimensional systems, but any LTI system that can be characterized by transfer functions of a single variable. It also covers robust stability of LTI systems under passive feedback.

Appendix

A.1. Some Results Pertaining to Schur Stability

Lemma A.1: Consider $f(s)$ as in (1.1). Then the LID of (2.3)–(2.4) satisfies Assumption 2.2 with $c(s) = s^{n/2}$ and ∂D the unit circle.

Proof: By definition,

$$h(s) = 0.5 \sum_{i=0}^{n} (a_i + a_{n-i}) s^{n-i}.$$

Thus for even n,

$$h(s) = 0.5 s^{n/2} \cdot \left\{ \sum_{i=0}^{n/2-1} (a_i + a_{n-i})(s^{n/2-i} + s^{i-n/2}) + 2a_{n/2} \right\}. \tag{A.1}$$

For odd n and $[\cdot]$ denoting integer part,

$$h(s) = 0.5 s^{n/2} \sum_{i=0}^{[n/2]} (a_i + a_{n-i})(s^{n/2-i} + s^{i-n/2}). \tag{A.2}$$

Thus

$$h(s) = s^{n/2} h^*(s)$$

with h^* obviously defined and real on the unit circle. Likewise,

$$g(s) = 0.5 s^{n/2} \sum_{i=0}^{[n/2]} (a_i - a_{n-i})(s^{n/2-i} - s^{i-n/2}) = s^{n/2} g^*(s) \tag{A.3}$$

with g^* purely imaginary on the unit circle.

A.2. Proof of Theorem 2.3

The proof follows from Lemma A.1 and a result in [22], which states that $f(s)$ of (1.1) is Schur iff $|a_n| < 1$, and zeros of $h(s)$ and $g(s)$ are all on the unit circle, separate each other, and are simple.

A.3. Bounds on I_n

Because of the recursive relations for I_n in terms of $\phi(n)$ given in Section 3.4, it suffices to bound

$$\sum(n) = \sum_{i=1}^{n} \phi(i).$$

a) Upper bound: By definition of the Euler function it follows that $\phi(n) \leqslant n-1$. Thus

$$\sum(n) \leqslant (n-1)n/2.$$

Hence I_n grows at a rate no faster than quadratic in n.

b) Lower bound: For large n there are approximately $n/(\log n)$ prime numbers that are less than or equal to n. Thus a lower bound for $\Sigma(n)$ is of the order

$$\sum_{i=1}^{n} i/\log i \geqslant (1/\log n) \sum_{i=1}^{n} i = n(n-1)/(2\log n) \geqslant O(n).$$

Thus I_n grows at a rate faster than linear in n.

The above bound is true for large n. We show below that $\forall n > 4$, $2n-4$ represents a lower bound on I_n. In [23], I_n is directly related to certain roots associated with the projection of $h^*(e^{j\omega})$ and $g^*(e^{j\omega})$ on the real interval $[-1,1]$. Using the ideas of [23] it can be shown that if n increases by 2, I_n increases by at least 4. Hence the result follows from the values of I_n deducible from [23] for $n \leqslant 4$.

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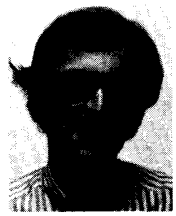

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