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Brief Paper

On Model Reduction of Discrete Time Systems*

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Key Words—Stability; model reduction; root property; discrete Schwartz form; lattice realization; order approximation.

Abstract—In this paper, further justification of model reduction by Badreddin–Mansour's method is presented. In particular, connections between polynomial root-location properties, coefficient properties and Schur–Cohn coefficient properties are established. Lattice realizations of a discrete system and its reduced model are given, which illustrate certain relationships between the original model and its reduced form. These relationships give insight into the validity of the method of reduction and offer insight on the order of approximation.

1. *Introduction*

IT IS DESIRABLE for realization, control, design, computation and other purposes to represent adequately, and without sacrificing the important characteristics, a high order system by a lower order system.

Model reduction of one-dimensional scalar discrete systems has been extensively discussed in the literature. A survey of the various methods of model reduction has been given by Bosley and Lees (1972). Furthermore, model reduction of multivariable (multi-input/multi-output) one-dimensional discrete systems has been the subject of recent investigations, for example (Bistritz and Shaked, 1984; Badreddin and Mansour, 1983 and 1984).

In the order reduction method of Badreddin and Mansour (1980) and Badreddin (1982), both the stability and the correctness of the steady state response of the reduced model are guaranteed if the original model is stable. The method is based on utilizing the discrete Schwartz form, which has come to be also known as the Mansour form (Dourdomas, 1980). Representation of discrete systems using the discrete Schwartz form is also presented in a paper of Takizawa *et al.* (1982). As indicated therein, this form of realization has certain useful features.

In this paper, the order reduction method of Badreddin and Mansour is further investigated and certain extensions are obtained. The composition of the paper is as follows.

In Section 2, connections between polynomial root location properties, polynomial coefficient properties and Schur–Cohn coefficient properties are established. (Such connections are used later to define the range of validity of the reduction method.)

In Section 3, a lattice realization of a discrete system in terms of Schur–Cohn coefficients is recalled. Such realizations are important in digital filter analysis and design. Also in this section, the lattice realization of the reduced model is given and is connected with the Badreddin and Mansour reduction formulas.

In Section 4, certain new relationships between the original and the reduced model are derived.

In Section 5, the ideas of the previous three sections are tied together so as to better define the range of applicability of the Badreddin and Mansour reduction method.

Section 6 contains concluding remarks.

2. *Connections between the roots, coefficients and Schur–Cohn coefficients of a polynomial.*

In this section, important relationships between the roots, coefficients and Schur–Cohn coefficients of a polynomial will be established. These relationships are very useful in studying the use of the Badreddin and Mansour method of model reduction, and in a later section, they will be used to clarify the range of validity of the method.

2.1. *Connection between the roots and coefficients.* Let

$$F(z) = \sum_{i=0}^{n} a_{n-i} z^i, \qquad a_0 \equiv 1. \tag{1}$$

Suppose there are *precisely* r roots of $F(z) = 0$, viz. $z_1, z_2, \ldots, z_r$, for which

$$|z_i| < \varepsilon \qquad i = \overline{1, r} \text{ and some } 0 < \varepsilon \ll 1, \tag{2a}$$

while the remaining roots are $O(1)$: more precisely, suppose that

$$\varepsilon^{-1} \gg |z_{r+1} z_{r+2} \ldots z_n| \gg \varepsilon. \tag{2b}$$

Of course, by requiring the root separation property of (2), the coefficients a_i in (1) necessarily must be constrained in some way. To understand this constraint, $F(z)$ is written in terms of the roots as follows:

$$F(z) = (z - z_1)(z - z_2) \ldots (z - z_r)(z - z_{r+1}) \ldots (z - z_n). \tag{3}$$

Equating coefficients,

$$\left.\begin{aligned}
|a_n| &= \left|\prod_{i=1}^{r} z_i \prod_{i=r+1}^{n} z_i\right| \leqslant K\varepsilon^r \\
|a_{n-1}| &= |0(\varepsilon^r) + 0(\varepsilon^{r-1})| \leqslant K\varepsilon^{r-1} \\
&\vdots \\
|a_{n-r+1}| &= \qquad\qquad\qquad \leqslant K\varepsilon \\
|a_{n-r}| &= \qquad\qquad\qquad \leqslant K \\
|a_1| &= \qquad\qquad\qquad \leqslant K
\end{aligned}\right\} \tag{4a}$$

while (2b) ensures also that

$$|a_{n-r}^{-1}| \leqslant K. \tag{4b}$$

In (4), the constant K satisfies $\varepsilon \leqslant K \leqslant \varepsilon^{-1}$. Note that a single K can be found for each inequality in (4) by choosing the largest value, if the various K are initially different.

Thus, a root property has been translated into a property of

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the coefficients. To establish the converse (in fact an almost-converse), proceed as follows:

assume $\varepsilon \ll 1$ and (4), and let

$$\begin{aligned} p(z) &= z^n + a_1 z^{n-1} + \cdots + a_{n-r} z^r \\ &= z^r[z^{n-r} + a_1 z^{n-r-1} + \cdots + a_{n-r}] \end{aligned} \tag{5}$$

and

$$q(z) = a_{n-r+1} z^{r-1} + \cdots + a_n. \tag{6}$$

Let $\alpha \in (0, 1)$ and consider the behaviour of $p(z)$ and $q(z)$ on the circle $|z| = \varepsilon^\alpha < 1$. The polynomial $z^{n-r} + a_1 z^{n-r-1} + \cdots + a_{n-r}$ has z in every term except the last one, i.e. a_{n-r}; then (4) implies

$$\begin{aligned} |z^{n-r} + \cdots + a_{n-r}| &\leqslant K' \qquad \text{on } |z| = \varepsilon^\alpha \\ |z^{n-r} + \cdots + a_{n-r}|^{-1} &\leqslant K' \qquad \text{on } |z| = \varepsilon^\alpha \end{aligned} \tag{7a}$$

or

$$|p(z)| \leqslant K' \varepsilon^{\alpha r} \qquad \text{on } |z| = \varepsilon^\alpha \tag{8a}$$

and

$$\varepsilon^{\alpha r} |p(z)|^{-1} \leqslant K' \qquad \text{on } |z| = \varepsilon^\alpha. \tag{8b}$$

Here K' is a constant of the same order as K, and in particular $\varepsilon \ll K' \ll \varepsilon^{-1}$. Similarly, from (6),

$$\begin{aligned} |q(z)| &\leqslant K\varepsilon\varepsilon^{\alpha(r-1)} + K\varepsilon^2\varepsilon^{\alpha(r-2)} + \cdots + K\varepsilon^r \\ &\leqslant K'(\varepsilon^{\alpha r + 1 - \alpha}). \end{aligned} \tag{9}$$

Thus, for sufficiently small ε, and α not equal to unity, it follows that $\varepsilon^{(1-\alpha)} \ll 1$, and noting (8), (9)

$$|q(z)| < |p(z)|. \tag{10}$$

Now

$$F(z) = p(z) + q(z) \tag{11}$$

and so by Rouche's theorem it follows that $F(z)$ and $p(z)$ will have the same number of roots inside $|z| < \varepsilon^\alpha$. Now the choice of $p(z)$ in (5) ensures that $p(z)$ has *precisely* r zeros inside $|z| < \varepsilon^\alpha$, so that $F(z)$ will also have r roots in $|z| < \varepsilon^\alpha$. This is the almost-converse of the coefficient property derived above from a root property.

2.2. *Connection between the coefficients and Schur–Cohn coefficients.* Construct the Marden–Jury table for the polynomial $F(z) = z^n + a_1 z^{n-1} + \cdots + a_n$ as follows:

$$\begin{array}{lllll} F_n(z) & 1 & a_1 & \cdots a_{n-1} & a_n \\ & a_n & a_{n-1} & \quad a_1 & \\ F_{n-1}(z) & b_0 & b_1 & \cdots b_{n-1} & \\ & \vdots & \vdots & \quad \vdots & \end{array} \tag{12}$$

Thus

$$F_{n-1}(z) = \sum_{i=0}^{n-1} b_i z^{n-1-i}, \tag{13}$$

where

$$b_0 = 1, \qquad b_i = \frac{a_i - a_n a_{n-i}}{1 - a_n^2}. \tag{14}$$

Suppose that

$$\left.\begin{aligned} &|a_n| \leqslant K\varepsilon^r \\ &|a_{n-1}| \leqslant K\varepsilon^{r-1} \\ &|a_{n-r}| \leqslant K, \ldots, |a_1| \leqslant K \text{ and } |a_{n-r}|^{-1} \leqslant K \end{aligned}\right\} \tag{15}$$

with $\varepsilon \ll K \ll \varepsilon^{-1}$. From (14) and (15) it follows that

$$|b_{n-1}| \leqslant K'\varepsilon^{r-1}. \tag{16}$$

Similarly,

$$\begin{aligned} &|b_{n-2}| \leqslant K'\varepsilon^{r-2} \\ &\vdots \\ &|b_{n-r+1}| \leqslant K'\varepsilon \\ &|b_{n-r}| \leqslant K' \qquad |b_{n-r}|^{-1} \leqslant K'. \end{aligned} \tag{17}$$

Here, $\varepsilon \ll K' \ll \varepsilon^{-1}$. This argument can clearly be repeated to obtain bounds on the coefficients of $F_{n-2}(z)$.

Note that the Schur coefficients Δ_i are given by

$$\Delta_n = a_n, \qquad \Delta_{n-1} = b_{n-1}, \qquad \ldots . \tag{18}$$

Therefore, for some $\bar{K}$

$$\begin{aligned} &|\Delta_n| \leqslant \bar{K}\varepsilon^r \\ &|\Delta_{n-1}| \leqslant \bar{K}\varepsilon^{r-1} \\ &\vdots \\ &|\Delta_{n-r+1}| \leqslant \bar{K}\varepsilon \\ &|\Delta_{n-r}| \leqslant \bar{K} \qquad |\Delta_{n-r}|^{-1} \leqslant \bar{K}. \end{aligned} \tag{19}$$

Therefore, the coefficient property implies conditions on the Schur–Cohn coefficients. The converse has been established by Mansour (1965).

Example. To substantiate the above relationships, the following example is presented. Let

$$\begin{aligned} F(z) &= (z - 0.001)(z - 0.6)(z - 0.8) \\ &= z^3 - 1.401z^2 + 0.4814z - 0.00048. \end{aligned} \tag{20}$$

In this case,

$$\varepsilon = 0.0011, \qquad r = 1, \qquad n = 3. \tag{21}$$

From (4),

$$\begin{aligned} |a_3| &= 0.00048 \\ |a_2| &= 0.4814 \\ |a_1| &= 1.401, \end{aligned} \tag{22}$$

which satisfies (4) with $K \simeq 2$. From (5),

$$\begin{aligned} p(z) &= z^3 - 1.401z^2 + 0.4814z \\ &= z[z^2 - 1.401z + 0.4814] \\ &= z[(z - 0.7969)(z - 0.604)]. \end{aligned} \tag{23}$$

Now, $z = 0$ is the only root of $p(z) = 0$ inside $|z| = \varepsilon^{0.99} \simeq 0.0012$ just as the number of roots of $F(z) = 0$ inside $|z| = 0.0012$ is only one. This verifies the connection between the coefficients and root property.

Construct the Marden–Jury table as follows:

$$\begin{array}{rllll} & 1 & -1.401 & 0.4814 & -0.00048 \\ \Delta_3 \leftarrow & 0.00048 & 0.4814 & 1.401 & 1 \\ \hline & 1 & -1.4008 & 0.4807 & \\ \Delta_2 \leftarrow & 0.4807 & -1.4008 & 1 & \\ \hline & 1 & -0.9460 & & \\ \Delta_1 \leftarrow & -0.9460 & 1 & & \end{array}$$

Hence,

$$\begin{aligned} |\Delta_3| &= 0.00048 \\ |\Delta_2| &= 0.4807 \\ |\Delta_1| &= 0.9460. \end{aligned}$$

This agrees with inequality (19). Had one begun with the Δ_i, the formulas of Mansour (1965) would have been used, viz.

$$|a_1| = |\Delta_1 + \Delta_1\Delta_2 + \Delta_2\Delta_3|$$
$$|a_2| = |\Delta_2 + \Delta_1\Delta_3 + \Delta_1\Delta_2\Delta_3|$$
$$|a_3| = |\Delta_3|$$

to obtain information about the coefficient sizes.

3. Lattice realization of the reduced order Badreddin–Mansour model

The lattice realization of digital filters is important in applications for several reasons, including low coefficient sensitivity, non-occurrence of zero input limit cycle in one's complement truncation (Takizawa *et al.*, 1982), and applicability in spectral analysis problems (Jones *et al.*, 1985). In this section, the lattice realization of the original model is presented, using the discrete Schwartz form and from it, a reduced model is obtained and shown to be exactly the mathematical model obtained by Badreddin and Mansour (1980) and Badreddin (1982).

Consider the original system presented in state space formulation based on the discrete Schwartz form:

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ \vdots \\ x_{n-1}(k+1) \\ x_n(k+1) \end{bmatrix} = \begin{bmatrix} -\Delta_1 & (1-\Delta_1^2) & 0 & \cdots & 0 \\ -\Delta_2 & -\Delta_1\Delta_2 & (1-\Delta_2^2) & & \\ \vdots & & & & \\ -\Delta_{n-1} & -\Delta_1\Delta_{n-1} & & & (1-\Delta_{n-1}^2) \\ -\Delta_n & -\Delta_1\Delta_n & -\Delta_2\Delta_n & \cdots & -\Delta_{n-1}\Delta_n \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ \vdots \\ x_{n-1}(k) \\ x_n(k) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} u_1(k) + \begin{bmatrix} \Delta_1 \\ \Delta_2 \\ \vdots \\ \Delta_{n-1} \\ \Delta_n \end{bmatrix} u_2(k) \qquad (24)$$

$$y(k) = [h_1 \quad h_2 \ldots h_{n-1} \quad h_n] \begin{bmatrix} x_1(k) \\ \vdots \\ x_n(k) \end{bmatrix} + e_1 u_1(k) + e_2 u_2(k). \qquad (25)$$

Badreddin and Mansour (1980) and Badreddin (1982) consider two different scalar approximation problems, defined by taking one of $u_1(k)$ and $u_2(k)$ identically zero in (24). It is convenient here for both problems to be considered simultaneously.

It is shown by Badreddin (1982) that the above, after model reduction by order 1, gives:

$$\begin{bmatrix} \hat{x}_1(k+1) \\ \vdots \\ \hat{x}_{n-1}(k+1) \end{bmatrix} = \begin{bmatrix} -\Delta_1 & (1-\Delta_1^2) & & & \\ & & 0 & & \\ & & & & (1-\Delta_{n-2}^2) \\ -\Delta'_{n-1} & -\Delta_1\Delta'_{n-1} & -\Delta_2\Delta'_{n-1} & \cdots & -\Delta_{n-2}\Delta'_{n-1} \end{bmatrix} \begin{bmatrix} \hat{x}_1(k) \\ \vdots \\ \hat{x}_{n-1}(k) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \hat{u}_1(k) + \begin{bmatrix} \Delta_1 \\ \vdots \\ \Delta_{n-1} \end{bmatrix} \hat{u}_2(k), \qquad (26)$$

where

$$\Delta'_{n-1} = \frac{\Delta_n + \Delta_{n-1}}{1 + \Delta_n\Delta_{n-1}} \qquad (27)$$

and

$$\hat{y}(k) = [h'_1 \quad h'_1 \ldots h'_{n-1}] \begin{bmatrix} \hat{x}_1(k) \\ \vdots \\ \hat{x}_{n-1}(k) \end{bmatrix} + e'_1\hat{u}_1(k) + e'_2\hat{u}_2(k), \qquad (28)$$

where

$$h'_i = h_i - \left(\frac{\Delta_n\Delta_{i-1}}{1 + \Delta_n\Delta_{n-1}}\right)h_n,$$
$$e'_1 = e_1, \qquad (29)$$
$$e'_2 = e_2 + \left(\frac{\Delta_n h_n}{1 + \Delta_n\Delta_{n-1}}\right)$$

for $i = 1, 2, \ldots, n-1$ and $\Delta_0 = 1$.

For the system of equations given in (24) and (25) it can be readily shown that Fig. 1 presents its digital filter realization.

To obtain a realization of the reduced model given in (26) through (28), one simply replaces the left-most delay in Fig. 1 by a unity-gain element. The result is drawn in Fig. 2.

This assertion is now verified. From Fig. 2,

$$z(k) = -(\Delta_n + \Delta_{n+1})[v(k) + w(k)] \qquad (30)$$

and

$$w(k) = -\Delta_n\Delta_{n-1}[v(k) + w(k)]. \qquad (31)$$

It follows that

$$z(k) = -\Delta'_{n-1}v(k). \qquad (32)$$

Also from Fig. 2 and from (31)

$$t(k) = -\Delta_n[v(k) + w(k)] = -\frac{\Delta_n}{1 + \Delta_n\Delta_{n-1}}v(k). \qquad (33)$$

Thus,

$$y(k) = \sum_{i=1}^{n-1} h_i x_i(k) + h_n t(k) + e_1 u_1(k) + e_2 u_2(k). \qquad (34)$$

Now, from Fig. 1,

$$v(k) = \Delta_{n-2}x_{n-1}(k) + \Delta_{n-3}x_{n-2}(k) + \cdots + \Delta_1 x_2(k) + x_1(k) - u_2(k). \qquad (35)$$

From (33) and the above,

$$h_n t(k) = -\frac{h_n\Delta_n}{1 + \Delta_n\Delta_{n-1}}[x_1(k) + \Delta_1 x_2(k) + \cdots + \Delta_{n-2}x_{n-1}(k)] + \frac{h_n\Delta_n}{1 + \Delta_n\Delta_{n-1}}u_2(k). \qquad (36)$$

Substituting (36) in (34), finally gives,

$$y(k) = \sum_{i=1}^{n-1} h_i x_i(k) - \frac{h_n\Delta_n}{1 + \Delta_n\Delta_{n-1}}\sum_{i=1}^{n-1}\Delta_{i-1}x_i(k) + e_1 u_1(k) + \left(e_2 + \frac{h_n\Delta_n}{1 + \Delta_n\Delta_{n-1}}\right)u_2(k) \qquad (37)$$

or

$$y(k) = \sum_{i=1}^{n-1} h'_i x_i(k) + e'_1 u_1(k) + e'_2 u_2(k), \qquad (38)$$

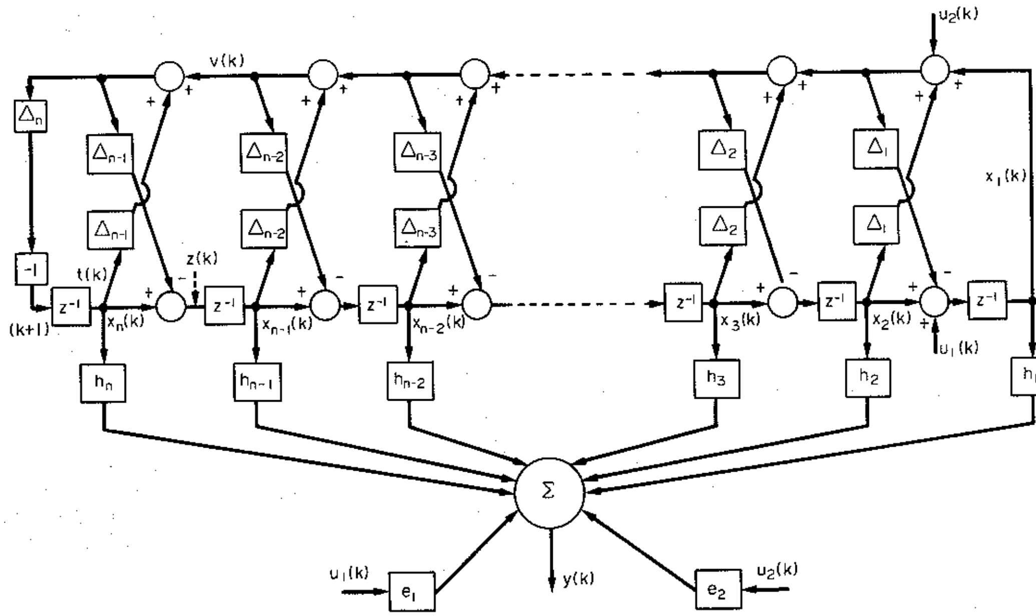

FIG. 1. Lattice realization of the discrete system.

where

$$h_i' = h_i - \frac{\Delta_n \Delta_{i-1}}{1 + \Delta_n \Delta_{n-1}} h_n,\ e_1' = e_1,$$

$$e_2' = e_2 + \frac{h_n \Delta_n}{1 + \Delta_n \Delta_{n-1}}\ i = 1, \ldots, (n-1),\ \Delta_0 = 1. \quad (39)$$

Comparing (26)–(29) with (38) and (39), it is verified that the realization of Fig. 2 corresponds to the reduced system.

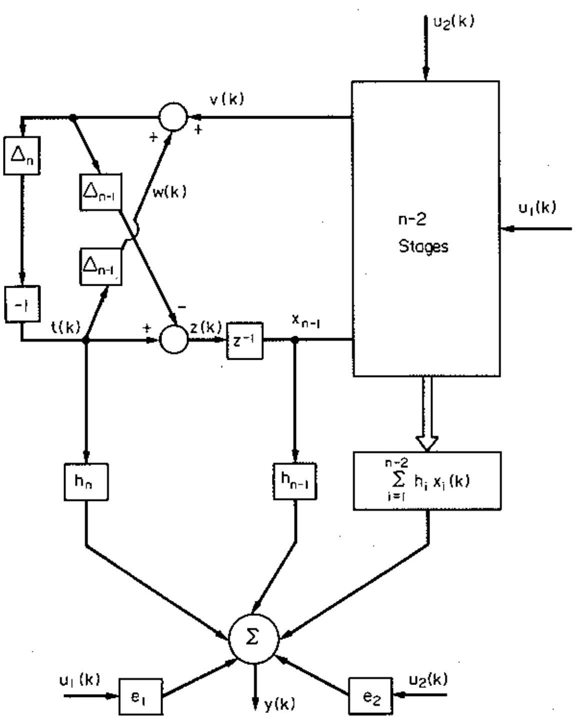

FIG. 2. Lattice realization of the reduced system.

4. *Connections between the original and the reduced system transfer functions*

In this section, certain relationships between the original and the reduced system transfer functions are stated. Such relationships will give indications concerning the error of the approximation at $z = 1$ and more generally at $z = e^{j\omega}$.

Let $W(z)$ and $V(z)$ be the transfer functions of the original and the reduced systems. (The pair obtained by taking $u_2(k) \equiv 0$ in (24) and (25) will be considered, as well as the pair obtained with $u_1(k) \equiv 0$ in (24) and (25).) Let Δ_n in (24) and (25) be a variable parameter, then the transfer functions can be written as $W(z, \Delta_n)$ and $V(z, \Delta_n)$.

Claim 1. $W(z, 0) = V(z, 0)$ for both forms of input coupling vector. This claim follows by straightforward linear algebra. (40)

Claim 2.

$$\left.\frac{\partial W}{\partial \Delta_n}(1, \Delta_n)\right|_{\Delta_n = 0} = \left.\frac{\partial V}{\partial \Delta_n}(1, \Delta_n)\right|_{\Delta_n = 0} \quad \text{for both forms of input coupling vector.} \quad (41)$$

Again, this claim follows by algebraic manipulation. The major steps for one of the two cases are outlined, viz. when the input coupling vector is

$$g = [1, 0, \ldots, 0]^{\mathrm{T}}. \quad (42)$$

Now

$$\frac{\partial W}{\partial \Delta_n} = [h_1, \ldots, h_n][zI - F_n(\Delta_n)]^{-1} \frac{\partial F_n(\Delta_n)}{\partial \Delta_n} [zI - F_n(\Delta_n)]^{-1} g$$

or

$$\left.\frac{\partial W}{\partial \Delta_n}\right|_{\Delta_n = 0} = [h_1, \ldots, h_n]$$

$$\cdot \left[\begin{array}{c|c} zI - F_{n-1}(\Delta_{n-1}) & \begin{matrix} 0 \\ 0 \\ \vdots \\ -(1 - \Delta_{n-1}^2) \end{matrix} \\ \hline 0 & z \end{array}\right]^{-1} \begin{vmatrix} 0 \\ \vdots \\ 1 \end{vmatrix}$$

$$\cdot [-1, -\Delta_1, \ldots, -\Delta_{n-1}][zI - F_n(\Delta_n)]^{-1} g. \quad (43)$$

Substituting for the inverted matrix and performing matrix multiplications, finally gives,

$$\left.\frac{\partial W}{\partial \Delta_n}\right|_{\Delta_n=0} =$$

$$\left\{[h_1, h_2, \ldots, h_{n-1}][zI - F_{n-1}]^{-1}\frac{(1-\Delta_{n-1}^2)}{z}\begin{bmatrix}0\\ \vdots\\ 0\\ 1\end{bmatrix} + \frac{h_n}{z}\right\}$$
$$\cdot[-1, -\Delta_1, \ldots, -\Delta_{n-2}][zI - F_{n-1}]^{-1}\tilde{g}_{n-1} \qquad (44)$$
$$\tilde{g}_{n-1} = [1, 0, \ldots, 0]^{\mathrm{T}}.$$

From (26)–(29),

$$\left.\frac{\partial V}{\partial \Delta_n}\right|_{\Delta_n=0} = [h_1, \ldots, h_{n-1}]\left\{[zI - F_{n-1}(\Delta_{n-1})]^{-1}\begin{bmatrix}0\\ \vdots\\ 0\\ 1\end{bmatrix}\right.$$
$$\left.\cdot[-1, -\Delta_1, \ldots, -\Delta_{n-2}](1-\Delta_{n-1}^2)\cdot[zI - F_{n-1}(\Delta_{n-1})]^{-1}\tilde{g}_{n-1}\cdot\right\}$$
$$- h_n[1, \Delta_1, \ldots, \Delta_{n-2}][zI - F_{n-1}(\Delta_{n-1})]^{-1}\tilde{g}_{n-1}. \qquad (45)$$

Now comparing (44) and (45), (41) is established as required, at least when the input coupling vector is as in (42). The argument for the other input coupling vector runs similarly. Also recall that, as established in Badreddin and Mansour (1980) and Badreddin (1982),

$$V(1, \Delta_n) = W(1, \Delta_n). \qquad (46)$$

5. *Interpretation of the results of sections* 1–4

Equations (40), (41) and (46) sum up key features of the approximation procedures. For all $z = e^{j\omega}$, ω real, it can be seen that near $z = 1$, the approximation error is $O(e^{j\omega} - 1)O(\Delta_n)$, which indicates better accuracy at low frequencies.

This is consistent with the ideas of Section 3 for the following reason. Section 3 demonstrates that in one particular realization of the high order transfer function, the low order approximation is obtained by replacing a unit delay (z^{-1}) by a unity gain element, i.e. the value of z^{-1} at $z = 1$. For the high order transfer function, the incoming signal to this unit delay is multiplied by Δ_n (and this is the only occurrence of Δ_n in the realization). The block diagram replacement thus suggests that the approximation will be better when Δ_n is small and at frequencies ω where $|e^{j\omega} - 1|$ is small, i.e. at low frequencies.

The connection with Section 2 is best thought of in terms of a reduction of model order of up to r. The material of Section 3 shows that approximation will be best when Δ_n, $\Delta_{n-1}, \ldots, \Delta_{n-r+1}$ are all small. That of Section 2 shows that this will follow when the denominator polynomial has r poles near the origin. What happens in the approximation process is that these r poles are wiped out, and the remaining $(n-r)$ pole positions slightly modified.

In summary, the Badreddin–Mansour scheme is likely to work well to reduce a system from nth to $(n-r)$th order when there are r poles close to the origin; further, the approximate transfer function will be a better fit near $z = 1$ (and an exact fit at $z = 1$) than away from $z = 1$.

6. *Conclusion*

In this paper, the method of model reduction of discrete systems based on the Badreddin and Mansour ideas is further elaborated. In particular, by using connections between the polynomial coefficients, the roots and the Schur–Cohn determinants (which are of independent interest), the range of validity of reduction is established. Furthermore, the lattice realization of the original system compared with that of the reduced one is presented. Finally, some connections between the transfer function of both systems are obtained. These also shed some light on the nature of the approximation. Extension of this method of reduction to two-dimensional systems seems promising and is the subject of future research.

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