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ROBUST SCHUR POLYNOMIAL STABILITY AND KHARITONOV'S THEOREM

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Abstract

The paper considers robust stability properties for Schur polynomials of the form

$$f(z) = \sum_{i=0}^{n} a_{n-i} z^i .$$

By plotting coefficient variations in planes defined by variable pairs a_i, a_{n-i} for each i and requiring in each such plane the region of obtained coefficients to be bounded by lines of slope 45°, 90° and 135°, we show that stability for all polynomials defined by corner points is necessary and sufficient for stability of all polynomials defined by any points in the region. Using this idea, one can construct several necessity and differing sufficiency conditions for the stability of polynomials where each a_i can vary independently in an interval $[\underline{a}_i, \bar{a}_i]$. As the sufficiency conditions become closer to necessity conditions the number of distinct polynomials for which stability has to be tested increases.

I. Introduction

The problem of robust polynomial stability is easily posed: suppose there is presented a set of polynomials

$$f(z) = \sum_{i=0}^{n} a_{n-i} z^i \qquad a_i \in [\underline{a}_i, \bar{a}_i] \qquad (1.1)$$

What is a finite set of conditions for all such f(z) to have all roots in a prescribed region? In this paper, we are interested in ensuring that all roots lie in $|z| < 1$.

The robust polynomial stability problem in continuous time, i.e. where the region of interest is the left half plane, has a known and elegant solution, due to Kharitonov [1]. The "weak" version of Kharitonov's theorem states that a necessary and sufficient condition for robust stability is that all corner polynomials are stable (corner polynomials are obtained when $a_i \in \{\underline{a}_i, \bar{a}_i\}\ \forall\, i$). The "strong" version states that a necessary and sufficient condition for stability is that a particular four of the 2^{n+1} corner polynomials are stable. Actually, when $n < 6$, stability of fewer than four corner polynomials constitutes a necessary and sufficient condition for robust stability, see [2].

Given this time-continuous stability result, an obvious conjecture that can be made concerning the discrete-time problem (when the region of interest is $|z| < 1$) is that stability of all corner polynomials is necessary and sufficient for robust stability. This is not correct, and there exist counterexamples demonstrating the falsity of this conjecture, see [3, 4]. Nevertheless, certain rather more specialized results are available. Let us now recall some. If $\underline{a}_i = \bar{a}_i$ for $i = 0,1,\ldots,(n-1) \llcorner 2$ (see footnote for the meaning of $\llcorner$), then the conjecture is true [3]. If $n = 2$ or 3 and $f(\cdot)$ is monic the conjecture is true [5]. If $n = 2$ or 3 and $f(\cdot)$ is not monic, or $n = 4$ or 5 with $f(\cdot)$ monic, then it is possible to define further points such that stability at these points and the corner points gives stability for all $f(\cdot)$ [5]. If $f(\cdot)$ is monic then stability at the point defined by

$$\tilde{a}_i = -\mathrm{Max}(|\underline{a}_i|; |\bar{a}_i|) \qquad \forall\, i$$

yields robust stability for all $|a_i| \le |\tilde{a}_i|$ and therefore also for $a_i \in [\underline{a}_i, \bar{a}_i]$ [6], [7].

$x \llcorner y$ rounds x/y to the nearest lower integer.

In this paper, we present results applicable for any value of n. Roughly speaking, we consider coefficient variations in planes defined by a_0 and a_n, a_1 and a_{n-1}, etc. If the coefficients vary inside a polygon in each plane, where the sides of the polygon have slopes of either 45°, 90° or 135°, then stability at the corner points is necessary and sufficient for robust polynomial stability.

It is not just the result itself that is of interest, but the methods used to derive it, since it is these methods which are likely to yield further extension. Accordingly, the methods themselves are given some exposure in the next section. As it turns out, they can be seen in retrospect to be natural transfers to the discrete-time problem of methods used to establish the (continuous-time) Kharitonov theorem.

The above mentioned stability result is presented in section 3, and it is then used to explain the construction of differing necessity and sufficiency conditions for robust polynomial stability where the coefficient variation is as indicated by (1.1).

One method for tackling robust discrete-time stability should be noted here as being unattractive – that based on bilinear transformation, or use of the fact that if f(z) has all zeros in $|z| < 1$, then

$$g(s) = (s-1)^n f\left(\frac{s+1}{s-1}\right)$$

has all zeros in $\mathrm{Re}(s) < 0$. The problem is that if the coefficients of f(z) lie in rectangular boxes with sides parallel to the axes, this is *not* the case for the coefficients of g(s), nor conversely. The bilinear mapping distorts regions in coefficient space to such an extent that is is difficult to make progress, although not impossible, see [8].

II. Approaches to Discrete-Time Robust Stability Testing

In proving Kharitonov's theorem, a major step in the argument invokes the two observations that f(s) is (continuous-time) stable if and only if [f(s)+f(-s)]/[f(s)-f(-s)] is lossless positive real, and that lossless positive real functions form a convex set. Our approach to discrete-time robustness involves duplicating these observations, mutatis mutandis.

Given f(z), let us define two polynomials $g_e(z)$ and $g_0(z)$, by

$$g_e(z) = \frac{1}{2}\,[f(z) + z^n f(z^{-1})] \tag{2.1a}$$

$$g_0(z) = \frac{1}{2}\,[f(z) - z^n f(z^{-1})] \tag{2.1b}$$

Then we have

Theorem 1 [9-12]: f(z) has all its roots in $|z| < 1$ if and only if $g_e(z)/g_0(z)$ is (discrete) lossless bounded real. Equivalent statements are:

(a) Zeros of $g_e(z)$ and $g_0(z)$ are simple, all lie on $|z| = 1$, alternate on $|z| = 1$, and $|a_0/a_n| < 1$,

(b) $g_0(z)/g_e(z)$ is (discrete) lossless bounded real.

Just as in continuous time, discrete lossless bounded real transfer functions also form a convex set.

With the convexity of lossless positive real functions, a straightforward consequence of the theorem, noted in [13], is the following:

Proposition 2: Let $f_i(z)$, i = 1,2 be two polynomials of degree m. Let $g_{ei}(z)$, $g_{0i}(z)$ for i = 1,2 be defined as in (2.1). Then all polynomials of the following four families

$$[\lambda g_{e1}(z) + (1-\lambda)g_{e2}(z)] + g_{0i}(z) \qquad i = 1,2 \quad \lambda \in [0,1] \tag{2.2a}$$

$$g_{ei}(z) + [\lambda g_{01}(z) + (1-\lambda)g_{02}(z)] \qquad i = 1,2 \quad \lambda \in [0,1] \tag{2.2b}$$

are discrete-time stable if and only if $g_{ei}(z) + g_{0j}(z)$ are stable for all i,j, i = 1,2 and j = 1,2.

As a corollary, we have the following:

Corollary 3: With hypotheses as in the above proposition, all polynomials of the following family are stable:

$$[\lambda g_{e1}(z) + (1-\lambda)g_{e2}(z)] + [\mu g_{01}(z) + (1-\mu)g_{02}(z)] \qquad \lambda \in [0,1], \quad \mu \in [0,1]$$

Proof: By the proposition, we have stability of $[\lambda g_{e1}(z) + (1-\lambda)g_{e2}(z)] + g_{0i}(z)$ for i = 1,2. Hence, $g_{0i}(z) / [\lambda g_{e1}(z) + (1-\lambda)g_{e2}(z)]$ is lossless positive real for i = 1,2. Hence, $[\mu g_{01}(z) + (1-\mu)g_{02}(z)] / [\lambda g_{e1}(z) + (1-\lambda)g_{e2}(z)]$ is lossless positive real, and the result follows.

Figure 1 depicts the above situation. Especially point G_1 (G_2) corresponds to $f_1(z)$ ($f_2(z)$). From Proposition 2

(2.2a) results stability for every point along the lines G_1G_3 and G_2G_4; from (2.2b) follows analogously the stability along the lines G_1G_4, G_2G_3 and from Corollary 3 the stability of the whole box $G_1G_2G_3G_4$.

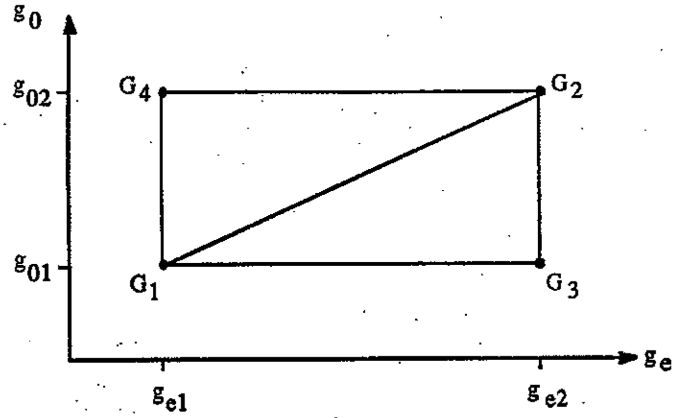

Figure 1: Sufficiency test for robust stability

Let us now translate some of these ideas into a robust stability result that is more in the form we desire. We present the primary results in the form of theorems:

Theorem 4: Consider a class of polynomials f(z) in which the coefficients vary over some set $\mathcal{A}$, not necessarily rectangular. Consider the associated classes of polynomials $g_e(z)$ and $g_0(z)$ in which the coefficients vary over sets $\mathcal{A}_e$ and $\mathcal{A}_0$. Suppose that $\mathcal{A}_e$ and $\mathcal{A}_0$ are convex polytopes; denote the polynomials associated with the corners by $g_{e1},...,g_{ep}$ and $g_{01},...,g_{0q}$. Then all f(z) with coefficients in $\mathcal{A}$ are stable if $g_{ei}(z) + g_{0j}(z)$ is stable for all $i \in \{1,2,...,p\}$ and $j \in \{1,2,...,q\}$.

Proof: Write $f(z) = g_e(z) + g_0(z)$ and recognize the coefficient vector for g_e, g_0 is a convex combination of coefficient vectors for $g_{e1},...,g_{ep}$ and $g_{01},...,g_{0q}$. Consequently,

$$g_e(z) = \sum_{i=1}^{n} \lambda_i g_{ei}(z) \qquad \lambda_i \geq 0 \quad \Sigma\lambda_i = 1 \qquad (2.3a)$$

$$g_0(z) = \sum_{j=1}^{q} \mu_j g_{0j}(z) \qquad \mu_j \geq 0 \quad \Sigma\mu_j = 1 \qquad (2.3b)$$

Stability of $g_{ei}(z) + g_{0j}(z)$ for all i,j implies lossless positive realness of g_{ei}/g_{0j} and g_{0j}/g_{ei} for all i,j and hence (by an extension of the argument proving the corollary) the lossless positive realness of $\Sigma\lambda_i g_{ei} / \Sigma\mu_j g_{0j}$. The claim is then easily seen.

Note that this is a sufficient result for the stability of all f(z). When is it necessary? One answer is as follows:

Corollary 5: Assume the same hypotheses as Theorem 4. Suppose also that every polynomial $g_{ei}(z) + g_{0j}(z)$ for $i \in \{1,2,...,p\}$ and $j \in \{1,2,...,q\}$ has a coefficient vector in $\mathcal{A}$. Then all f(z) with coefficient vector in $\mathcal{A}$ are stable if and only if all $g_{ei}(z) + g_{0j}(z)$ are stable.

Let us note two distinct applications of these ideas. First, consider an affine set of polynomials

$$f(z) = f_1(z) + v f_2(z) \qquad v \in [\underline{v}, \bar{v}] \qquad (2.4)$$

We can find a *sufficient* condition for the stability of every polynomial in this set, in terms of the stability of a finite set of polynomials, in the following way.

Let

$$f_1(z) + \underline{v} f_2(z) = g_{e1}(z) + g_{01}(z) \qquad (2.5a)$$
$$f_1(z) + \bar{v} f_2(z) = g_{e2}(z) + g_{02}(z) \qquad (2.5b)$$

Then we have

Proposition 6: Stability of $g_{ei}(z) + g_{0j}(z)$ for i = 1,2 and j = 1,2 implies stability of the affine set (2.4).

Proof: One can either observe that this is a special case of Theorem 4, or one can recognize that

$$f_1(z) + v f_2(z) = [\lambda g_{e1}(z) + (1-\lambda) g_{e2}(z)] + [\lambda g_{01}(z) + (1-\lambda) g_{02}(z)]$$

where

$$\lambda = \frac{v - \underline{v}}{\bar{v} - \underline{v}} \quad \text{so that} \quad \lambda \in [0,1] .$$

The result follows by applying Corollary 3 with the identification $\mu = \lambda$.

Of course, the checking of stability of (2.4) can also proceed using root locus ideas, or even an extended-type of Schur-Cohn criterion. The result of Proposition 6 can be regarded as quick and weak, quick because only four polynomials have to be checked, but weak since only a sufficiency result is available.

We can obtain tighter conditions at the expense of introducing more points, see Figure 2. Let v_i, $i = 0,1,...,\mu+1$, be a sequence of $\mu+2$ numbers with $v_i < v_{i+1}$ and the endpoints $v_0 = \underline{v}$, $v_{\mu+1} = \bar{v}$. Then we have

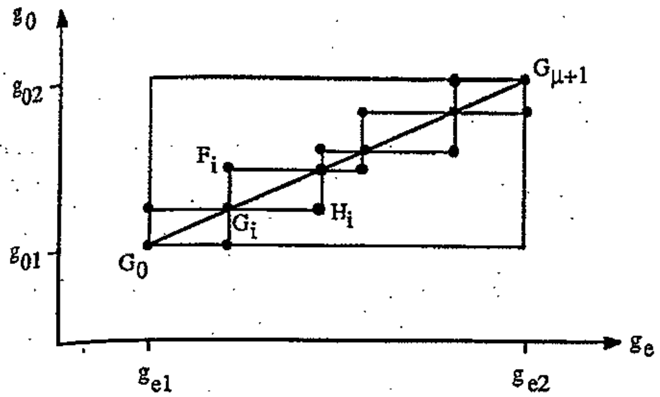

Figure 2: Construction of tighter sufficiency test.

Corollary 7: Let $f_i(z)$, $i = 1,2$, be two discrete-time stable polynomials of degree m with $g_{ei}(z)$, $g_{0i}(z)$, for $i = 1,2$, defined as in (2.1). Then stability of

- $(g_{e1}(z) + v_i g_{e2}(z)) + (g_{01}(z) + v_i g_{02}(z))$ (2.6)

  for all $i = 1,2,\ldots,\mu$, is necessary for stability of the affine set (2.4).

- $(g_{e1}(z) + v_i g_{e2}(z)) + (g_{01}(z) + v_{i+1} g_{02}(z))$ (2.7a)
  $(g_{e1}(z) + v_{i+1} g_{e2}(z)) + (g_{01}(z) + v_i g_{02}(z))$ (2.7b)

  for all $i = 0,1,\ldots,\mu$, is sufficient for stability of the affine set (2.4).

Proof: Necessity is obvious because (2.6) represents points of the affine set (2.4). These points correspond to points G_i of Figure 2.

The sufficiency conditions (2.7) are a straightforward consequence of Proposition 6 and 2. The line G_iG_{i+1} of Figure 2 is stable if beside the endpoints also F_i and H_i correspond to (2.7a) and (2.7b), respectively, are stable (Proposition 6). However, stability of G_i, $i = 1,2,\ldots,\mu$, follows from stability of F_{i-1} and H_i or F_i and H_{i-1}, respectively (Proposition 2).

A much more general result now follows:

Proposition 8: Consider a class of polynomials

$$f(z) = \sum_{i=0}^{n} a_{n-i} z^i$$

in which for each $i \neq n/2$, a_i and a_{n-i} vary inside a region of the form depicted in Figure 3. (No particular significance attaches to the fact that the region lies in the first quadrant.) If n is even, $a_{n/2}$ varies in an interval $[\underline{a}_{n/2}, \bar{a}_{n/2}]$. Then all f(z) are stable if and only if every member of the finite set of f(z) defined by every possible combination of corner points (and interval end-points in case $i = n/2$) is stable.

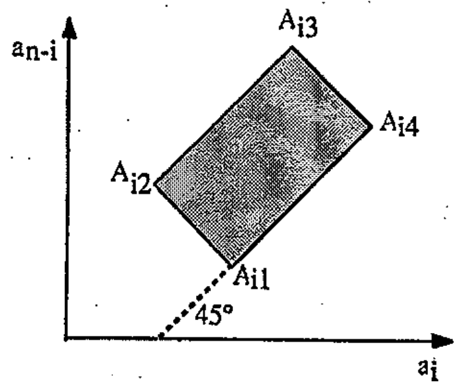

Figure 3: Region of variation of a_i, a_{n-i}

Proof: It is not hard to verify that the sets $\mathcal{A}_e$, $\mathcal{A}_0$ over which the coefficients of $g_e(z)$, $g_0(z)$ vary are rectangular boxes, with edges parallel to the axes. More precisely, the region depicted in Figure 3 is of the form

$$\underline{\alpha}_i \leq a_i + a_{n-i} \leq \bar{\alpha}_i$$
$$\underline{\beta}_i \leq a_i - a_{n-i} \leq \bar{\beta}_i$$

Consequently, the coefficient in $g_e(z)$ of z^i and of z^{n-i} lies in the interval $[\underline{\alpha}_i, \bar{\alpha}_i]$. Similarly, the coefficient of $g_0(z)$ of z^{n-i} lies in the interval $[\underline{\beta}_i, \bar{\beta}_i]$ and of z^i lies in the interval $[-\bar{\beta}_i, -\underline{\beta}_i]$.

Evidently, $\mathcal{A}_e$ and $\mathcal{A}_0$ are convex polytopes, and a pairing of any corner of $\mathcal{A}_e$ with any corner of $\mathcal{A}_0$ is equivalent to picking out one of A_{i1} through A_{i4} for $i = 0,1,\ldots,(n-1)\lfloor 2$ and one of $\underline{a}_{n/2}$ or $\bar{a}_{n/2}$ in case n is even. The result then follows by Corollary 5.

Remark: In case $n = 4$, the cardinality of the finite set is 32: 4 choices in the (a_0,a_4) plane, 4 choices in the (a_1,a_3) plane and 2 choices in the a_2 interval. In general, there are 2^{n+1} corner points.

The above result appears to the authors to be the closest parallel to (a weak form of) Kharitonov's theorem. Of course, the disadvantage is that the coefficient variation for f(z), though it involves rectangular boxes, does not involve rectangular boxes with edges parallel to the axes. A second but lesser disadvantage is that it is in general not possible to avoid consideration of all 2^{n+1} corner points, i.e. there is no equivalent of a strong Kharitonov theorem.

III. Robust Polynomial Stability

We return now to the problem of original interest, that of characterizing the robust stability of

$$f(z) = \sum_{i=0}^{n} a_{n-i} z^{i} \qquad a_i \in [\underline{a}_i, \bar{a}_i] \qquad (3.1)$$

We will not give a precise necessary and sufficient condition; we will, however, give various necessity and differing sufficiency conditions.

Proposition 8 provides a basis for giving such conditions (which we shall later improve upon). Consider Figure 4. Let k_i be the number of the corner points A_{ij} of the covering box in the plane (a_i, a_{n-i}); i.e. for $i = n/2$ we have $k_i = 2$ (interval endpoints), and $k_i = 4$ otherwise. Then we have:

Theorem 9: If stability holds for all possible corner point combinations

$$\{A_i \,;\, i = 0,1,\ldots,n\lfloor 2\} \qquad (3.2)$$

with

$$A_i \in \{A_{ij} \,;\, j = 1,\ldots,k_i\} \qquad (3.3)$$

then robust stability follows for all

$$a_i \in [\underline{a}_i, \bar{a}_i] \qquad i = 0,1,\ldots,n.$$

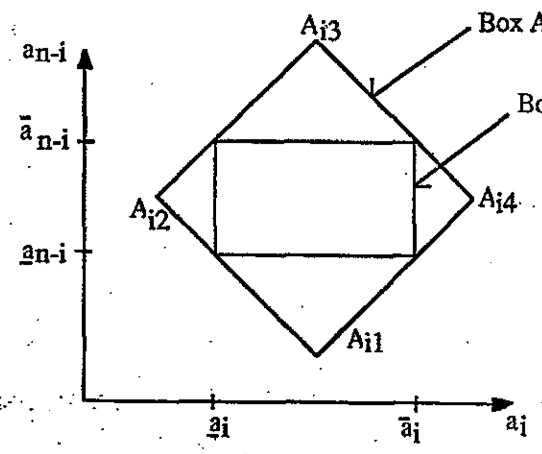

Figure 4: Construction of sufficiency test for stability in axis-parallel rectangular box

Proof: The parameter variations mentioned in Proposition 8 (box A_i of Figure 4) strictly cover the interesting parameter variations (3.1) (box B_i). Therefore Proposition 8 gives a sufficiency condition for robust stability.

We can obtain a tighter sufficiency condition at the expense of introducing more points, see Figure 5 with $k_i = 12$. If stability holds for all possible corner point combinations (3.2), (3.3), then this also is a sufficient condition for robust stability.

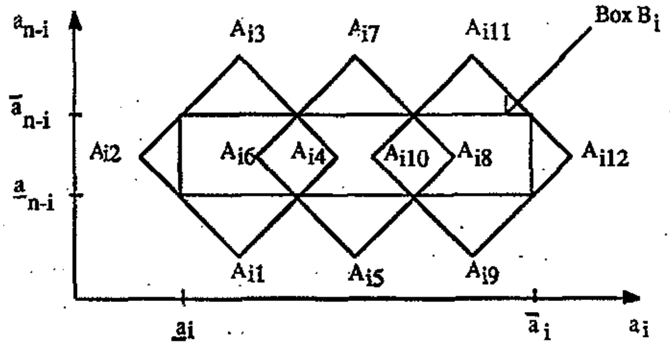

Figure 5: Construction of less conservative sufficiency test.

The choice of the covering rectangular boxes is not unique — Figure 6 shows another possibility with the same number of corner points.

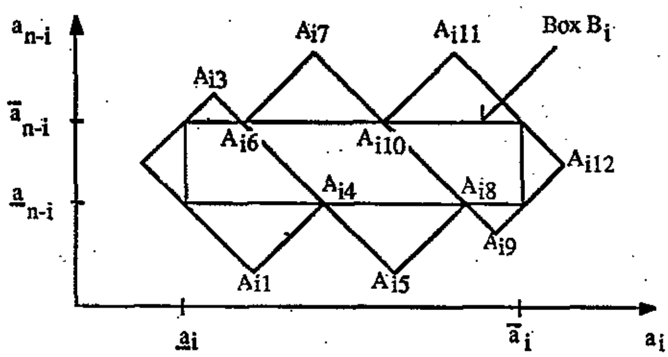

Figure 6: Construction of another sufficiency test.

Necessity conditions are obtainable by inscribing the boxes $B_i = \{\underline{a}_i \le a_i \le \bar{a}_i, \underline{a}_{n-i} \le a_{n-i} \le \bar{a}_{n-i}\}$ with slanted rectangles. Stability for all combinations of corner points of these rectangles as in Figures 7 or 8 is necessary for the robust stability in the boxes B_i. Notice that stability at the corner points of the slanted rectangles is necessary and sufficient for stability inside the shaded region. The unshaded region inside B_i is thus a measure of the extent to which the necessity condition fails to be sufficient.

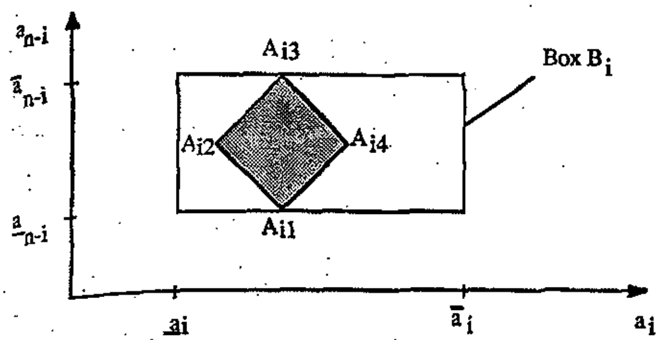

Figure 7: Construction of necessity test.

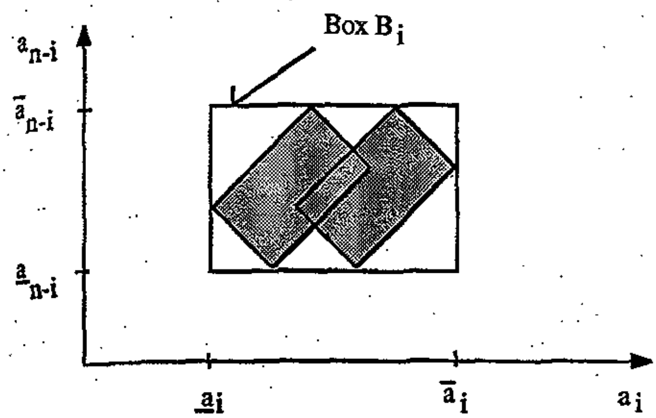

Figure 8: Construction of more complex necessity test.

We can improve on these conditions by combining the ideas of Section 2 with the result of Hollott and Bartlett (1986). The main result is as follows.

Theorem 10: Consider a class of polynomials

$$f(z) = \sum_{i=0}^{n} a_{n-i} z^i$$

in which for each $i \neq n/2$, a_i and a_{n-i} vary inside a single (or finite union of) twodimensional polygonal region(s) bounded by lines of slopes 45°, 90° and 135° (see Figure 9), or on a line of slope 45°, 90° or 135°. If n is even, $a_{n/2}$ varies in an interval $[\underline{a}_{n/2}, \bar{a}_{n/2}]$. Then all f(z) are stable if and only if the finite set of f(z) defined by every possible combination of corner points are stable.

Proof: Consider the set of polynomials defined by corner points for $i \neq I1$ for some fixed I1, and defined by requiring a_{I1}, a_{n-I1} to lie on one of the lines of slope 45° or 135° bounding the coefficient region in (a_{I1}, a_{n-I1}) space. In the space of coefficients of $g_e(s)$ and $g_o(s)$, the corresponding set of points is a straight line parallel to an axis joining two of the corner points in one of these spaces. It follows (by the convexity property of lossless positive real transfer functions) that stability at the corner points implies stability of this set of polynomials.

Similarly, we can show that if a_{I1}, a_{n-I1} lie on a bounding line of slope 45° or 135° in (a_{I1}, a_{n-I1}) space and a_{I2}, a_{n-I2} lie on a bounding line of slope 45° or 135° in (a_{I2}, a_{n-I2}) space while other coefficients are at corners we have a stable polynomial. The procedure can be continued. Thus stability at all corner points implies stability on all region-bounding lines of slope 45° or 135°.

Now consider a polynomial all of whose coefficients lie on such lines except that (a_{I1}, a_{n-I1}) either lies on a vertical line which itself defines part of the allowed coefficient region, or $(a_{I1}, \bar{a}_{n-I1})$ lies on a vertical line and is strictly inside such a region. In either case, for some $\underline{a}_{n-I1}$ and a_{n-I1}, associated with the bottom and top of this line, the polynomials with a_{n-I1} replaced by $\underline{a}_{n-I1}$ and by $\bar{a}_{n-I1}$ are both stable. Using the argument of [3], it follows that all convex combinations of these polynomials are stable, and in particular the polynomial defined by (a_{I1}, a_{n-I1}).

Next, we can introduce the possibility of (a_{I2}, a_{n-I2}) lying between points on two lines of slope 45° or 135°, and so on, and repeat the argument. This proves the theorem.

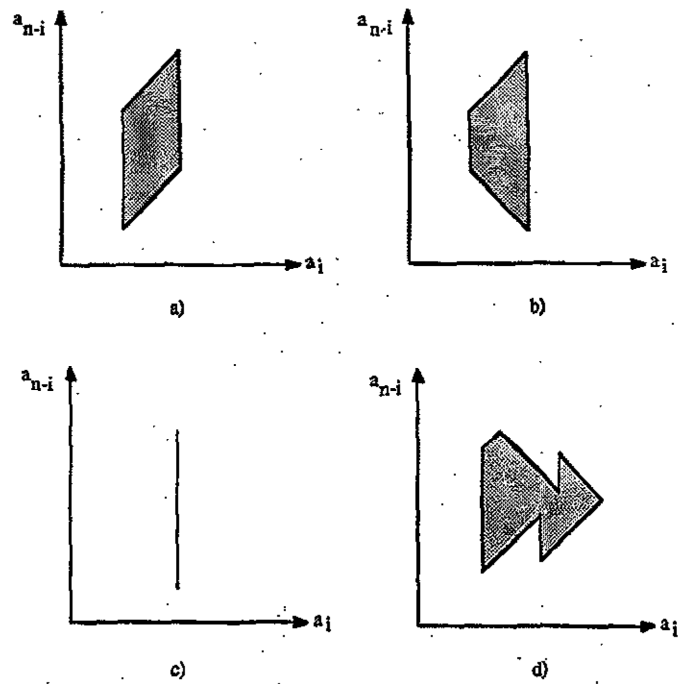

Figure 9: Region of allowed variation of a_i, a_{n-i}.

With this theorem, it is possible to obtain tighter sufficiency and (differing) necessity conditions for stability given coefficient variation in a rectangular box parallel to the axes. Figure 10 illustrates sufficiency and Figure 11 necessity. Stability at all possible corner points implies (Figure 10) and is implied by (Figure 11) stability in the rectangular box.

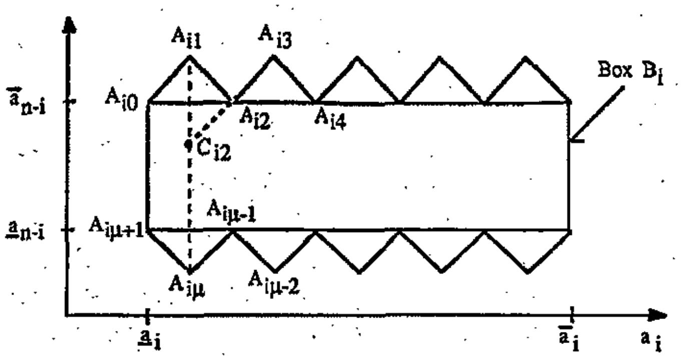

Figure 10: Construction of sufficiency test.

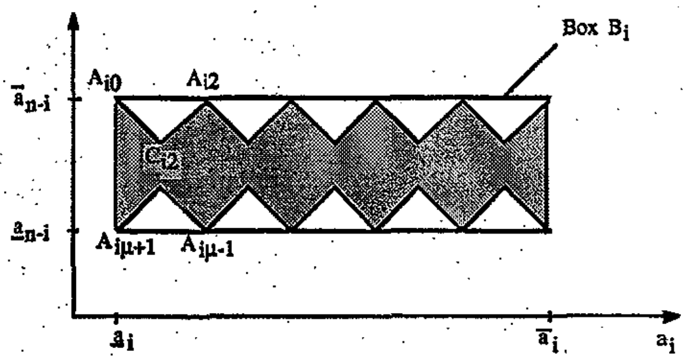

Figure 11.: Construction of necessity test.

The important advantage over the earlier sufficiency condition, depicted with Figure 5, is that no longer is it necessary to test for stability at certain points inside the rectangle $\underline{a}_i \le a_i \le \bar{a}_i$, $\underline{a}_{n-i} \le a_{n-i} \le \bar{a}_{n-i}$, the points labelled A_{i4}, A_{i6}, A_{i8} and A_{i10} in Figure 5 for example.

Once again, the unshaded part of box B_i in Figure 11 represents the extent to which the necessity condition fails to be sufficient. Likewise in Figure 10, the area defined by the points A_{ij} outside box B_i measures the extent to which the sufficiency condition fails to be necessary.

We can reduce the effort necessary for stability checking of all corner point combinations by combining the ideas of Theorem 10 with those presented in [14]. For simplicity we give the following specialization of a result of [14]:

Corollary 11: For the stability of the set f(z) of (3.1), it is necessary and sufficient that one has stability for all polynomials for which $a_i \in [\underline{a}_i, \bar{a}_i]$, $a_j \in \{\underline{a}_j, \bar{a}_j\}$ $\forall\, j \neq i$, $i = 0,1,\ldots,n$

Consider Figure 10. For robust stability it is therefore sufficient to check the stability of the edges of B_i for all combinations of corner points of B_j, $j \neq i$. But due to [3], stability of the vertical edges follows from the stability of the corresponding endpoints. We therefore need to check only the stability of the horizontal edges. However, by Corollary 7, we can easily check the stability of these lines. Moreover, we can substantially reduce the number of necessary stability tests in the following way:

Theorem 12: Let $\mathcal{F}_i$ describe the outside corner points in the plane (a_i, a_{n-i}) of Figure 10, i.e.

$$\mathcal{F}_i = \{A_{i0}, A_{i1}, A_{i3}, \ldots, A_{i\mu-2}, A_{i\mu}, A_{i\mu+1}\}$$

and $\mathcal{H}_i$ analogously for Figure 11.
Then the stability of all polynomials for which

$$a_j \in \{\underline{a}_j, \bar{a}_j\} \qquad j \neq i,\ j \neq n-i$$

and

$$(a_i, a_{n-i}) \in \mathcal{F}_i \qquad i = 0,1,\ldots,n \lfloor 2 \tag{3.4}$$

is sufficient for the stability of all polynomials f(z) of the set (3.1).

A corresponding necessary condition for stability of f(z) follows for (a_i, a_{n-i}) to be drawn from $\mathcal{H}_i$ instead of $\mathcal{F}_i$ (3.4).

Proof: Suppose the stability of $A_{ij} \in \mathcal{F}$ (with all possible combinations of corner points of B_j, $j \neq i$). Then with [3] stability of the line $A_{i1}A_{i\mu}$ follows. The edge $A_{i2}A_{i3}$ is a part of the line $C_{i2}A_{i3}$. From the Proposition 8 and the stability of points C_{i2} and A_{i3} stability of A_{i2} results. Analogously we proceed for $A_{i\mu-1}$ and then repeat the argument for the line $A_{i3}A_{i\mu-2}$ with A_{i4} and $A_{i\mu-3}$ and so on. This proves the theorem.

As pointed out in [14], root locus ideas can be used also to check the stability of a one-dimensional affine family of polynomials $f_1(z) + \nu f_2(z)$. Alternatively working with say the Schur-Cohn determinants, one requires positivity of a finite collection of polynomials in ν for $\nu \in [\nu_1, \nu_2]$ (each polynomial corresponding to one of the determinants). Positivity can be checked for example using Sturm's theorem, which involves only a finite number of rational calculations.

IV. Conclusions

By seeking to parallel the arguments used in proving Kharitonov's theorem for continuous-time robust polynomial stability, we have presented a necessary and sufficient condition for a form of discrete-time robust polynomial stability which does not involve coefficient variations in rectangular boxes parallel to the axes. However, for such regions, separate necessity and sufficiency conditions for stability can be derived, involving only a finite number of polynomials. These results can be improved on by invoking another result on discrete-time robust stability, in which only one half of the coefficients are allowed to vary. (This result incidentally can be regarded as appealing to discrete-time lossless positive real functions also).

The number of corner points and thus polynomials whose stability has to be checked grows exponentially with the polynomial degree, in contrast to the continuous-time case. This is certainly a disadvantage.

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