Correspondence

Corrections to "Comment on 'Algorithm for Construction of Variable Length Code with Limited Maximum Word Length' "

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The above correspondence[1] contains some errors due to an incomplete editorial change of a reference into a footnote. The following corrections should be made:

- In the sixth line of Section II: "As in the correspondence [1]" should become "As in the correspondence.[1]"
- In the caption for Fig. 2: "the algorithm in [1]" should become "the algorithm in.[1]"
- In the header of the third column of Table I: "Algorithm [2]" should become "Algorithm [1]."
- In the header of the fourth column of Table I: "Algorithm [1]" should become "Algorithm.[1]"
- In the header of the third column of Table II: "Algorithm [2]" should become "Algorithm [1]."

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[1] J. De Lameillieure and I. Bruyland, "Comment on 'Algorithm for construction of variable length code with limited maximum word length'," *IEEE Trans. Commun.*, vol. COM-34, pp. 1252-1253, Dec. 1986.

Error Recovery of Decision Feedback Equalizers on Exponential Impulse Response Channels

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Abstract—**In this correspondence we determine an upper bound on the error recovery time of a decision feedback equalizer (DFE) operating on a high signal to noise ratio channel of exponential impulse response to be used for binary transmission. This bound is related to the channel time constant and we show that exponential impulse response channels form a favourable class of channels with regard to DFE error recovery properties.**

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I. Introduction

The problem of error recovery and error probability of decision feedback equalizers (DFE's) has been the subject of a number of papers [1]–[8]. The main difficulty with the analysis of error properties of DFE's has been the inclusion in the analysis of the effects of error propagation which arise because the DFE uses past, possibly incorrect decisions in cancelling intersymbol interference (ISI). The best known results which incorporate error propagation are those of Duttweiler *et al.* [1] where bounds on error probability are given. (More recently this style of analysis was extended by O'Reilly *et al.* [5], [6].) These results are closely related to error recovery time bounds given by Cantoni *et al.* [2] as was demonstrated in [4]. All these bounds *appear* to be conservative, and thus from a practical viewpoint appear to be pessimistic. Also they do not seem to reflect the good performance of DFE's observed in practice.

Recently, it was shown [3], [4] that the various bounds given in [1], [2], and [6] are in fact tight in the sense that there do exist channels where the actual error performance coincides with the bounds. It then becomes important to establish for what kind of channels error performance much better than these bounds can be achieved, i.e., we must identify stronger hypotheses on the type of channels used in the theoretical analysis which both reflect properties of real channels and give attractive error performance. This technical note represents an extreme case of this approach where we consider only channels with an exponential impulse response. We also restrict ourselves to studying high signal to noise ratio channels because these are of greatest practical interest and also because this assumption simplifies the analysis.

We define the *channel recovery time* as the maximum time taken for the eye pattern to open when initially closed. (The maximum is over all initial states and all input sequences, and is computed assuming no noise.) This definition differs slightly from that adopted in [2] and [3] where only finite impulse response models for the channel were considered. The distinction is not significant. The specific problem we address is: Given an initial state where the eye is closed, then *how long* do we have to wait before we are *guaranteed* the eye has opened and will remain open? This channel recovery time that we derive forms an upper bound on the recovery time for any particular error.

II. Problem Formulation

We begin with some definitions. A communication channel $H(z)$ will be represented as an infinite impulse response (IIR) filter

$$H(z)=h_0+h_1z^{-1}+h_2z^{-2}+\cdots+h_nz^{-n}+\cdots$$

where z is the z transform variable. As a special case of this model set, we consider the *exponential impulse response channel* where the h_i's are given by

$$h_i=h_0\alpha^i; \quad i=0, 1, 2, \cdots \tag{2.1}$$

with $0<\alpha<1$. Note that the transfer function associated with (2.1) is a proper first-order system, being given by $h_0z(z-\alpha)^{-1}$.

Because the channel is of first order it turns out that only a single state variable will be required to describe the dynamics of a DFE on such a channel.

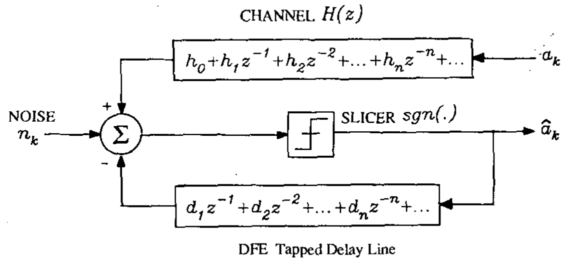

Fig. 1. Model of channel and decision feedback equalizer.

The overall system that we consider is depicted in Fig. 1. Binary data a_k is sent down the channel $H(z)$ and is corrupted by channel noise n_k and intersymbol interference (ISI). A DFE attempts to invert the channel by modeling and then cancelling the ISI using past decisions $\hat{a}_k$. These past decisions need not be correct and therefore *error propagation* becomes an important component in the analysis of the performance of DFE's. Our analysis will consider a worst case error propagation situation.

We make the following simplifying assumptions for our analysis:

A1: The DFE feedback tap weights d_i (Fig. 1) correspond to the desired channel values, i.e., adaptation has taken place and ideally we have $d_i = h_i$ for all $i \in \mathbb{Z}^+$.

A2: The time scale of error recovery (error propagation) is sufficiently separated from that of any adaptation process; thus even if the DFE is adaptive, we may take the DFE feedback taps d_i as being *fixed* at h_i for our recovery analysis.

A3: The channel noise may cause an initial error state but during the subsequent error recovery is *negligible.* This effectively is a high signal to noise ratio assumption.

From Fig. 1 the output of the DFE slicer (sgn($\cdot$) function) is given by

$$\hat{a}_k = \text{sgn}\left(h_0 a_k + \sum_{i=1}^{\infty} (h_i a_{k-i} - d_i \hat{a}_{k-i}) + n_k\right).$$

Then with assumptions A1)–A3) and the exponential channel values (2.1), this equation reduces to

$$\hat{a}_k = \text{sgn}\left(h_0 a_k + h_0 \sum_{i=1}^{\infty} \alpha^i e_{k-i}\right); \quad e_{k-i} \triangleq a_{k-i} - \hat{a}_{k-i} \tag{2.2}$$

where $e_{k-1} \in \{-2, 0, +2\}$ form the decision errors.

We define a state variable S_k (or the "residual ISI") via

$$S_k \triangleq h_0 \sum_{i=1}^{\infty} \alpha^i e_{k-i}. \tag{2.3}$$

Then it is a simple algebraic exercise to verify that S_k satisfies the key recursion

$$S_{k+1} = \alpha S_k + h_0 \alpha e_k. \tag{2.4}$$

This result is analogous to the similar result derived by Zador [7] which also treats the exponential impulse response channel. These early results were subsequently simplified and an approximation derived for the error rate enhancement due to error propagation for exponential impulse response channels) [8]. [7] is the earliest known to the authors which uses the theory of Markov chains to analyze DFE's; a mathematical tool common to more recent papers treating error properties of DFE's [1]–[6]. Our techniques are essentially deterministic and we do not require the use of Markov chains.

We make two observations based on (2.2)–(2.3) regarding the possible values taken by the decision $\hat{a}_k$, given the state S_k and the data a_k:

B1: When $a_k = \text{sgn}\, S_k$, the correct decision is always made, i.e., $\hat{a}_k = a_k$.

B2: When $a_k = -\text{sgn}\, S_k$, a correct decision will be made if and only if $|S_k| < h_0$. When $|S_k| < h_0$ we refer to the eye as being *open* (and *closed* otherwise). Only with the eye open can error-free transmission be guaranteed.

We can now formulate the problem we posed earlier in terms of the residual ISI S_k defined in (2.3).

Let $|S_0| > h_0$ (initially the eye is closed). Then i) for all $\{a_i\}$, $i \geq 0$ (input sequences), ii) for all e_i, $i < 0$ (initial error states consistent with $|S_0| > h_0$), find a K such that for all $k \geq K$ there holds $|S_k| < h_0$ (the eye remains open).

Our solution can be related to a time constant associated with the channel.

III. Upper Bound of the Error Recovery Time

We *define* the time constant of the channel as the number J such that $\alpha^J = 0.01$. Here

$$J \triangleq \frac{\ln(0.01)}{\ln \alpha}. \tag{3.1}$$

Next we determine a bound on the time for the eye to open when initially closed also in terms of α. Without loss of generality, we take the initial error state as being *positive* and nondegenerate (i.e., the eye closed). We write this as $S_0 > h_0$.

The following properties are easily demonstrated from (2.2)–(2.4), our previous observations B1 and B2, and our positivity assumption on S_0:

P1: When $a_k = +1$ then $S_{k+1} = \alpha S_k$ and this implies the value of the state decreases and remains positive (and $\hat{a}_k = a_k$).

P2: When $a_k = -1$ then

$$S_{k+1} = \begin{cases} \alpha S_k - 2\alpha h_0 & (\text{with } \hat{a}_k \neq a_k) \quad \text{if } S_k > h_0 \\ \alpha S_k & (\text{with } \hat{a}_k = a_k) \quad \text{if } S_k < h_0 \end{cases}$$

and this implies the value of the state decreases and remains positive, or, if $S_k < 2h_0$, that $|S_{k+1}| < h_0$, leading to $|S_{k+i}| = \alpha |S_{k+i-1}|$ for $i > 1$.

Therefore, irrespective of the sign of the data a_k, $\{|S_k|\}$ tends toward 0 at least as fast as α^k. Also it is clear from P1 and P2 that once the eye opens it remains open. Therefore, the desired upper bound on the error recovery time K simply corresponds to the *first* time instant that the eye opens. Surprisingly from P2, we note that $\{|S_k|\}$ decreases faster, i.e., the DFE recovers faster, when we make *incorrect* decisions compared to *correct* decisions.

Next we bound the possible size of S_0, i.e., we find the worst case initial error state. Equation (2.3) shows that it will be maximal when all past decisions are in error and of the same sign (a most unlikely situation). Therefore, letting $e_{k-i} = 2$ for all $i \in \mathbb{Z}^+$ in (2.3) yields

$$S_0 \leq 2h_0 \sum_{i=1}^{\infty} \alpha^i = \frac{2\alpha}{1-\alpha} h_0$$

(Note, for nondegeneracy, i.e., $S_0 > h_0$ we need $\alpha > 1/3$). Then the desired bound K is obtained by examining the worst case recovery (worst case input sequence), consisting of a succession of correct decisions where $S_{k+1} = \alpha S_k$ starting from the upper limit on S_0. Hence the desired K satisfies (such a K can always be found since $0 < \alpha < 1$)

$$S_K = \alpha^K S_0 \leq \alpha^K \frac{2\alpha}{1-\alpha} h_0 < h_0. \tag{3.2}$$

The strict inequality in (3.2) yields

$$2\alpha^{K+1} < 1 - \alpha.$$

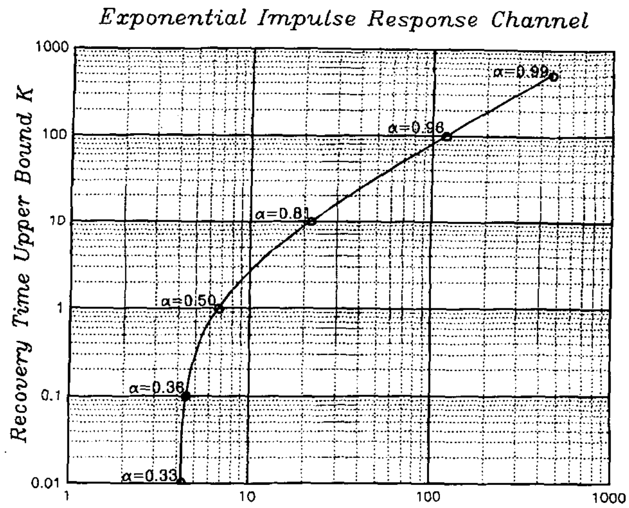

Fig. 2. Recovery time upper bound versus channel time constant.

Therefore, our upper bound K satisfies

$$K > \frac{\ln (1-\alpha) - \ln 2}{\ln \alpha} - 1. \tag{3.3}$$

This completes our analysis upon comparing K in (3.3) to J in (3.1). Fig. 2 shows a plot of K versus J for α ranging from 0.33 to 0.99. We conclude from this figure that for exponential impulse response channels where $\alpha < 0.99$, the time required for error recovery is less than the order of a time constant that we associated with the channel. (Fig. 2 also shows the recovery performance improves as α is decreased, as one would expect).

Remarks

1) Assumptions on the data a_k regarding whiteness and the probability distribution are not required in the above derivation in contrast to [1]–[4], [7], and [8]. Therefore, the above result holds in a general setting where the data may be correlated, etc.

2) The same bound K is valid also for $-1 < \alpha < 0$ (which is a most unlikely situation practically). In this case $\{|S_k|\}$ also forms a monotonically decreasing sequence.

3) Let the DFE tapped delay line be represented by a finite impulse response model with N taps (rather than an IIR filter). Further, assume $d_i = h_i$ for $i = 1, 2, \cdots, N$ along with (2.1) and our previous definition for S_k (2.3). Then the unmodeled channel impulse response values contribute at most $\alpha^{N+1}(1 - \alpha)^{-1}h_0$ to the ISI and the modifications to the analysis are straightforward, given sufficiently large N.

4) The maximum error recovery bound for multilevel transmission with symbols $\{-M + 1, -M + 3, \cdots, M - 1\}$ can be shown to be $(M - 1)$ times the result for the binary case (3.3).

IV. Conclusions

A DFE operating on a high signal to noise ratio exponential impulse response channel used for binary transmission has a maximum (deterministic) error recovery time of the order of a time constant that we can associate with the channel. A qualitatively similar result can be derived for multilevel transmission. This implies in *practical* terms that DFE's will operate satisfactorily on such a restricted class of channels, contrary to be general class of channels considered in [1]–[6]. This is an encouraging result and is a pointer to the future successful unification of practical and theoretical results by imposing strong constraints on the class of channels to be included in the error recovery analysis. Further investigations aimed at expanding the class of channels for which firm results are available are presently in hand.

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Extension to the Rude-CSMA Analysis

ROBERT E. FELDERMAN

Abstract—**This paper extends the rude-CSMA analysis by removing the restriction that each node have the same average arrival rate and average service time. Additionally, some minor errors are corrected.**

Introduction

We extend the analysis of the rude-CSMA algorithm [1] to allow each node to have different average arrival rates γ_i and different average service times μ_i.

In the Appendix of the paper [1], the authors show that the Markov process defined by the two rate equations (1) and (2) is a reversible process, and then use this fact to find the equation for $\Pi(S, x, y)$. Since we now have different rate equations than in the paper, we will show that this new process is reversible and will derive a new equation for $\Pi(S, x, y)$.

Our new rate equations are

$$r_1^i(S(t)) = \mu_i \tag{1'}$$

and

$$r_0^i(S(t)) = \gamma_i x^{N_0^i(S(t))} y^{N_1^i(S(t))}. \tag{2'}$$

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