On the Local Stability Properties of Adaptive Parameter Estimators with Composite Errors and Split Algorithms

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Abstract—We examine the stability characteristics of a generalized error system structure for adaptive parameter estimation systems. This error system form encompasses the structure of a number of particular applications of adaptive parameter estimation theory which fall outside the framework of more familiar error system models. In the basic model, one recursively updates the parameter estimate vector with a term which is the product of a small step size, a filtered version of the regressor vector, and the system prediction error. The prediction error is a filtered inner product of the regressor and parameter error vectors. By contrast, the prediction error form entering our generalized error system is a sum of differently filtered products of corresponding entries in the regressor and parameter error vectors, termed a composite error. Similarly, the algorithm form, which we call a split algorithm, updates each parameter estimate individually by a term composed as the product of the step size, a filtered regressor element, and the composite error. In the update of different parameter estimates, the filtering operation applied to the appropriate regressor elements may be different, thereby “splitting” the algorithm.

This paper analyzes the consequences for error system stability which derive from the added generality. For a particular choice of split algorithm, the generalized error system has stability properties similar to those for the basic error system. However, for other split algorithm selections, fundamental differences between the two error system forms arise. We demonstrate that, when using averaging theory techniques to analyze the error system, one must augment stability conditions for the basic error system in order to ensure stability for the generalized error system. We rigorously establish the inadequacy of regressor spectral restrictions and persistent spanning conditions to guarantee local error system stability of the generalized structure, but we demonstrate alternative conditions which will yield such local stability. To illustrate the concepts involved, we examine the recursive identification of parameters in a parallel-form realization of a linear system.

I. INTRODUCTION

A USEFUL description of many parameter estimation systems is given by

\[ e(k) = H \left[ X^T(k) \hat{\Theta}(k) \right] \]

\[ \tilde{\Theta}(k+1) = \hat{\Theta}(k) + \mu F(x(k)) e(k) \]

(1.1a)

(1.1b)

Here, \( x(k) \) denotes the \( i \)th entry of the system regressor \( X(k) \), \( \tilde{\Theta}(k) \) denotes a recursively updated parameter estimate from the parameter estimate vector \( \hat{\Theta}(k) \), and \( \hat{\Theta}(k) \) is a vector of parameter errors formed by the difference between the nominal or true parameter values \( \Theta \) and the estimates \( \hat{\Theta}(k) \). The prediction error \( e(k) \) in (1.1a) is a measurable signal equal to the filtered inner product of the regressor and parameter error vectors. The operator \( H \) is assumed to be linear and time-invariant, and in many instances is parametrized by the unknown \( \Theta \). The adaptive algorithm (1.1b) describes the evolution of the parameter estimates, with the form of the update term given by the product of a small step size \( \mu \), a filtered regressor entry \( F(x(k)) \), and the prediction error \( e(k) \). Notice that the same filtering operation \( F \), also assumed to be linear and time-invariant, appears in the update of each parameter \( \hat{\Theta}_i(k) \).

The paradigm of (1.1), or a variation on its basic structure, appears in the study of a number of adaptive systems. (See, for example, fundamental adaptive system structures in [1]–[7]. In particular, we note [8] as a detailed study of (1.1), with the addition of filtering the prediction error in the algorithm (1.1b) and with the possibility of time-varying operators.

Nonetheless, situations arise in the field of adaptive systems which (1.1) is inadequate to describe. A particular example is the adaptive noise canceler described in [9], in which the prediction error appears as the sum of two differently filtered inner products of regressor and parameter error subvectors. Another example arises in the recursive identification of the parameters of a parallel-form realization of a linear system (see, e.g., [10]), described in Section V of this paper. Such situations have motivated the development of a more general adaptive system model which is able to accommodate the description of these more complicated systems [11], [12].

Consider

\[ e(k) = \sum_{i=1}^{n} H_i [x_i(k) \tilde{\Theta}_i(k) ] \]

\[ \hat{\Theta}_i(k+1) = \hat{\Theta}_i(k) + \mu F_i [x_i(k)] e(k), \quad i = 1, \ldots, n \]

(1.2a)

(1.2b)

as a simplified version of the most general description of the new error models in [11]. Comparing (1.2) with (1.1), one notes that the essential difference is that potentially different filtering operations \( F_i \) and \( H_i \) now act on the regressor elements in (1.2b) and regressor/parameter error products in (1.2a). This modification allows the model (1.2) to capture the description of the adaptive system in [9], for example, or the parallel-form identifier noted previously and described in Section V. We term (1.2) a Split Algorithm Composite Error, or SPACE, system. The composite error (1.2a) is a composite of differently filtered regressor and parameter error products, and the split algorithm update law (1.2b) is split into potentially different algorithm specifications. We will sometimes refer to the simpler equations (1.1) as a non-SPACE system.

Despite the seemingly modest differences between (1.1) and
(1.2), the type of condition which has been successfully used to guarantee "good behavior" of the estimator (1.1) is insufficient to guarantee the same good behavior for (1.2). For our purposes, by good behavior we mean local stability at $\Theta = 0$ of the error system describing the evolution of the parameter errors in each equation. Averaging techniques are useful in the study of (1.1) in error system form [5], [8], [13]–[15]. Essentially, for periodic excitation, local stability of (1.1) will follow from satisfaction of two conditions: 1) the regressor $X(k)$ must be persistently spanning [16], and 2) the operator composition $HF^{-1}$ (assuming $F$ is stably invertible) must be strictly positive real, or SPR. (Being SPR means $HF^{-1}$ is asymptotically stable and $\text{Re} \{HF^{-1}(e^{i\omega})\} > 0$ for all $\omega \in [0, \pi]$.). Furthermore, local stability of (1.1) can still result if $HF^{-1}$ satisfies a strictly positive real condition only over a subset $\Omega$ of the frequency domain (i.e., for all $\omega \in \Omega$, $\text{Re} \{HF^{-1}(e^{i\omega})\} > 0$). If a persistently spanning regressor has frequency content confined to $\Omega$, then local stability of (1.1) follows.

However, the arguments leading to these results break down when applied to the stability analysis of (1.2). We show that when differences exist between the individual compositions $H_l F_l^{-1}$, no spectral restrictions will prevent the existence of persistently spanning regressors for which the error system is locally unstable. Therefore, for most systems described by (1.2), more than just spectral restrictions and persistently spanning conditions are needed to guarantee local stability of the error system for (1.2). An exception is a particularly special case of (1.2), in which the $H_l F_l^{-1}$ operators are essentially identical. For this case, one may reapply the analysis used for (1.1). Note that the $F_l$ and $H_l$ operators need not be the same in this special case; only the compositions $H_l F_l^{-1}$ must agree.

For practical applications, this requirement of operator matching can be a problem. In many situations, the operators $H_l$ appearing in the prediction error are unknown or are parametrized by the unknown $\Theta$ (see [11]). Thus, a system designer will not be able to ensure exact equality of the $H_l F_l^{-1}$ compositions. One does not expect, however, that small differences between these operators would cause catastrophically poor behavior. Nor would one postulate the nonexistence of regressor signals which may locally stabilize $\Theta = 0$, even when the operator differences are larger. The problem, then, is to characterize what types of regressor signals yield locally stable SPACE systems for a given set of operator compositions $H_l F_l^{-1}$.

The approach we take to this problem focuses on situations in which the $H_l F_l^{-1}$ compositions (with $H_l$ and $F_l$ from (1.2)) are approximately equal, but not exactly equal, to some nominal operator composition $HF^{-1}$. We make rigorous the following claim: if a regressor $X(k)$ provides an adequate degree of stabilization for a nominal non-SPACE error system determined by operator $HF^{-1}$, then a SPACE system excited by the same $X(k)$ would retain local stability if the operators $H_l F_l^{-1}$ in the SPACE system are close enough to $HF^{-1}$.

The concepts of dominant persistent excitation [17] and of an average SPR condition [14], [18] are useful in interpreting our notion of an adequate degree of stabilization. Essentially, these ideas tell us that if most of the energy of a (persistently spanning) regressor lies in frequencies for which $HF^{-1}$ satisfies an SPR condition, then the (non-SPACE) error system (1.1) will be locally stable. Here, the degree to which this stability condition is satisfied for a nominal error system then indicates how close a match one needs between the SPACE system operators $H_l F_l^{-1}$ and the nominal operator $HF^{-1}$ to achieve SPACE system stability.

We organize the paper as follows. Section II discusses non-SPACE system stability analysis using averaging theory. The stability conditions involved are limited to persistently spanning regressors, restrictions on the regressor frequency content, and strictly positive real operator conditions. We also discuss the concepts of dominant persistent excitation [17] and the average SPR condition [18] in this context, and we discuss the notion of degree of stabilization provided by a given regressor. In Section III, we show that just persistent spanning and spectral restrictions on the regressor are insufficient to grant stability for the general SPACE system.

Section IV addresses the connection between the degree of stabilization of a nominal error system and bounds on SPACE system operator differences, when establishing local error system stability. We conduct the analysis using spectral densities of the signals involved. This approach facilitates the expression of the relationship between the degree of nominal error system stability and allowed differences between the frequency responses of the $H_l F_l^{-1}$ operators.

We demonstrate in Section V the application of these concepts to adaptive systems through examination of the recursive identification of the parameters in a parallel-form realization of a linear system. Finally, we discuss future directions for this SPACE system theory in the concluding section.

II. AVERAGING ANALYSIS FOR NON-SPACE SYSTEMS

In this section, we consider results and concepts which arise from the analysis of non-SPACE adaptive systems using averaging theory. The principal concepts involved include a persistently spanning (PS) condition on the regressor and a strict positive real (SPR) condition on a key operator in the adaptive system. The following main stability theorem relates the positivity of the operator as a function of frequency to the spectral content of the regressor in deriving conditions for adaptive system stability. An important characteristic of these stability conditions is the relative lack of a need to use the structural properties of the regressor itself. In fact, this is an asset of the analysis, permitting its application to a variety of adaptive systems whose error systems share only the same form, and not necessarily the same particular internal structure. The results of this section are not new (see, e.g., [8], [13], [18]). However, they are phrased in a modestly original fashion, providing a backdrop for the analysis of SPACE systems in Section III. We confine our analysis to the periodic case, though extensions to almost periodic and some nonperiodic excitation are possible. Note that in general, and especially in adaptive control applications, periodic regressor signals may not necessarily arise until parameter convergence. In our linearized analysis at $\Theta = 0$, however, periodic excitation results in periodic regressors if the system is stable.

To begin, we write (1.1) in error system form as

$$\Theta(k + 1) = \begin{bmatrix} I - \mu & F \vdots & \vdots \end{bmatrix} \begin{bmatrix} x_1(k) \\ \vdots \\ x_n(k) \end{bmatrix} + \begin{bmatrix} H \\ \vdots \\ H \end{bmatrix} \begin{bmatrix} x_1(k) \\ \vdots \\ x_n(k) \end{bmatrix} \Theta(k) + O(\mu^2). \quad (2.1)$$

The approximation leading to the $O(\mu^2)$ term comes out of consideration of $\Theta(k)$ as slowly varying (from a small step size $\mu$) in comparison to $X(k)$ and in relation to the bandwidth of $H$. The
in (1.1b), so that $\Theta$ no longer appears in the argument of $H$. (See [8] or [14] for explicit determination of the $O(\mu^2)$ perturbation.) We then focus on the linear homogeneous portion of (2.1) which takes the form

$$\hat{\Theta}(k+1) + [I - \mu R(k)]\hat{\Theta}(k)$$

(2.2)

with

$$R(k) = \begin{bmatrix} \vdots & \vdots & \vdots \\ F & [x_1(k)] & \vdots \\ \vdots & \vdots & \vdots \\ F & [x_n(k)]\end{bmatrix}$$

(2.3)

Assuming $F$ is stably invertible (note that $F$ takes the form $R(k)$, we may set $\Omega(k)$ assuming $F$ is stably invertible (note that $F$ is a user-chosen operator), we may set $\psi(k) = F[x(k)]$, with $Y(k) = [y_1(k) \ldots y_n(k)]$, and alternatively write

$$R(k) = \begin{bmatrix} y_1(k) \\ \vdots \\ y_n(k) \end{bmatrix} \begin{bmatrix} HF^{-1} & \vdots & HF^{-1} \end{bmatrix} \begin{bmatrix} y_1(k) \\ \vdots \\ y_n(k) \end{bmatrix}$$

(2.4)

Well-known averaging theory is useful in the stability analysis of (2.2) with periodic excitation (and thus periodic $R(k)$). Let

$$\overline{R}_k = \frac{1}{T} \sum_{k=k_0+1}^{k_0+T} R(k)$$

(2.5)

denote the average value of $R(k)$ over period $T$. Since $R(k)$ is periodic, $\overline{R}_k = \overline{R}$ for all $k_0$. We then have the following ([14]).

**Lemma 2.1:** $\exists \mu > 0$ such that (2.2) is exponentially stable for all $\mu \in (0, \mu^*)$ if and only if

$$\text{min } \text{Re } \{\lambda_i(\overline{R})\} > 0.$$ 

(2.6)

**Lemma 2.2:** If $\text{min } \text{Re } \{\lambda_i(\overline{R})\} < 0$, then $\exists \mu > 0$ such that for all $\mu \in (0, \mu^*)$ (2.2) is unstable.

We can apply Lemmas 2.1 and 2.2 to the adaptive system (2.1) by appealing to local stability of that equation for small enough $\mu$ given exponential stability for the linear part (given boundedness of the $O(\mu^2)$ term, see [8]). Note that in the aforementioned analysis, we have presumed that the regressor $X(k)$ does not depend on the parameter estimates and thus the parameter errors $\Theta(k)$. If such were the case, linearization of the error system equation at $\Theta = 0$ would be in order (see, for example, comments in [13] and [14]). (We also note that similar results may be obtained when the excitation is not periodic [14, 15].)

To apply the previous lemmas to (2.1), we need the following definitions. We say $\{Y(k)\}$ is *persistently spanning* (PS) [16] if

$$\exists N, \alpha, \beta > 0 \text{ such that } \forall k_0 \beta I \geq \sum_{k=k_0+1}^{k_0+N} Y(k)Y^T(k) \geq \alpha I$$

(2.7)

and we note that a linear time-invariant operator $M$ is *strictly positive real* (SPR) if and only if

$$M(e^{i\omega}) + M^H(e^{i\omega}) = 2 \text{ Re } \{M(e^{i\omega})\} > 0$$

$$\forall \omega \in [0, 2\pi)$$

(2.8)

and $M$ is asymptotically stable [19].

Let $\overline{R}_k[X]$ denote the time average of $R(k)$ in (2.4). In terms of (2.7) and (2.8), a standard result is that if periodic $Y(k)$ is PS, and $HF^{-1}$ is SPR, then $\overline{R}_k[X]$ satisfies the condition (2.6) [14]. Therefore, PS $Y(k)$ and SPR $HF^{-1}$ are sufficient to yield (at least) local stability of the error system (2.1). However, we can do better, by recognizing that we need only require that $\rho(\omega) = \text{Re } \{HF^{-1}(e^{i\omega}) + HF^{-1}(e^{-i\omega})\} > 0$ for the frequencies contained in the spectrum of $Y(k)$ (a type of restricted SPR condition). We formalize this statement, beginning with the following definitions.

**Definition 2.1:** $E_S$ is the set of periodic $n$-vector valued functions $X(k)$ such that each component of $X(k)$ is a finite sum of sinusoids with frequencies lying in $\Omega$. $\nabla \nabla \nabla$

**Definition 2.2:**

$$E_{PS} = \{X(k): X(k) \text{ is PS}\}.$$ 

(2.9)

$$\nabla \nabla \nabla$$

**Theorem 2.1:** Let $\Omega$ be any open set in $(-\pi, \pi)$. Then for all $X \in E_S \cap E_{PS}$, $\exists \mu > 0$ such that $\forall \mu \in (0, \mu^*)$ system (2.1) is locally stable about $\Theta = 0$ if $HF^{-1} = M$ is asymptotically stable and satisfies $M(e^{i\omega}) + M^H(e^{i\omega}) > 0$ $\forall \omega \in \Omega$. Furthermore, if $M(e^{i\omega}) + M^H(e^{i\omega}) < 0$ for some $\omega \in \Omega$, $\exists X \in E_S \cap E_{PS}$ for which $\exists \mu > 0$ such that $\forall \mu \in (0, \mu^*)$ (2.1) is locally unstable about $\Theta = 0$.

**Proof:** The proof of the first part of the theorem uses the idea of average strict positive reality as expressed in [14, 18]. The PS and restricted SPR conditions guarantee that (2.6) holds for $\overline{R}_k[X]$, so that by Lemma 2.1 the stability result holds. To prove the second part of the theorem, it is easy to construct a regressor sequence to show the instability result (for details, see [12]).

**Theorem 2.1** shows the use of PS and SPR conditions in establishing local stability of (2.1) for an entire class of regressors. With $\Omega_M = \{\omega: M(e^{i\omega}) + M^H(e^{i\omega}) > 0\}$, where $M = HF^{-1}$, (2.1) is locally stable for all $X \in E_0 \cap E_{PS}$. However, $X \in E_0 \cap E_{PS}$ is not necessary for local stability. For instance, having the power in $X$ lie predominantly in $\Omega_M$ is sufficient to assure local stability. In the average $\overline{R}_k[X]$, the "positive contribution" at frequencies in $\Omega_M$ then outweighs the "negative contribution" at other frequencies, so that (2.6) is satisfied and the error system is locally stable. This effect is an interpretation of the average SPR condition of [18]. A similar concept is the dominant persistent excitation idea of [17], in which one wants the frequency content of the regressor to lie predominantly in a range (typically lower frequencies) where the adaptive system is well modeled. This "good" excitation, if strong enough, will offset any destabilizing effects of excitation of the unmodeled, or parasitic system modes.

From a broader perspective, these conditions each delineate a class $E_S$ of *stabilizing excitation*. $E_S$ contains those regressors $X(k)$ for which $\overline{R}_k[X]$ satisfies the positivity condition (2.6). For a given regressor $X$, the maximum value of $\gamma$ for which

$$\text{min } \text{Re } \{\lambda_i(\overline{R}_k[X])\} > \gamma$$

(2.10)

holds is the *degree of stabilization* provided by $X$. The degree
\(\gamma\) indicates how contractive the homogeneous part of (2.1) is, when the system is excited by \(X\).

Theorem 2.1 proves that \(E_S \geq E_{\text{tol}} \cap E_{\text{PS}}\). The average SPR condition of [18] defines another subset of \(E_S\). Each of these results relies on a combination of persistently spanning regressors, operator positivity, and spectral restrictions. However, we see in the next section that such good fortune does not bless us when we consider the more general SPACE case. Considering conditions which only deal with operator positivity, PS regressors, and spectral restrictions on the regressor will be insufficient, in a certain sense, for establishing stability of SPACE systems.

III. SPECTRALLY-RESTRICTED EXCITATION AND SPACE SYSTEM STABILITY

We now turn our attention back to the general error system in (1.2) involving split algorithms and composite errors. We will assume throughout that the operators \(F_1, F_n^{-1}\), and \(H_i\) are asymptotically stable, fixed, linear operators with rational transfer functions. For convenience of notation, we set \(M_i = H_i F_n^{-1}\). When applying the concepts of the preceding section to this case, we are interested in the eigenvalues of the time average of (3.4), for periodic \(X(k) \in E_n\), which we denote by \(\bar{R}_e[X]\). The following lemmas describe some necessary conditions for the eigenvalues to have positive real parts for all \(X \in E_n \cap E_{\text{PS}}\).

Lemma 3.1: If one of the \(M_i\) does not satisfy the property that \(M_i(e^{i\omega_0}) + M_i^H(e^{i\omega_0}) \geq 0\) for all \(\omega \in \Omega\), then \(3 \times 3 \in E_{\text{PS}}\) such that \(\min \{\min \Rightarrow \text{Re} [\lambda_i(\bar{R}_e[X])] \} < 0\).

Proof: Suppose for \(\omega_0 \in \Omega\) that \(M_i(e^{i\omega_0}) + M_i^H(e^{i\omega_0}) < 0\). If we let the \(i\)th component of \(X\) be a sinusoid of frequency \(\omega_0\) (with the other components equal to zero), it is easy to show that \(\bar{R}_e[X]\) has a negative eigenvalue. As in the proof of Theorem 2.1, we may then augment the zero entries of \(X(k)\) with sinusoids of different frequencies in \(\Omega\) to achieve \(X \in E_{\text{PS}}\) without affecting the negative eigenvalue (see [12] for details).

Lemma 3.2: Suppose that the phase responses of \(M_1, \ldots, M_n\) do not exactly agree on an open set \(\Omega\). Then \(3 \times 3 \in E_{\text{PS}}\) such that

\[
\min_i \text{Re} [\lambda_i(\bar{R}_e[X])] < 0. \tag{3.5}
\]

Proof: By assumption there exists \(\omega_0 \in \Omega\) for which the phase responses of \(M_i(e^{i\omega_0})\) and \(M_i(e^{i\omega_0})\) differ, for some \(i, j\) pair. Since \(\Omega\) is open and the phase responses are almost everywhere continuous, we can find rational \(\omega_0\) with this same property. Let \(\omega_0 = \omega_0\) for convenience. Assume that \(i = 1, j = 2\), and set \(M_1(e^{i\omega_0}) = m_1 e^{i\eta_1}\) for \(i = 1, 2\), with \(m_1 \neq 0\). Then we have \(\eta_1 \neq \eta_2\).

Now let

\[
Y(k) = \begin{bmatrix}
\cos (\omega_0 k + \frac{\eta_1 - \eta_2}{2}) \\
0 \\
0
\end{bmatrix}
\tag{3.6}
\]

One may calculate that the \(2 \times 2\), upper-left-hand block of \(\bar{R}_e[X]\) is given by

\[
\frac{1}{2} \begin{bmatrix}
m_1 \cos (\eta_1) & m_2 \cos (\frac{\eta_1 + \eta_2}{2}) \\
\frac{1}{2} m_1 \cos (\frac{\eta_1 + \eta_2}{2}) & m_2 \cos (\eta_2)
\end{bmatrix}
\tag{3.7}
\]

with the remaining entries of \(\bar{R}_e[X]\) equal to zero. The determinant of the nonzero \(2 \times 2\) block of \(\bar{R}_e[X]\) in (3.7) is

\[
\det (\bar{R}_e[X]) = \frac{m_1 m_2}{4} \left( \cos \eta_1 \cos \eta_2 - \cos^2 \left( \frac{\eta_1 + \eta_2}{2} \right) \right)
\tag{3.8}
\]

Since \(\eta_1 \neq \eta_2\), this determinant is negative, implying \(\bar{R}_e[X]\) has a negative eigenvalue. To achieve \(Y(k) \in E_{\text{PS}}\), one may augment the zero entries of \(Y(k)\) in (3.6) with sinusoids of frequency different from \(\omega_0\). Doing so does not change the negative eigenvalue in \(R_e[X]\), thus completing the proof. \(\square\)
Lemma 3.1 requires each \( M_i \) to satisfy a restricted SPR condition for SPACE system stability. Lemma 3.2 implies that without a precise match between the phase responses of the operators \( \{ H_jF_{j-1} \} \), one cannot guarantee local stability of the SPACE system for all (persistently spanning) regressors which have a particular spectral restriction. We formalize these statements in the following.

**Theorem 3.1:** If for some \( i, j \) pair the phase responses of \( H_jF_{j-1} \) and \( H_iF_{i-1} \) do not exactly agree on \( \Omega \), or if for some \( i \) \( H_iF_{i-1} \) does not satisfy an SPR condition restricted to frequencies in \( \Omega \), then there exists periodic \( X \in \mathbb{E}_0 \cap \mathbb{E}_g \) and \( \mu^* \) such that \( \forall \mu \in (0, \mu^*) \), (3.1) is locally unstable at the origin.

**Proof:** Follows from Lemmas 3.1, 3.2, and 2.2.

The discussion of several points regarding Theorem 3.1 is in order. First, a phase response matching among \( \{ M_i = H_jF_{j-1} \} \) over any \( \Omega \) is achieved, given \( \{ H_i \} \), only for very particular choices of \( \{ F_j \} \). For example, in the case that \( M_i \) and \( M_j \) have the same phase responses and both are stable and minimum phase, then they can differ only by a positive scaling constant. Thus, the rather negative result of Theorem 3.1 tells us that we do not have a stability result, similar to Theorem 2.1 for non-SPACE systems, for a broad class of SPACE systems. A second point is that if \( H_iF_{i-1} = \cdots = H_kF_{k-1} \), then the SPACE system stability question becomes a non-SPACE one, since the operator matrix in (3.4) has in this case identical operators all along the diagonal, as in (2.4). We have the following theorem for this special case of SPACE systems.

**Theorem 3.2:** If \( H_iF_{i-1} = M \) for all \( i \) and \( M \) is asymptotically stable with \( M(e^{i\omega}) + M^H(e^{i\omega}) > 0 \) satisfied \( \forall \omega \in \Omega \), then for all \( X \in \mathbb{E}_0 \cap \mathbb{E}_g \), \( \exists \mu^* \) such that \( \forall \mu \in (0, \mu^*) \) system (3.1) is locally stable at the origin.

If \( M(e^{i\omega}) + M^H(e^{i\omega}) < 0 \) for some \( \omega_0 \in \Omega \), then there exists \( X \in \mathbb{E}_0 \cap \mathbb{E}_g \) for which \( \exists \mu^* \) such that \( \forall \mu \in (0, \mu^*) \), (3.1) is locally unstable at the origin.

**Proof:** With \( H_iF_{i-1} = M \) for all \( i \), \( \overline{R}_d[X] = \overline{R}_d[MX] \), so that the proof follows from a direct application of Theorem 2.1.

We are therefore faced with the seeming requirement of exactly matching each \( H_jF_{j-1} \) with all the other \( H_iF_{i-1} \) operators in order to achieve local stability. However, in applications the \( H_i \) operators are often parameterized by the unknown parameters and are therefore themselves unknown [11]. It is thus unlikely that one will be able to specify each algorithm filter \( F_i \) to match precisely its counterpart \( H_i \) in the composite error. In the event of even a slight mismatch, the phase responses of the operators will differ, and Theorem 3.1 then states that there is the potential for instability given any spectral restriction on the regressor. One alternative approach is based on using a gradient descent algorithm to minimize the square of the prediction error \( e \) in (1.2a). Such an algorithm results in choosing \( F_i \) in (1.2b) as \( \hat{H}_i \), a time-varying approximation of \( H_i \) based on current parameter estimates [20]. This method offers local stability once \( \hat{H}_i \rightarrow H_i \), as then each \( H_iF_{i-1} \) composition equals the identity operator. Nonetheless, since one has good behavior with an exact match of the \( H_iF_{i-1} \) operators, one does not expect catastrophic behavior when there are slight mismatches between \( H_iF_{i-1} \) and \( H_jF_{j-1} \).

### IV. SPACE STABILITY VIA DEGREE OF NOMINAL NON-SPACE STABILIZATION

We now develop concepts which relax the requirement that \( H_iF_{i-1} = M \) for SPACE system stability. The central idea is the interplay between the degree of stabilization of a nominal non-SPACE system and the closeness of the operators \( \{ H_iF_{i-1} \} \) in securing a positivity property for the average of \( \mathcal{R}(k) \). We focus on use of the average SPR condition [18] to establish a degree of stabilization for the non-SPACE system. Then, by restricting the difference between the frequency responses of the operator \( M = H_iF_{i-1} \) of the non-SPACE system (2.1) and the operators \( M_j = H_jF_{j-1} \) of the SPACE system (3.1), we conclude local stability of that class of SPACE systems. Although we provide quantitative bounds on the operator differences for stability, these bounds are quite conservative.

We find that a spectral analysis facilitates the development of these results. For our periodic (filtered) regressors \( Y(k) \), we write the fourier series expansion as

\[
Y(k) = \sum_{n=0}^{N-1} C_Y(n) e^{j2\pi nk/N} \tag{4.1}
\]

and the (discrete) power density spectrum as

\[
\Phi_{YY}(n) = C_Y(n)C_Y^H(n), \quad n = 0, \cdots, N - 1. \tag{4.2}
\]

Using

\[
\frac{1}{N} \sum_{k=0}^{N-1} Y(k)Y^T(k) = \sum_{n=0}^{N-1} \Phi_{YY}(n) \tag{4.3}
\]

the PS condition for a periodic vector \( Y(k) \) is simply restated as

\[
\sum_{n=0}^{N-1} \Phi_{YY}(n) > 0. \tag{4.4}
\]

For the averaging analysis approach to SPACE system stability, we are concerned with positivity properties of matrices with the form

\[
\overline{R}_d[X] = \operatorname{avg} \begin{bmatrix} y_1(k) \\ y_n(k) \\ \vdots \\ y_N(k) \end{bmatrix} \begin{bmatrix} M_1 & \cdots & M_n \\ \vdots & \ddots & \vdots \\ M_1 & \cdots & M_n \end{bmatrix} \begin{bmatrix} y_1(k) \\ y_n(k) \\ \vdots \\ y_N(k) \end{bmatrix}^T \tag{4.5}
\]

In terms of (4.2), we have an expression for \( \overline{R}_d[X] \) analogous to (4.3) given by

\[
\overline{M}^H(2\pi n/N)\Phi_{YY}(n) = \sum_{n=0}^{N-1} \Phi_{YY}(n) \cdot \overline{M}^H(2\pi n/N) \tag{4.6}
\]

where \( \overline{M}_j(\omega) = H_jF_{j-1}(e^{j\omega}) \) and \( \mathcal{R}(\omega) = \text{diag} \{ M_1(\omega), \cdots, M_n(\omega) \} \). With a generalized spectral analysis [21], similar expressions may be derived using continuous power spectrums when such quantities exist [12]. Doing so allows expansion of the analysis beyond the periodic case.

Clearly, if \( \mathcal{R}(\omega) \) and \( \Phi_{YY}(\omega) \) interact such that the integral in (4.6) has eigenvalues with positive real parts, then we achieve stability for the adaptive system. In the case where all the \( \overline{M}_j(\omega) \) are equal and the case when each \( M_j(\omega) \) is a constant independent of \( \omega \), the interaction leading to stability may be simply defined. We describe these situations in the following.
Case 1: \( M_1 = \cdots = M_n = M \). This is precisely the case when the SPACE system simplifies back to the non-SPACE format of Section II. Then (4.6) may be written as

\[
\tilde{R}_x[X] = \frac{1}{2} \sum_{n=0}^{N-1} M^H(2\pi n/N)\Phi_{YY}(n) + \frac{1}{2} \tilde{R}_x[X]^T.
\]

A sufficient condition which guarantees that the real parts of the eigenvalues of \( \tilde{R}_x[X] \) are positive is that

\[
1 = \frac{1}{2} \sum_{n=0}^{N-1} M(2\pi n/N)\Phi_{YY}(n) + \frac{1}{2} \tilde{R}_x[X]^T \geq 0.
\]

i.e., \( \tilde{R}_x[X] + \tilde{R}_x[X]^T \) is positive definite. Expression (4.8) is just a frequency domain restatement of the average SPR condition [18]. The interpretation one gives to this condition is that the power of the excitation at frequencies for which \( \text{Re} [M(o)] > 0 \) must outweigh the power at frequencies for which \( \text{Re} [M(o)] < 0 \).

Also compare (4.8) to Theorem 3.2. There \( X \in E_0 \) implies \( \text{Re} [M(o)] > 0 \) for all \( o \) where \( \Phi_{XX}(o) \neq 0 \). This fact, together with \( X \in E_{PS} \) implying (4.4), yields satisfaction of (4.8). Thus, (4.8) enables the restatement of Theorem 3.2 in terms of spectral densities.

Case 2: \( M_i(o) = m_i \) for each \( i \). Here since \( \mathcal{A} \) is constant, it may be pulled out of the summation in (4.6):

\[
\tilde{R}_x[X] = \left[ \sum_{n=0}^{N-1} \Phi_{YY}(n) \right] \mathcal{A}.
\]

\( \tilde{R}_x[X] \) is then the product of two matrices, the first of which is positive definite given satisfaction of the PS condition. \( \mathcal{A} \) will be positive definite if and only if \( m_i > 0 \) for each \( i \). In this particularly simple case, one only needs positive scalar gains for stability. Canceling the dynamics of \( H_i \) by \( F_i^{-1} \) with any positive leftover scale factor is sufficient for stability. Note that no phase shift occurs from \( \mathcal{A} \) at any frequency in this case, so that a PS condition on the regressor \( X(k) \) is sufficient for local stability of the parameter estimator.

Nonetheless, in practical situations one will not attain either of the preceding cases. Since the operators \( H_i \) may be unknown to the designer who must specify each \( F_i \), having \( H_i F_i^{-1} = M \) for all \( i \), or having \( H_i F_i^{-1} \) a scalar, may be unrealistic. What we show in the following, however, is that given an adequate degree of satisfaction of a PS condition (as in Case 1) for a nominal operator \( M(o) \), then positivity of \( \tilde{R}_x[X] \) will occur if each \( M_i(o) \) differs only slightly from the nominal \( M(o) \). Then all the algorithm designer needs do is to establish approximate equality between the \( M(o) \) operators.

Rewrite \( \tilde{R}_x[X] \) from (4.7) as

\[
\tilde{R}_x[X] = \sum_{n=0}^{N-1} M^H(2\pi n/N)\Phi_{YY}(n) + \sum_{n=0}^{N-1} \Phi_{YY}(n)\Delta^H(2\pi n/N) \quad (4.10)
\]

where

\[
\Delta(o) = \begin{bmatrix} \Delta_1(o) & 0 \\ 0 & \Delta_n(o) \end{bmatrix} = \begin{bmatrix} M_1(o) - M(o) & 0 \\ 0 & M_n(o) - M(o) \end{bmatrix}.
\]

A sufficient condition for \( \min_o \text{Re} \lambda(\tilde{R}_x[X]) > 0 \) is \( \tilde{R}_x[X] + \tilde{R}_x[X]^T > 0 \). We have

\[
\frac{1}{2} (\tilde{R}_x[X] + \tilde{R}_x[X]^T) = \sum_{n=0}^{N-1} \text{Re} [M(2\pi n/N)]\Phi_{YY}(n) + \sum_{n=0}^{N-1} \Phi_{YY}(n)\Delta^H(2\pi n/N) + \Delta(2\pi n/N)\Phi_{YY}(n) \quad (4.12)
\]

Picking \( M(o) \) so that each \( \Delta_i(o) \) is in some sense small, we see that if

\[
\sum_{n=0}^{N-1} \text{Re} [M(2\pi n/N)]\Phi_{YY}(n) > \gamma I > 0 \quad (4.13)
\]

then having

\[
-\gamma I < \frac{1}{2} \sum_{n=0}^{N-1} \Phi_{YY}(n)\Delta^H(2\pi n/N) + \Delta(2\pi n/N)\Phi_{YY}(n) \quad (4.14)
\]

will yield positive definiteness of (4.12). Notice that as the \( \Delta_i(o) \) shrink to zero, (4.14) will be satisfied for any \( \gamma \). Equations (4.12)–(4.14) thus relate the degree of stabilization \( \gamma \) arising from an average SPR condition (for a nominal non-SPACE system) to operator differences \( \Delta(o) \).

From (4.14), we see that we therefore have a kind of "stability margin" provided by the nominal degree of stabilization in (4.13). With \( \gamma \) set to the maximum value for which (4.13) is satisfied, we have a bound on the term involving \( \Delta \) which will yield the desired positivity of \( \tilde{R}_x[X] \) if met. Therefore, a certain degree of tolerance to variations in \( \Delta(o) \) is provided by the level of excitation given by \( \Phi_{YY} \), weighted by \( \text{Re} [M(o)] \). Smallness of the \( \Delta(o) \) transfer functions implies that all the \( M(o) \) are a close match to the nominal \( M(o) \).

Formally, we have the following.

**Theorem 4.1:** Given \( M(o) \), the spectral density \( \Phi_{YY}(n) \) of an \( N \)-periodic (filtered) regressor \( Y(k) \), and \( \tilde{R}_x[X] \) and \( \Delta(2\pi n/N) \) defined by (4.10) and (4.11) with

\[
\sum_{n=0}^{N-1} \text{Re} [M(2\pi n/N)]\Phi_{YY}(n) \geq \gamma I > 0 \quad (4.15)
\]

then \( \exists \delta(M, \Phi_{YY}) > 0 \) such that if

\[
|\Delta_i(2\pi n/N)| < \delta \quad \forall i, n = 0, \ldots, N-1 \quad (4.16)
\]
we have
\[
\min \Re \lambda_i(\hat{R}, [X]) > 0. \tag{4.17}
\]

The maximum value of \(\delta\) from (4.16) is proportional to \(\gamma\) in (4.15).

**Proof:** To prove (4.17) it is sufficient to show that (4.12) is positive definite. If the largest magnitude of the eigenvalues of the second term of (4.12) is less than \(\gamma\), then (4.12) will be positive definite. We have, for an arbitrary vector \(z\)
\[
\left\| \frac{1}{\gamma} \sum_{n=0}^{N-1} \left[ \Phi Y_y(n) \Delta^H(2\pi n/N) + \Delta(2\pi n/N) \Phi Y_y(n) \right] z \right\|_2^2 \\
\leq \frac{1}{\gamma} \sum_{n=0}^{N-1} \left\| \Phi Y_y(n) \right\|_2^2 \\
\leq d \left\| z \right\|_2 \left\| \Phi Y_y(n) \right\|_2 \\
= d \left\| \phi Y_y(n) \right\|_2 \left\| \lambda_i(\phi Y_y(n)) \right\|_2.
\]


Therefore, the magnitudes of the eigenvalues of the second term of (4.12) are bounded by \(dC\), where
\[
C = \sum_{n=0}^{N-1} \left\| \Phi Y_y(n) \right\| < \infty
\]
is a finite constant depending on \(\Phi Y_y(n)\), and \(d\) is given by (4.19). Thus, for \(d \) small enough, we will have \(dC < \gamma\), which implies that (4.12) is positive definite, so that (4.17) holds.

Setting \(\delta = \gamma/C\), having (4.19) hold for \(d \leq \delta\) yields the desired result. Because \(\gamma\) and \(C\) depend on both \(M(2\pi n/N)\) and \(\Phi Y_y(n)\), \(\delta\) will share this dependence. \(\nabla \nabla \nabla \nabla\)

An interpretation of this theorem is as follows. For a non-space system, satisfaction of (4.13) for some \(\gamma > 0\) will yield local stability for the parameter estimation system. In the SPACE case, if we can “match” all the operator compositions \(H_r F_i^{-1}\) through appropriate choice of each \(F_i\), then we retain that same condition for parameter estimator stability. Since such an exact match is a practical impossibility, the stability requirement becomes altered slightly. Now we have an excitation condition (4.13) for a nominal transfer function \(M(\omega)\), to which we attempt to match the compositions \(H_r F_i^{-1}\). The degree of satisfaction of (4.13) then provides a margin for the errors \(\Delta_i(\omega) = M_i(\omega) - M(\omega)\) which preserves stability, given by (4.14).

The bound \(\delta\) for \(\Delta_i(\omega)\) given in Theorem 4.1 is quite restrictive. The dependence of \(\delta\) on \(M\) and \(\Phi Y_y\) is complex, which hinders formulation of a less stringent and more accurate bound. Notice that the regressor plays a double-edged role in determining system stability. First, it is integral to establishing a healthy degree of nominal persistent excitation, and second, it affects the size of the “margin for error” in matching the different operators \(M_i(\omega)\). Furthermore, even with the simplified bound specified in Theorem 4.1, there are difficulties in estimating \(\delta\) given \(M\) and some knowledge of the regressor \(X\).

Nonetheless, this analysis lends rigor to the claim that closeness of the operators \(\{M_i(\omega)\}\) enables local stability of the corresponding SPACE system via a sufficient degree of stabilization through the average SPR condition.

From a frequency domain point of view, one would like the spectral content of the regressor to lie in the range where the nominal operator \(M(\omega)\) satisfies the SPR condition. One must also minimize the differences between \(M(\omega)\) and the actual operators \(M_i(\omega)\) over that frequency range. This approach agrees with basic engineering intuition dictating that one should limit the signals’ bandwidths to ranges where the system response is well known. For SPACE systems, suppose one has good, though not exact, knowledge of the response of the composite error operators \(H_r F_i^{-1}\) over some range of frequencies. Then, with an appropriate choice of the algorithm operators \(F_i(\omega)\), one can assure closeness of the operators \(M_i(\omega)\) over that frequency range. Restricting the regressor’s spectral content to this range of frequencies will then likely yield satisfaction of the stability conditions.

V. IDENTIFICATION OF A PARALLEL-FORM LINEAR SYSTEM

The theory of the preceding sections provides some insights into the following system identification problem: we wish to identify the parameters in a parallel-form representation of a linear system
\[
T(q^{-1}) = \sum_{i=1}^{n} \frac{b_i}{1 - a_i q^{-1}}
\tag{5.1}
\]
as depicted in Fig. 1. In (5.1), \(b_i\) and \(a_i\) may be complex in order to accommodate the potential for complex pole (and zero) locations. For the identification structure of Fig. 1, the prediction error \(e(k)\) has a composite form.

Much of the work in identification of pole-zero-transfer functions has focused on direct form realizations (see, e.g., [1], [22]). Proofs of the parameter estimate convergence may depend
in part on the boundedness of the (possibly filtered) regressor used in the adaptive algorithm. This regressor boundedness depends on maintaining the stability of time-varying regressor filters or the stability of the adapted model itself, if it generates regressor signals used in the estimator [4]. Checking the stability of the filters of the adapted model can be costly from a computational point of view when direct-form realizations form the underlying system model [23]. However, with the parallel-form realization, this stability check is quite simple. One only needs to maintain \( |a_i| < 1 \). (Choice of a mechanism for maintaining stability is a separate issue which we will not discuss here.) This feature of the parallel realization is one motivation for its use in adaptive filtering applications [10], [12], [24], [25]. Similar properties for the lattice form make that realization attractive as well [20], [26], [27].

Most analysis of adaptive algorithms for these alternative realizations has dealt with gradient descent methods (see the aforementioned references). Our SPACE system formulation of results of this paper in a local stability analysis.

For simplicity, let the coefficients in (5.1) be real, requiring that the singularity locations are real. In general, in order to use real arithmetic for the arbitrary, possibly complex, singularity case, one must consider second-order sections. Another point to note is that the parallel-form structure of Fig. 1 degenerates when the model has repeated poles. We therefore restrict attention to models with distinct poles.

The output \( y(k) \) in Fig. 1 is constructed as

\[
y(k) = \sum_{i=1}^{n} y_i(k)
\]

where

\[
y_i(k) = b_iu(k) + a_iy_i(k-1).
\]

The adapted model creates an output estimate

\[
\hat{y}(k) = \sum_{i=1}^{n} \hat{y}_i(k)
\]

with

\[
\hat{y}_i(k) = \hat{b}_i(k)u(k) + \hat{a}_i(k)\hat{y}_i(k-1).
\]

Define regressor vectors

\[
X_i(k) = [u(k)\hat{y}_i(k-1)]^T
\]

and parameter vectors

\[
\Theta_i = [b_i ~ a_i]^T
\]

\[
\hat{\Theta}_i(k) = [\hat{b}_i(k) ~ \hat{a}_i(k)]^T
\]

\[
\tilde{\Theta}_i(k) = [b_i - \hat{b}_i(k) ~ a_i - \hat{a}_i(k)]^T.
\]

Then

\[
e_i(k) = y_i(k) - \hat{y}_i(k)
= u(k)[b_i - \hat{b}_i(k)] + \hat{y}_i(k-1)[a_i - \hat{a}_i(k)]
+ a_i[y_i(k-1) - \hat{y}_i(k-1)]
\]

or

\[
e_i(k) = \frac{1}{1 - a_iq^{-1}}[X_i^T(k)\tilde{\Theta}_i(k)].
\]
WTLLUMSON
E,\textsuperscript{14} another where
one where the
system). The region of predicted local stability is a subset of
correct
the
depending of the predicted stability region on the actual
excitation, with the region changing as the frequencies in (5.15)
the depicted boundaries.) Notice that each region in Fig. 2 is

\begin{equation}
\frac{0.3}{1 - 0.7q^{-1}} + \frac{0.35}{1 - 0.65q^{-1}}.
\end{equation}

The corresponding adaptive filter again has two parametrizations equivalent to (5.16)

\begin{align}
1) & \Theta = [0.3 \ 0.7 \ 0.35 \ 0.65] \\
2) & \Theta' = [0.35 \ 0.65 \ 0.3 \ 0.7]
\end{align}

which are thus fixed points of the algorithm (5.11). Once more we calculate the local stability of these points, for different \(\bar{a}_1\), \(\bar{a}_2\) in (5.11), with input (5.15). We set the frequencies \(\omega_1\), \(\omega_2\) to 0.3 and 0.8. Fig. 4 depicts the regions of stability and instability of (1) in (5.17) for this input.

Notice, however, that the algorithm filter setting \(\bar{a}_1 = a_1\) is much closer to the instability boundary than the corresponding parametrization was in Fig. 3. Application of Theorem 4.1 yields a much smaller region of predicted stability (as seen in Fig. 4), probably a reflection of the proximity of the instability region. A possible explanation for this differences lies in the character of the frequency responses of each parallel section in (5.12) and (5.16). The filter of (5.12) consists of one lowpass and one highpass section, while that of (5.16) consists of two lowpass filters.

For the situation of Fig. 3, the input (5.15) with \((\omega_1, \omega_2) = (0.8, 2.2)\) places each frequency in the passband of one section of (5.12) and in the stopband of the other. The effect of the lowpass–highpass situation, and the corresponding regressor filtering in (5.11) with \(\bar{a}_1 = a_1\), is to reduce the crosscorrelation between \(F_{1}[X_1]\) and \(H_{2}[X_2]\), and between \(F_{2}[X_2]\) and \(H_{1}[X_1]\), since at each frequency one of the signals in each of these pairs is attenuated. The result is that off-diagonal elements of the average excitation matrix \(\bar{R}\) are reduced in magnitude, so that the SPACE system character of the problem is downplayed; and a large region of local stability about \(\bar{a}_1 = a_1\) results.

Corresponding to Fig. 4, with the filter of (5.16) and \((\omega_1, \omega_2) = (0.3, 0.8)\), comparable power remains in each sinusoidal component of \(F_{1}[X_1]\) and \(H_{1}[X_1]\), so that we do not have a similar diagonalizing or decoupling effect in this case. There is less distinguishability between the two parallel sections of (5.16), since the poles are close to one another. Thus, there is more

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig2.png}
\caption{Regions of predicted local stability. (a) \((\omega_1, \omega_2) = (0.8, 2.2)\). (b) \((\omega_1, \omega_2) = (0.6, 2.4)\). (c) \((\omega_1, \omega_2) = (1.0, 2.4)\). (d) \((\omega_1, \omega_2) = (1.0, 2.0)\). (e) \((\omega_1, \omega_2) = (0.6, 2.2)\).}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig3.png}
\caption{Regions of local stability (unshaded) and local instability (shaded). \(\bar{\Theta} = [0.7 \ 0.7 \ -0.7]^{T}\), with \((\omega_1, \omega_2) = (0.8, 2.2)\).}
\end{figure}
potential for destabilizing interaction in $\bar{R}$ for this situation, as seen in Fig. 4. Nonetheless, we still see a broad range of choices for $\bar{a}_1$ and $\bar{a}_2$ which provide local stability for the adaptive filter. Keep in mind that the magnitude of the frequency components in the various filtered regressor elements indicates a potential for destabilization via undesirable interactions in the average of $\bar{R}$. The differing phase relations, however, figure prominently in realizing this potential (recall the role of phase in Theorem 3.2).

A curious aspect of this second example is that, for (5.16) with $(\omega_1, \omega_2) = (0.3, 0.8)$ in input (5.15), there exist pairs $(\bar{a}_1, \bar{a}_2)$ for which neither $\Theta$ nor $\Theta'$ is locally stable, and no pair for which both fixed points are locally stable. For values of $\bar{a}_1$ and $\bar{a}_2$ for which both fixed points are locally unstable, the input $u(k) = \cos(0.3k) + \cos(0.8k)$ is an example of excitation yielding a FS regressor without providing good behavior of the adaptive system. For other pairs of $(\bar{a}_1, \bar{a}_2)$, though at least one of the fixed points is locally stable for this input, some different excitation signal will destabilize the error system.

VI. CONCLUSIONS

We have shown in this paper that local stability properties of SPACE adaptive systems are different from the local stability properties of their simpler non-Space counterparts. Our analysis of SPACE systems uses averaging techniques similar to those successfully applied to non-Space systems, but an extension of the non-Space analysis is necessary to describe SPACE system stability. Unlike in the non-SPACE system case, persistent spanning of a regressor whose frequency content is appropriately restricted is not sufficient to establish local stability of the general SPACE system. The theorems of Section III indicate that this type of condition may guarantee such stability only with a very particular choice of algorithm operators. This choice basically requires exactly matching the algorithm operators to the operators in the composite error. Usually, achievement of this precise algorithm operator selection is unlikely in practical situations. Therefore, a relaxing of this condition is needed to assure local stability.

Our examination of the averaged update stability conditions in Section IV indicates an approach leading to local SPACE system stability. The main thrust of our result is that one may also relax the operator matching conditions through appropriate excitation of the adaptive system. If a regressor achieves a strong degree of stabilization for a particular nominal error system with a non-Space structure, then stability follows for a class of SPACE systems excited by the same regressor. This class simply contains those SPACE systems whose error system operators are close to the nominal operator characterizing the nominal (and stable) non-Space system. We have provided some quantitative bounds on operator differences whose satisfaction will result in SPACE system stability given an adequate degree of stabilization of the nominal (non-SPACE) error system.

However, as the identification example of Section V indicates, these bounds are very conservative. For a given system excitation, we have a locally stable parameter estimator for a much wider range of algorithm filters than determined by Theorem 4.1. Future work should concentrate on better illuminating the relationships between regressor structure, operator specification, and excitation conditions for local stability of the error system.

One feature of adaptive systems which has been underexplored in the past is the built-in structure of the regressor. For example, in the regressor (5.6) for the parallel-form model, the signals in the regressor are the input and output of a system whose specification we know. In direct form models, a number of regressor entries are simply delayed versions of other entries. Regressor structure has played a role in the characterization of input signals which lead to a FS regressor [29], but interrelationships between regressor elements have figured only indirectly in determining currently applied stability conditions for adaptive systems. For SPACE systems, regressor structure may be an important key to the establishment of local stability. This potential utility of exploiting the regressor structure is illustrated in [12].

One topic which concerns the results presented in this paper is the translation of persistent spanning conditions on the filtered regressor to conditions on external signals. Work in [29] addresses this problem for non-Space systems. However, for the SPACE system case, the difference in the operators filtering the regressor elements will affect how external signal richness influences persistent spanning of the filtered regressor. Knowing the effect of external signals on the spanning properties of the regressor is significant for establishing satisfaction of our stability conditions in an adaptive system.

Another issue of interest is the possible effect of filtering the prediction error within the split algorithms. The possibility for this additional filtering was mentioned in [11]. Though addition of different error filtering in each parameter update may unnecessarily complicate the adaptive system behavior, identical filterings may help ameliorate effects caused by differences in the regressor filters from the desired nominal operators.

REFERENCES


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