

TECHNICAL NOTE

H_∞ -OPTIMAL CONTROLLER DISCRETIZATION

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SUMMARY

A redesign method for discretizing a continuous-time controller is proposed. The resulting hybrid control system, for example with continuous-time plant and discrete-time controller, is stable, and performance including the system's inter-sampling behaviour can be optimized by approximating some chosen reference transfer function of the continuous-time control system. In order to obtain a tractable problem, the continuous-time part of the hybrid system and the reference transfer function are approximated by a discrete-time system with arbitrary fast sampling. After lifting the resulting periodic system, the approximation problem can be formulated as a standard H_∞ -problem which is solved using standard software for H_∞ -controller design.

KEY WORDS Digital control Sampled data systems H_∞ controller design Intersampling behaviour

1. INTRODUCTION

Since most controller design procedures allow direct discrete-time controller design, the question arises: is there any need for a controller discretization method? Several considerations suggest that it might be reasonable to design first a continuous-time controller, which is afterwards converted into a discrete-time one. Direct design of discrete-time controllers requires a predetermination of the sampling time in order to model plant, noise, process disturbances etc. Upper bounds for the sampling time can only be obtained after considering closed-loop bandwidth (Franklin *et al.*,⁵ Ch. 10), and, depending on specifications, this may only be available *after* controller design. Because this obvious dilemma does not occur in continuous-time controller design, it may be reasonable to design first a continuous-time controller, which is in a second step approximated by a discrete-time one. A second reason is that the two-step approach allows a transparent discrete-time controller design even for a relatively large sampling time because intrinsic performance limitations are well separated from performance degradation due to sampling-time selection. Furthermore, performance degradation can be determined with an elegant, quantitative description of the inter-sample response, normally not included in direct discrete-time controller design. Only a subset of direct discrete controller design procedures take into account the inter-sample response of the plant in any way at all. These include linear-quadratic methods where the discrete-time index

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is constructed from a continuous-time index.⁵ A third reason for first executing a continuous-time design is that physical insight is more easily retained.

Before discussing existing discretization methods a list of requirements for controller discretization is proposed. First of all the very basic property of closed-loop stability should be guaranteed. In most applications we are also interested in the performance of the resulting control system including its inter-sampling behaviour. In addition the discretization method should provide the possibility of including a possible time delay in the control law required for controller realization. Furthermore, the determination of the discrete-time controller should be a numerically tractable problem to be able to discretize also large-scale systems.

An overview of existing discretization methods is given in Hanselmann.⁶ The isolated or open-loop methods, for instance bilinear transformation, hold input approximation and signal invariant transformation, do not take the closed-loop use of the controller into account and can therefore guarantee neither stability nor performance. Discretizations are easy to calculate but need an excessively small sampling time to perform well (in comparison to the sampling time when other discretization methods are used). Problems of these discretization methods when used for controller design with relatively large sampling time were reported by Katz.⁷ Methods for discrete-time implementation of linear state-feedback laws are proposed by Kuo and Peterson,¹² Yackel *et al.*¹⁶ and Kleinman and Rao.¹¹ A closed-loop redesign method is proposed by Rattan and Yeh¹³ and Rattan,¹⁴ but none of these approximation methods can guarantee the elementary property of closed-loop stability after controller discretization. Kennedy and Evans⁸ propose a controller redesign based on pole-zero matching of the complementary sensitivity function. Closed-loop stability and performance at the sampling instants can be achieved but inter-sampling behaviour is unaddressed. An extension to multivariable systems, which might not be straightforward, is not given. In Keller and Anderson⁹ the discretization problem is posed as a controller approximation problem, using ideas from Anderson and Liu.¹ The resulting controller leads to closed-loop stability but performance optimization was not included.

The drawback to all these methods is that they do not use the knowledge of the class of all controllers stabilizing the discrete-time plant, i.e. the hold-input discretization of the plant. It is clear that stability of the discrete-time system is necessary for stability of the hybrid system, i.e. the continuous-time plant and the discrete-time controller. Given the class of stabilizing controllers, a particular discrete-time controller can be determined by approximating (possibly optimally) some selected mapping associated with the continuous-time system, which is chosen to represent the system's performance, i.e. controller redesign. As an error measure, the \mathcal{L}_2 -induced operator norm of the difference between the two mappings is chosen. Now because of the mixture of continuous and discrete-time entities, some of the mappings in the hybrid system are operators which have no transfer-function representation. In order to get a tractable problem, the continuous-time part of the hybrid system is approximated by a discrete-time system with arbitrary fast sampling. Also the selected reference transfer function is discretized with respect to the same sampling period. This can be done in a chosen (sensible) frequency range as accurately as desired by hold-input discretization. In this approximation the former hybrid system is replaced by a N -periodic system, if the small (fast) sampling time is chosen to be submultiple N of the controller sampling time. By lifting the N -periodic control system a time-invariant transfer-function representation is obtained. The approximation problem can then be rewritten as a standard H_∞ -problem with exogenous and endogenous inputs and outputs.

In Section 2 of this study the discretization problem is formulated. In Section 3 the discrete-time approximation with respect to a relatively small sampling time is proposed. The lifting

operation is exhibited and the standard H_∞ -problem is derived. It is shown that the class of stabilizing controllers of the fast sampled system is equivalent to the class of stabilizing controllers of the low sampling rate discrete-time plant. How additional controller delay may be included in the controller determination is shown in Section 4. The proposed approximation of hybrid control systems offers also a method for direct discrete-time controller design. This is discussed in Section 5. In Section 6 an example is presented and Section 7 contains concluding remarks.

2. PROBLEM FORMULATION

Let $P(s)$ be the plant transfer function, $C(s)$ the known continuous-time controller, $C_1(z)$ the unknown discrete-time controller operating at sampling time τ_1 and let $H(s)$ represent a zero-order hold. As a measure of performance of the control system, the complementary sensitivity function $T(s) = PC(I + PC)^{-1}$ is chosen. Let T_h be the operator of the hybrid system corresponding to $T(s)$ and let $W(s)$ be a stable, strictly proper transfer function shaping the reference signal $r(s)$. The following criterion for performance deterioration is proposed:

$$J_h \triangleq (T_h - T)W \tag{1}$$

A corresponding block diagram is drawn in Figure 1.

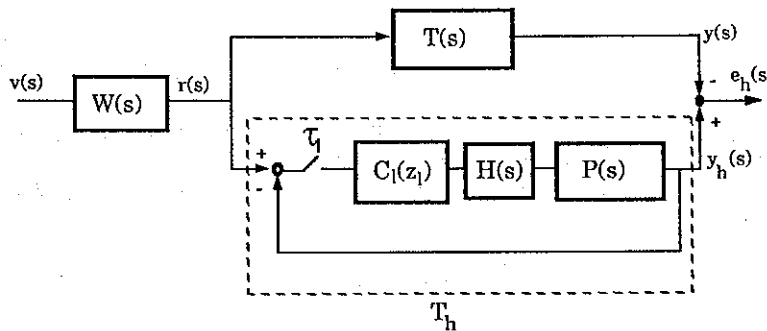


Figure 1. Representation of J_h

Let \mathcal{S}_h be the set of controllers stabilizing T_h and \mathcal{S}_1 the set, stabilizing the discrete-time plant, i.e. the hold-input discretization of $P(s)$ with respect to the controller sampling time τ_1 as already defined in the introduction. A discrete-time controller C_1 can be determined through the following redesign problem:

$$C_1 = \arg \min_{C_1 \in \mathcal{S}_h} \| J_h \| \tag{2}$$

where $\| \cdot \|$ is the \mathcal{L}_2 -norm-induced operator norm.

For $\| J_h \|$ to be finite, $W(s)$ is a stable, strictly proper transfer function. $W(s)$ may be chosen as an anti-aliasing filter for the input $v(s)$ or more generally as a weighting factor for the approximation of $T_h - T$.

3. APPROXIMATION OF J_h AND TRANSFORMATION INTO AN H_∞ -PROBLEM

The optimization problem (2) contains the operator T_h which has no transfer function representation; therefore it cannot be easily solved. In order to obtain a more tractable

problem the criterion J_h is modified. The continuous-time transfer function $T(s)$ and the operator T_h are replaced by their hold-input discretization with respect to an arbitrarily small sampling time τ_s . τ_s is chosen to be a submultiple of τ_1 , the sampling time of the final control system.

$$N\tau_s = \tau_1 \quad (3)$$

Also the weighting $W(s)$ is replaced by a discrete-time one.

Because the resulting system has two different time scales, the notation must be clarified. Discrete-time operators mapping sequences with time-scale $k\tau_s$ are denoted with subscript 's', the subscript 'l' is used for the time scale $k\tau_1$ and the subscript 'h' denotes hybrid systems and their outputs.

Following the lines of the proof of Proposition 3.1 in Keller and Anderson⁹ it can be shown, that the modified error measure $\|J_{hs}\|$ with

$$J_{hs} \triangleq (T_{hs} - T_s(z_s))W_s(z_s) \quad z_s = e^{s\tau_s} \quad (4)$$

converges to $\|J_h\|$ as τ_s goes to zero, if $\|J_h\| < \infty$. This convergence is more of theoretical interest, because in applications inter-sampling behaviour may be sufficiently characterized even for N , i.e. the number of points between the sampling instants, being relatively small.

$T_s(z_s)$ and $W_s(z_s)$ are easy to calculate, whereas the determination of T_{hs} is not obvious. The operator $T_h: y_h = T_h r$ can be written as

$$y_h(s) = PH(s)C_1(z_1)(I + P_1(z_1)C_1(z_1))^{-1}r_1^*(s) \quad (5)$$

with $z_1 = e^{s\tau_1}$, $r_1^* = \sum_{k=-\infty}^{\infty} r(s - jk\omega_1)$, ($\omega_1 = 2\pi/\tau_1$): sampled input at sampling time τ_1 , $P_1(z_1)$: hold-input discretization of $P(s)$ with respect to τ_1 .

Because of the sampling device at the input of T_h , the hold-input discretization of T_h is simply the sampled output $y_s^*(s)$. With $u_1(z_1) \triangleq C_1(I + P_1C_1)^{-1}r_1^*(s)$ there is:

$$y_h(s) = PH(s)u_1(z_1) \quad (6)$$

Since $u_1(z_1)$ is already ω_s -periodic, i.e. even ω_1 -periodic, there is (see, for example, Franklin *et al.*⁵):

$$\begin{aligned} y_{hs}(z_s) &= (PH)_s^*(s)u_s^*(z_1) = (PH)_s^*(s)u_1(z_1) \\ &= (PH)_s(k, z_s)u_1(z_1) \quad k = 0, \dots, N-1 \end{aligned} \quad (7)$$

The resulting hold-input discretization of T_h is an N -periodic system. N -periodic systems can by means of lifting be represented by time-invariant transfer-function matrices in the variable z_1 . These transfer-function matrices operate on blocks of N successive inputs and are producing a block of N successive outputs. A more detailed description of the lifting operation and a well-known state-space representation of the resulting transfer functions can be found in connection with a similar application in Keller and Anderson.⁹ The lifting operation is an isomorphism from $l_2[0, \infty)$ to $l_2[0, \infty)$ and does therefore not alter $\|J_{hs}\|$. In drawing the approximation of the system in Figure 1 in terms of the lifted transfer functions, we denote with the diacritical tilde lifted operators. Subscripts are used (only for lifted operators) if the notation becomes ambiguous. In the lifted representation a transfer matrix of a sampler \tilde{S} can be chosen to be a row vector of length N having 1 as first element and zero on other entries elsewhere and the corresponding hold element \tilde{H} (controller without time-delay) is a column vector of ones. (In the multivariable case, the ones are replaced by identity matrices, the zeros

by matrices with zero entries, both of appropriate dimensions.)

$$\tilde{H} \triangleq \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} \quad \tilde{S} \triangleq [1 \ 0 \ \dots \ 0] \quad (8)$$

With this (simplest) selection of \tilde{S} and \tilde{H} , the sampling instant is being chosen to coincide with the time of the first element of the block representation. Other selections are possible but, as can be shown, lead to the same solution for C_1 .

The lifted transfer function of $PH_s(k, z_s)$, i.e. the hold element followed by $P_s(z_s)$, is:

$$P\tilde{H}(k, z_s) = \tilde{P}(z_1)\tilde{H} \quad (9)$$

There is a connection between the hold-input discretization $P_1(z_1)$ and an N -block representation $\tilde{P}(z_1)$ of the hold-input discretization $P_s(z_s)$. Since $P_1(z_1)$ is a hold-input discretization it can be represented with respect to z_s as a sequence of a hold element, $P_s(z_s)$ and a sampler, sampling every N^{th} signal.

$$P_1(z_1) = \tilde{S}\tilde{P}(z_1)\tilde{H} \quad (10)$$

The lifted operator \tilde{J} , shown in Figure 2, is now representable as a transfer function matrix:

$$\tilde{J} = (\tilde{P}\tilde{H}C_1(I + \tilde{S}\tilde{P}\tilde{H}C_1)^{-1}\tilde{S} - \tilde{T})\tilde{W} \quad (11)$$

In this expression all transfer functions are functions in the variable z_1 . Furthermore \tilde{J} can be written as a linear fraction:

$$\tilde{J} = \mathcal{F}\ell\{\tilde{G}, C_1\}, \quad \tilde{G} = \begin{bmatrix} -\tilde{T}\tilde{W} & \tilde{P}\tilde{H} \\ \tilde{S}\tilde{W} & -\tilde{S}\tilde{P}\tilde{H} \end{bmatrix} \quad (12)$$

with

$$\mathcal{F}\ell\left\{\begin{bmatrix} \tilde{G}_{11} & \tilde{G}_{12} \\ \tilde{G}_{21} & \tilde{G}_{22} \end{bmatrix}, K\right\} \triangleq \tilde{G}_{11} + \tilde{G}_{12}K(I - \tilde{G}_{22}K)^{-1}\tilde{G}_{21} \quad (13)$$

Several interesting conclusions can be drawn from (12). The system in Figure 2 is stabilizable if \tilde{G} in (12) is stabilizable with the inputs and outputs of \tilde{G}_{22} (see, for example, Francis³). Since \tilde{T} and \tilde{W} are assumed to be stable, this condition boils down to $\tilde{S}\tilde{P}\tilde{H}$ being stabilizable and detectable. Because $\tilde{S}\tilde{P}\tilde{H} = P_1(z_1)$, this condition is equivalent to the well-known stabilizability

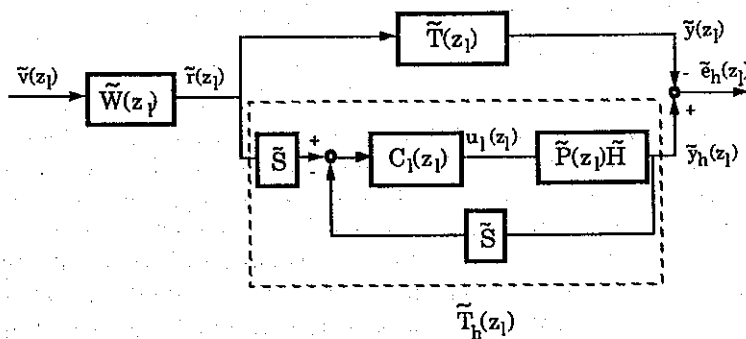


Figure 2. System, represented with lifted transfer functions

and detectability conditions for a direct discrete-time controller design. Furthermore, the set of controllers stabilizing the fast sampled system represented in Figure 2 is the set stabilizing $\tilde{S}\tilde{P}\tilde{H}$. Since $\tilde{S}\tilde{P}\tilde{H} = P_1(z_1)$ this class is independent of N .

Since $\|J_{hs}\| = \|\tilde{J}\| = \|\tilde{J}\|_\infty$ the controller C_1 is found for some suitably large N by solving the following standard H_∞ -minimization problem:

$$C_1 = \arg \min_{C_1 \in \mathcal{A}_1} \|\tilde{J}\| \quad (14)$$

The H_∞ -problem is of the third kind ($N > 1$). For solvability, we require no zeros of \tilde{G}_{12} and \tilde{G}_{21} on the unit circle, together with a stabilizability property. Zeros of G_{21} in (12) on the unit circle can be avoided through a sensible selection of $W(s)$. Because of the discretization of $P(s)$ the zeros \tilde{G}_{12} in (12) are difficult to predict. Since the transfer matrix \tilde{P} is multiplied by \tilde{H} from the right, possible zeros of \tilde{P} at $z = 1$ ($P(s)$ strict proper) generically disappear for N sufficiently large, i.e. larger than the order of P .

In order to verify the convergence of $\|J_{hs}\|$ to $\|J_h\|$ for $N \rightarrow \infty$ we have to show that the hybrid control system in Figure 1 is \mathcal{L}_2 -stable for any controller C_1 stabilizing \tilde{G} . The system in Figure 1 is \mathcal{L}_2 -stable, if it is exponentially stable and if $W(s)$ is stable and strictly proper (Chen and Francis²). It is exponentially stable, if $P_1(z_1) = \tilde{S}\tilde{P}\tilde{H}$ is stabilizable and detectable and if $(P_1(z_1), C_1(z_1))$ is a stable pair (Francis and Georgiou⁴). A possible solution to (12–14) always results in an exponentially stable system and the requirements on $W(s)$ are met by assumption.

Therefore, there follows

$$\lim_{N \rightarrow \infty} \|\tilde{J}\| = \|J_h\| \quad (15)$$

4. CONTROLLERS WITH TIME DELAY

The realization of controllers usually does not allow an immediate change of actuator values after the input is sampled. The computation of new values or a measurement in a process control loop may cause a time delay, which should be incorporated in the controller design. The time delay may often be much smaller than a sampling period. Including a whole sampling period of delay may unnecessarily reduce the system's performance. Within the proposed controller redesign procedure, time delays, which are a multiple of the small sampling time τ_s , can easily be taken into account. Consider a time delay L of length $i\tau_s$ with transfer function z_s^{-i} . In order to insert a time-delay in the controller design, we have to calculate its lifted representation. Using the transfer matrix representation of a lifted transfer function presented in Khargonekar *et al.*¹⁰ there results for a scalar system:

$$\tilde{L}(z_1) = \begin{bmatrix} 0_{i \times (N-i)} & z_1^{-1} I_{i \times i} \\ I_{(N-i) \times (N-i)} & 0_{(N-i) \times i} \end{bmatrix} \quad (16)$$

This block representation shows obviously how the delay z_s^{-i} affects a block of N successive input elements. The first i elements of the output of the lifted delay are the i last elements of the last input block. The remaining $N - i$ elements of the output are the first $N - i$ elements of the current lifted input vector shifted i elements down because of the delay. A similar representation can be obtained for the multivariable case with the additional possibility of different delays in the channels. This may be useful, if there are different delays in measurements. The lifted transfer function can now be inserted in the control loop either before the sampler \tilde{S} or after the hold element \tilde{H} . In the block representation these are the only

positions where it is possible to introduce a delay of less than the sampling time τ_1 . \tilde{L} can be incorporated either in \tilde{S} or \tilde{H} to obtain the H_∞ -problem (14).

5. DIRECT DISCRETE-TIME CONTROLLER DESIGN WITH CONTINUOUS-TIME CONTROLLER SPECIFICATIONS

Using the approximation idea, presented in this study, it is also possible to perform a direct discrete-time controller design for a continuous-time plant using H_∞ ideas. Figure 3 represents a very general problem formulation. If this hybrid system is, as before, approximated by a fast sampled, N -periodic system, followed by lifting, the following H_∞ -design problem follows:

$$C_1 = \arg \min_{C_1 \in \mathcal{S}} \|\tilde{J}\|_\infty \tag{17}$$

$$\tilde{J} = \mathcal{F}l\{\tilde{G}, C_1\}, \quad \tilde{G} = \begin{bmatrix} \tilde{W}_3 \tilde{W}_4 & 0 & \tilde{W}_3 \tilde{P} \tilde{H} \\ \tilde{W}_1 \tilde{W}_4 & \tilde{W}_1 \tilde{W}_2 & \tilde{W}_1 \tilde{P} \tilde{H} \\ \hline -\tilde{S} \tilde{W}_4 & \tilde{S} \tilde{W}_2 & -\tilde{S} \tilde{P} \tilde{H} \end{bmatrix} \tag{18}$$

The system in Figure 3 is stable, if it is exponentially stable and if $W_2(s)$ and $W_4(s)$ are stable, strictly proper transfer functions (and obviously if $W_1(s)$ and $W_3(s)$ are bounded) (Chen and Francis²). $W_2(s)$ and $W_4(s)$ may be chosen in a similar way as W in Section 2.

For the formal H_∞ setting, it can be shown that:

$$\mathcal{F}l\{\tilde{G}, \tilde{H}C_1\tilde{S}\} = \mathcal{F}l\{\hat{G}, C_1\} \quad \hat{G} = \begin{bmatrix} \tilde{G}_{11} & \tilde{G}_{21}\tilde{H} \\ \tilde{S}\tilde{G}_{21} & \tilde{S}\tilde{G}_{22}\tilde{H} \end{bmatrix} \tag{19}$$

Although inter-sampling behaviour of the control system is reflected through (18) into the direct discrete-time controller design, the problem of sampling-time selection still remains. A possible determination of the sampling time could result from a further investigation of the properties of lifted transfer functions. The idea is to choose the small sampling time τ_s reasonably small and increase N as far as the performance deterioration remains acceptable. Based on bounds of the H_∞ -problem (18) it might be possible to estimate the performance reduction without solving the H_∞ -problem. These bounds could provide a measure for the determination of N and therefore also the controller sampling time τ_1 .

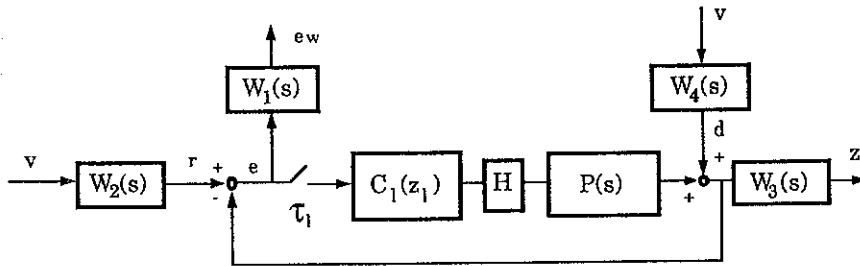


Figure 3. General H_∞ -problem formulation

6. EXAMPLE

A discrete-time controller was determined for the H_∞ -optimal controller of a large space structure designed by Safonov *et al.*¹⁵ The continuous-time controller design is based on a

reduced order partially controlled plant model of order 4 with two inputs and two outputs. Safonov *et al.* propose a conformal mapping for controller discretization, and performance of the resulting control system is verified with the original 116-state plant model. The transformation used is

$$s = 6000 \left(\frac{z - 1}{1 + z/0.3} \right)$$

This corresponds to a 3000 Hz sampling rate. There is no really systematic procedure for determining the parameters in the transformation, but there results a good discrete-time controller meeting the design specifications.

In order to separate the effects of controller discretization and model reduction the discrete-time controllers of Safonov and of this paper are compared using the fourth-order controller design model. The later discrete-time controller was determined by approximating the complementary sensitivity function, as per (1). The approximation error weighting was chosen to be a transfer function times the identity matrix. The entries of the fourth-order transfer functions consist of a second-order Butterworth filter and a second-order shaping function. The frequency plot of the singular values of the complementary sensitivity function and of the weighting are drawn in Figure 4. The plot shows that below 100 rad/s a small error is forced through a relative large value of the weighting function. This was chosen to meet the design specification on the magnitude of the sensitivity function S to be smaller than 0.01, as set out in Reference 15. The inclusion of this design specification on S in the approximation of T is possible, since the sum of S and T is one. The Butterworth filter was used to cut off higher frequencies.

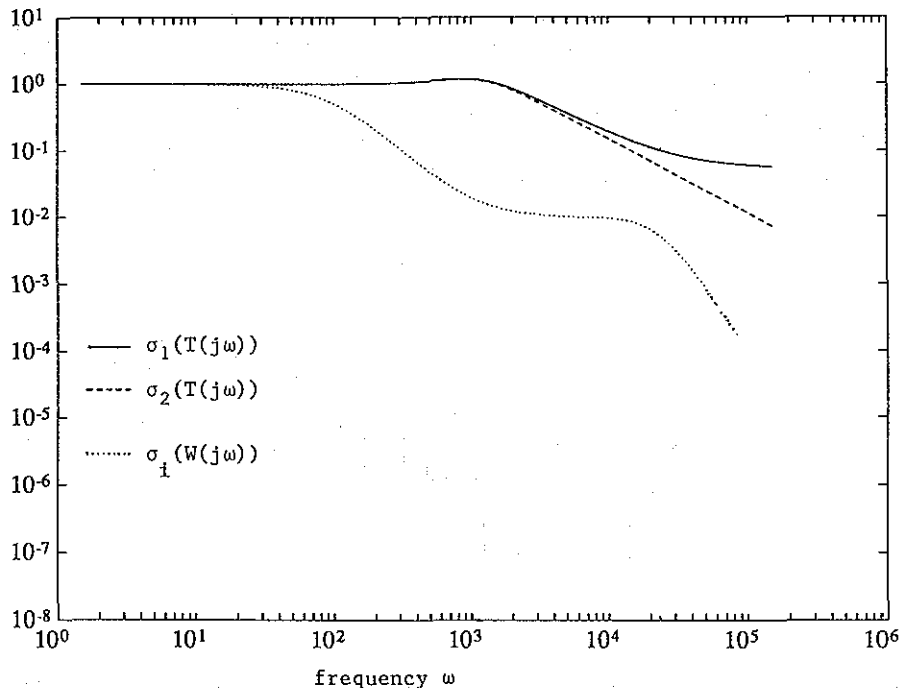


Figure 4. Singular value Bode plot of $T(s)$ and weighting $W(s)$.

There was a strong trade-off between three goals: (1) the specification for S ; (2) robustness requirements (the closed-loop singular value Bode plot of T must be bounded above by 3 db, see Safonov *et al.*¹⁵; (3) minimization of inter-sampling ripple. Controller discretization with different weightings showed that it was possible to achieve two of the goals but at the cost of the third. The proposed weighting in Figure 4 is a compromise between the goals. In Figure 5 the frequency plot of the largest singular of T_h is shown for different controllers. The continuous-time controller is compared with Safonov's controller and the controller obtained by our proposed method. The figure shows the aspects (2) and (3). It can be seen, that Safonov's controller leads to much higher maximal values of the singular values. At half the sampling frequency a remarkable resonance peak can be seen for Safonov's discrete-time controller. Therefore, a somewhat undesirable inter-sampling behaviour of the control system must be expected. Figures 6 and 7 show the time responses of the system's output y_1 and y_2 to an impulse at the first reference signal r_1 . The plots verify our claim that with the proposed discretization method inter-sampling behaviour is reflected in controller discretization. In comparison to Safonov's controller, inter-sampling ripple is significantly reduced. For the investigation of aspect (1) the sensitivity function S is plotted in Figure 8. The plot shows that Safonov's controller leads to a better disturbance attenuation. It shows further that the design specifications are not fulfilled with our discrete-time controller.

The discretization of the controller for twice the proposed sampling frequency shows that the trade-off between the above-mentioned aspects is significantly reduced, i.e. a controller meeting all performance requirements and with almost no inter-sampling ripple is obtained.

The H_∞ -optimal discrete-time controller was of order 22. Using unweighted balanced truncation it was possible to reduce the controller order to 8. The model-data can be found

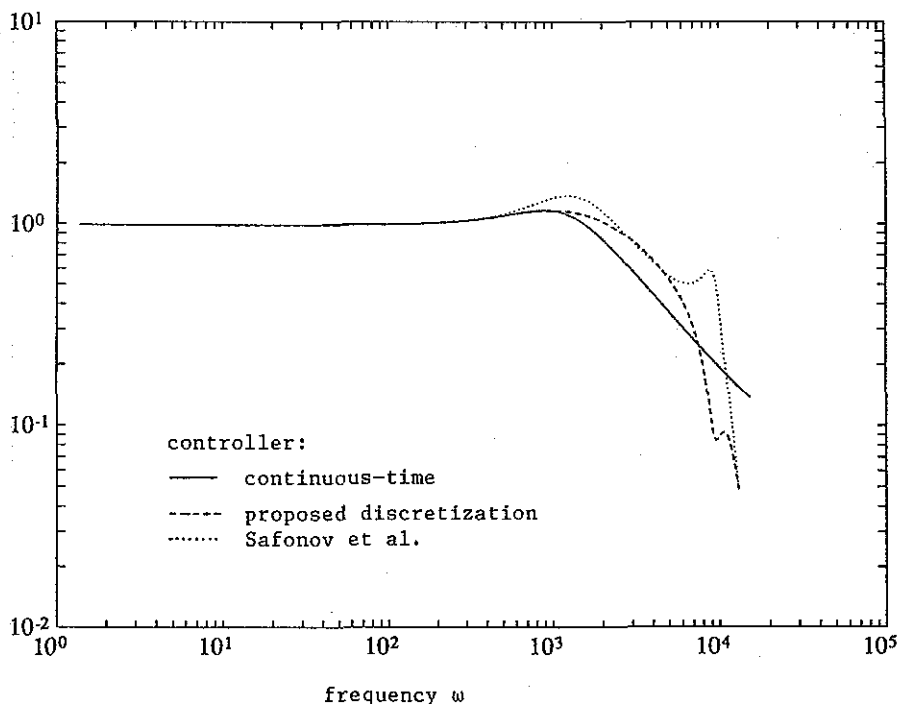
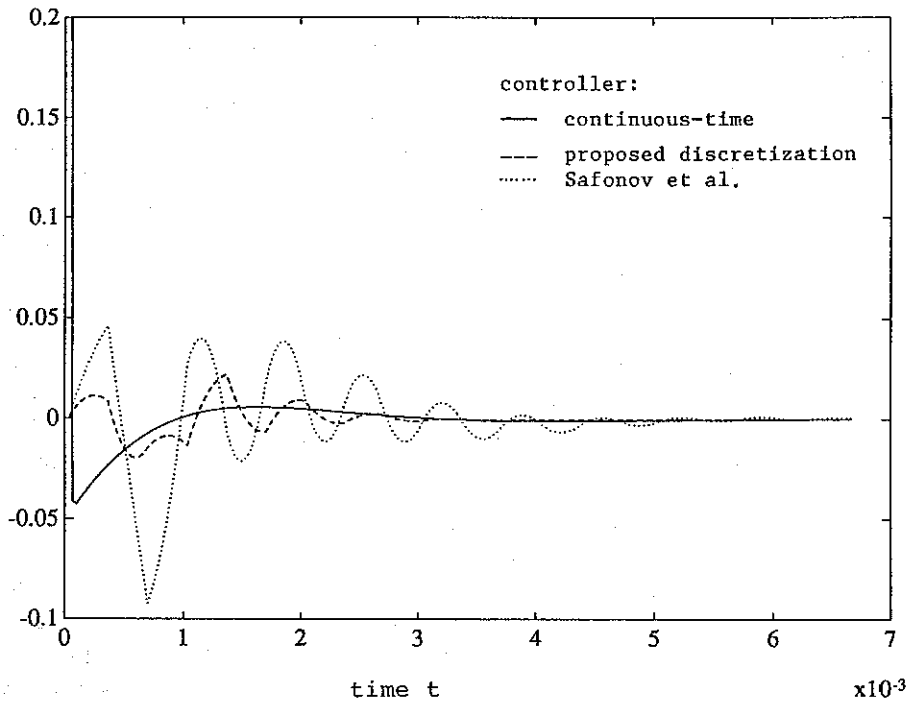
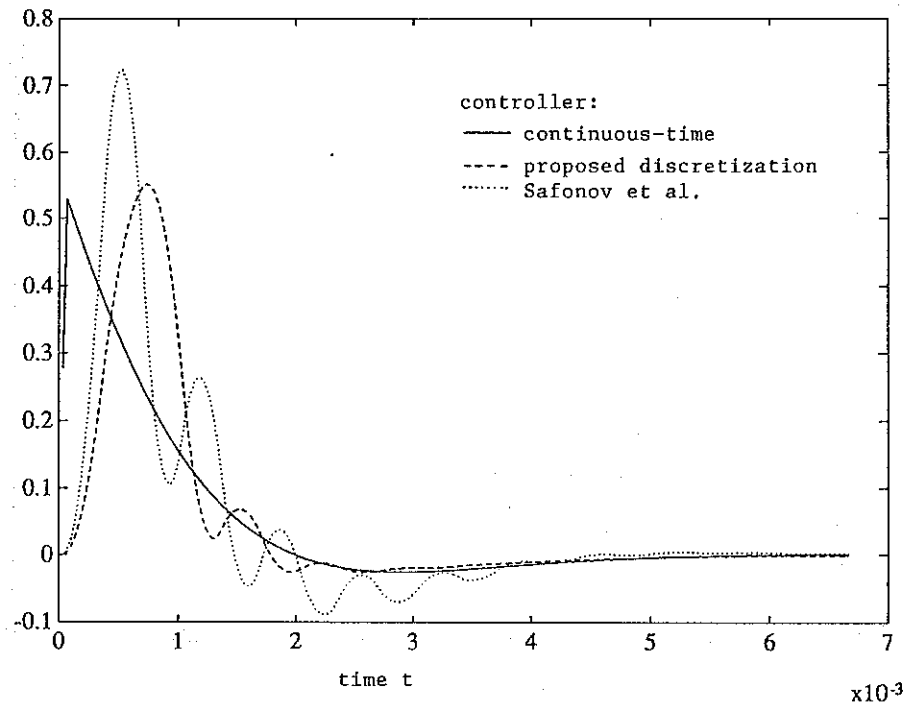


Figure 5. Bode plot of the largest singular value of $T(s)$ and T_h .

Figure 6. Impulse response from r_1 to y_1 Figure 7. Impulse response from r_1 to y_2

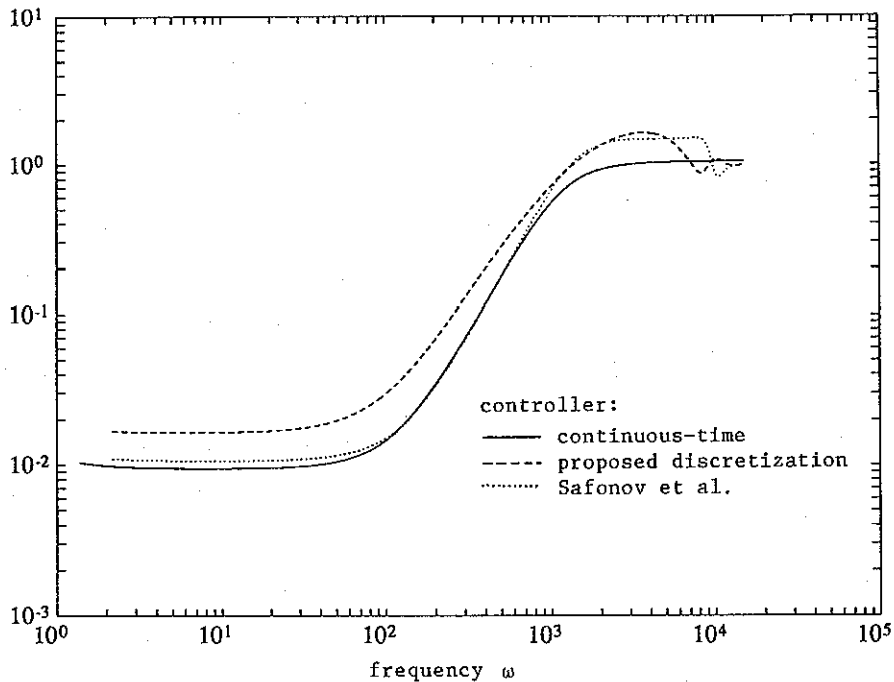


Figure 8. Bode plot of the largest singular value of the sensitivity function

in Reference 15; a state-space realization of the reduced-order controller is given in the Appendix.

7. CONCLUSION

The proposed method offers a powerful tool for controller discretization. The resulting hybrid control system is stable and its performance, including inter-sampling behaviour is optimized. Trade-offs resulting from the selection of sampling time are recognized, while trying to achieve a satisfactory approximation of the chosen continuous-time reference transfer function. If no satisfactory approximation can be achieved, the sampling time can be adjusted by just repeating the controller discretization and not the whole controller design procedure. Inter-sampling behaviour is reflected in the discretization step and does not therefore require a possibly iterative cycle of controller design (or discretization) followed by simulation. Also time-delays, necessary for controller implementation, can easily be incorporated in the controller discretization. The determination of the discrete-time controller is straightforward and can be performed with standard software used in H_∞ -controller design. A drawback of the proposed redesign procedure is, that the resulting controller can be of relatively large order, e.g. the sum of plant order, order of the weighting and order of the approximated transfer function. The example shows that the order can usually be significantly reduced by applying some of the order reduction methods proposed in Anderson and Liu.¹ Also a direct discrete-time controller design, which includes inter-sampling behaviour, was proposed. Several aspects of this approach, as for example sampling time predetermination, have to be investigated. A possibly very useful additional application may be the controller design for mixed systems, e.g. systems having continuous-time and discrete-time parts.

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APPENDIX: STATE-SPACE MATRICES OF DIGITAL CONTROLLER

$$A = \begin{bmatrix} -8.1619e-01 & -1.6604e-01 & -3.9153e-01 & 9.1375e-02 & -5.4415e-03 & -1.0489e-03 & -5.5671e-04 & -5.4554e-03 \\ 1.5536e-01 & -8.0026e-01 & -7.5841e-02 & -4.0256e-01 & -1.4862e-03 & 5.8965e-03 & 7.2824e-03 & -1.1813e-03 \\ 3.1599e-01 & 2.8702e-01 & -2.8929e-01 & 7.3621e-02 & -2.4860e-02 & 1.9235e-02 & 3.2715e-02 & -2.2951e-02 \\ -2.3421e-01 & 3.0849e-01 & -1.3529e-01 & -2.1327e-01 & 2.6197e-02 & 2.4397e-02 & 2.5029e-02 & 3.5343e-02 \\ 4.8612e-03 & -5.4897e-03 & -2.5021e-02 & 2.5146e-02 & 9.9065e-01 & 3.2269e-05 & 5.9403e-03 & -2.7139e-02 \\ -2.8730e-03 & -3.2966e-03 & 1.9087e-02 & 2.5246e-02 & -8.4437e-04 & 9.9228e-01 & -2.7496e-02 & -6.2811e-03 \\ 5.1639e-03 & 1.2424e-03 & -3.3623e-02 & -2.4647e-02 & -7.0229e-03 & 2.7220e-02 & 9.4166e-01 & 2.2162e-03 \\ -3.2786e-03 & 6.6131e-03 & 2.1698e-02 & -3.5629e-02 & 2.6888e-02 & 7.3486e-03 & -2.2164e-03 & 9.4007e-01 \end{bmatrix}$$

$$B = \begin{bmatrix} 7.0949e+01 & 6.1012e+01 \\ -5.9505e+01 & 7.3519e+01 \\ 3.2328e+01 & 1.1780e+02 \\ -1.1638e+02 & 2.4194e+01 \\ -1.0749e+00 & -1.1880e+00 \\ -9.7189e-01 & 1.4088e+00 \\ 1.1309e+00 & -2.7974e+00 \\ 2.0609e+00 & 1.0408e+00 \end{bmatrix}$$

$$C = \begin{bmatrix} 3.7431e+01 & -8.9202e+01 & -5.4624e+00 & 1.1740e+02 & 1.0636e+00 & 9.9996e-01 & 1.3698e+00 & 1.9442e+00 \\ 8.8244e+01 & 2.0464e+01 & -1.2373e+02 & -6.9443e+00 & 1.1317e+00 & -1.4449e+00 & -2.7250e+00 & 1.2007e+00 \end{bmatrix}$$

$$D = \begin{bmatrix} 1.3571e+04 & 1.5373e+03 \\ -1.6789e+01 & 1.3886e+04 \end{bmatrix}$$