A General Theory and Synthesis Procedure for Low-Sensitivity Active RC Filters

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Abstract — The fundamental requirements for low-sensitivity analog filter structures are shown to be the bounded-real (BR) property of the transfer function and its implementation by a structure that "preserves" the BR property for incremental changes in the parameters of the network. Using these properties, a low-sensitivity realization of any stable BR analog transfer function is developed. The final structure is in the form of a singly constrained analog active RC "two-pair" implemented as a cascade of two-pairs where each two-pair is characterized by a lossless bounded-real (LBR) transfer matrix. The proposed theory is shown to include the class of low sensitivity wave active RC structures related to the doubly terminated lossless two-ports. The new theory has been verified by computer simulation.

I. INTRODUCTION

CHARACTERISTICS of physical devices are subject to change for many reasons. For example, the transistor parameters may vary due to a change in the temperature, bias levels, or humidity; also, parameters may drift because of aging. Likewise, the parameters of the passive components may change from their nominal values because of variations of external and internal conditions. A detrimental effect of the parameter variations is the displacement of the poles and zeros of the network function from their nominal positions causing the response of the network in practice to deviate from its desired value. As a result, there has been considerable interest in the development of active RC filter structures with low sensitivity to parameter variations during the last two decades [1], [2]. The most significant contribution in this direction was the observation by Orchard [3] that certain suitably doubly terminated lossless ladder two-ports have inherently low passband sensitivity with respect to the element values. This led to numerous approaches, such as the inductance simulation approaches [4]–[6], FDNR based schemes [7], and the flowgraph based approaches [8], [9], for developing active RC filter structures that simulate in some sense the topological properties of the doubly terminated lossless two-ports. Even though all of these methods do lead to very low sensitivity active RC filter structures, they are basically limited to implementing transfer functions that are realizable as doubly terminated lossless ladders. However, there are cases where the transfer function of interest may not be implementable in such form, and hence, the question is whether low-sensitivity realizations are possible for such transfer functions. If so, the next problem is how to realize such structures.

II. REQUIREMENTS FOR LOW SENSITIVITY

Let the analog transfer function \( H(s) \) to be realized be a real rational function with the following properties:

(i) \( H(s) \) is stable, i.e., has all poles in the open left half s-plane, and

(ii) \(|H(j\omega)| \leq 1\), \( \forall \omega \). (1)

Such a function is called a bounded-real (BR) function. Assume that \(|H(j\omega)| = 1\) at a frequency \( \omega = \omega_1 \) in the passband. Fig. 1 shows the magnitude response of such a transfer function. Note that any stable transfer function can be made into a BR function simply by scaling.

Assume also that the structure implementing \( H(s) \) is such that \(|H(j\omega)|\) does not exceed unity for any \( \omega \) regardless of the actual values of the circuit parameters \( \{\alpha_r\} \) as long as they are in the immediate neighborhood of their nominal values \( \{\alpha_{0r}\} \) and as along as the structure remains stable. Now, let a parameter \( \alpha_r \) be perturbed incrementally from its nominal value:

\[
\alpha_{0r} \rightarrow \alpha_{0r} + \Delta \alpha_r. \tag{2}
\]

Then, \(|H(j\omega_r)|\) cannot increase as a result of this perturbation, and, therefore, the plot of \(|H(j\omega)|\) as a function of \( \alpha_r \) takes the form shown in Fig. 2 in the vicinity of \( \alpha_{0r} \). This property holds for each parameter of the network, and hence, the first-order sensitivity of \(|H(j\omega)|\) with respect to any parameter \( \alpha_r \) is zero at any frequency \( \omega \) where \(|H(j\omega)|\)
Fig. 2. Demonstration of zero sensitivity.

is equal to one, i.e.,

$$S_{r}(H(j\omega)) = \frac{\alpha_{r}}{|H(j\omega)|} \left. \frac{\partial |H(j\omega)|}{\partial \alpha_{r}} \right|_{\omega = \omega_{0}} = 0. \quad (3)$$

If there are a number of closely spaced maxima of $|H(j\omega)|$ of unity value in the passband, we can expect very low sensitivity properties in this band.

Summarizing, if the following properties are satisfied by the structure and its associated transfer function, then the structure is guaranteed to exhibit low passband sensitivity:

\begin{align*}
\text{P(1):} & \quad |H(j\omega)| \leq 1, \quad \forall \omega. \\
\text{P(2):} & \quad |H(j\omega_{i})| = 1, \quad i = 1, 2, \cdots, N. \\
\text{P(3):} & \quad \text{Property P(1) holds regardless of the exact values of the circuit parameters as long as they remain in the immediate neighborhood of their nominal values.}
\end{align*}

Property P(3) implies that the boundedness property P(1) is "structure-induced." Now, P(1) is equivalent to the condition:

$$|Y(j\omega)| \leq |X(j\omega)| \quad \text{for all finite energy}^{1} \text{ input } x(t) \quad (4)$$

where $Y(j\omega)$ and $X(j\omega)$ are, respectively, the Fourier transforms of the response $y(t)$ and the excitation $x(t)$. Applying Parseval's relation, we thus obtain from (4):

$$\int_{-\infty}^{\infty} |y(t)|^2 dt \leq \int_{-\infty}^{\infty} |x(t)|^2 dt \quad (5)$$

for all finite energy inputs, or in other words, the output energy delivered by the network is at most equal to the input energy delivered to the network by the source. Moreover, if (4) and (5) are satisfied with equality sign for all finite energy inputs, then the structure is said to be lossless. As a result, a stable real rational transfer function $H(s)$ for which $|H(j\omega)| = 1$ for all values of $\omega$, is called a lossless bounded-real (LBR) function. Note that such a function is simply a stable all-pass function. (The concept of LBR functions and matrices is well known in classical network theory [24].)

An useful extension of the above concept is the lossless bounded-real (LBR) $2 \times 2$ transfer matrix $\mathcal{F}(s) = [T_{ij}(s)]$ defined by the conditions: each element $T_{ij}(s)$ is stable, i.e. has all poles in the open left half $s$-plane and

$$\mathcal{F}(s) \mathcal{F}(s) = I \quad (6)$$

where $\mathcal{F}(s)$ is a scalar BR function but not necessarily LBR. The two-input two-output network (two-pair) realizing an LBR transfer matrix will be called an LBR two-pair if it ensures the LBR property of the transfer matrix independent of the actual values of the circuit parameters (as long as they are in a certain range). Before proceeding further, we state a few properties of BR functions which are useful in the development of a synthesis procedure.

\begin{enumerate}
\item \textbf{Property 1}: For a BR function $G(s)$ with $G(0) = \pm 1$, the quantity $G(s)(dG(s)/ds)$ evaluated at $s = 0$ is negative.
\item \textbf{Property 2}: For a BR function $G(s)$ with $G(\infty) = \pm 1$, the quantity $G(s)(dG(s)/ds^{-1})$ evaluated at $s = \infty$ is negative.
\item \textbf{Property 3}: For a BR function $G(s)$ with $G(j\omega_{0}) = \pm 1$, the quantity $G(s)(dG(s)/ds^{-1})$ evaluated at $s = j\omega_{0}$ is real and negative.
\item \textbf{Property 4}: If $G(s)$ is BR then so is $G[F(s)]$ where $F(s)$ is positive real. In particular, if $G(s)$ is LBR and $F(s)$ is a reactance function then $G[F(s)]$ is LBR. More generally, if $\mathcal{F}(s)$ is an LBR two-pair then $\mathcal{F}[F(s)]$ is an LBR two-pair whenever $F(s)$ is a reactance function.
\end{enumerate}

Proofs of these properties follow along similar lines as in the case of digital BR functions [11], and are, therefore, omitted here.

III. \textbf{THE SYNTHESIS APPROACH}

It should be noted that the basic requirements for low-sensitivity analog networks is quite similar to that recently developed for digital filters [11]-[13] where the boundedness conditions of the transfer function $G(z)$ was imposed on its magnitude on the unit circle $z = e^{j\omega}$. Following a similar approach as outlined in these works, we develop here a synthesis procedure by which an $m$th-order BR transfer function $G_{m}(s)$ is realized by "extracting" a lower-order LBR two-pair such that the constraining transfer function $G_{m-1}(s)$ is BR and is of lower order (Fig. 3). This process is continued, and after successive extractions, we are then left with a cascade of LBR two-pairs constrained by a
constant transfer function of magnitude less than one. The overall cascade is of the form shown in Fig. 8(a) and we call it the \Pi-cascade. It can be shown that the \Pi-cascade of two LBR two-pairs is an LBR two-pair and, therefore, the entire structure is an LBR two-pair terminated in a BR constant.

### 3.1. LBR Two-Pair Structures

A two-pair (Fig. 4) can be described by a chain matrix, defined as:

\[
\begin{bmatrix}
X_1(s) \\ Y_1(s)
\end{bmatrix} = \begin{bmatrix}
A(s) & B(s) \\ C(s) & D(s)
\end{bmatrix} \begin{bmatrix}
Y_2(s) \\ X_2(s)
\end{bmatrix} = \Pi(s) \begin{bmatrix}
Y_1(s) \\ X_1(s)
\end{bmatrix}
\]

where the chain parameters \(A(s), B(s), C(s),\) and \(D(s)\) are related to the transfer matrix parameters by

\[
\begin{aligned}
A &= \frac{1}{T_{21}}, & B &= -\frac{T_{22}}{T_{21}} \\
C &= \frac{T_{11}}{T_{21}}, & D &= \frac{T_{12}T_{21} - T_{11}T_{22}}{T_{21}} \\
T_{11} &= \frac{C}{A}, & T_{12} &= \frac{AD - BC}{A} \\
T_{21} &= \frac{1}{A}, & T_{22} &= -\frac{B}{A}
\end{aligned}
\]  

(9)

In this paper we deal exclusively with reciprocal two-pairs, characterized by \(T_{12}(s) = T_{21}(s)\) or equivalently by

\[
A(s)D(s) - B(s)C(s) = 1
\]

for all \(s\). The “extraction” of a two-pair from a function \(G_m(s)\) (Fig. 3) leaves behind a remainder \(G_{m-1}(s)\) where \(G_m\) and \(G_{m-1}\) are related as

\[
G_{m-1}(s) = \frac{C(s) - A(s)G_m(s)}{B(s)G_m(s) - D(s)}
\]

\[
G_m(s) = \frac{C + DG_{m-1}(s)}{A + BG_{m-1}(s)}.
\]

(11)

It can be shown that a reciprocal two-pair is LBR if and only if

\[
\begin{aligned}
A(s) &= D(-s) = D(s) \\
B(s) &= C(-s) = C(s) \\
A(s)D(s) - B(s)C(s) &= 1
\end{aligned}
\]

(12a, 12b, 12c)

with all zeros of \(A(s)\) strictly in the left half \(s\)-plane for stability. With the help of (12) it can be shown that a first-order reciprocal LBR two-pair must take one of the following two forms:

#### Type I

**Chain Parameters:**

\[
\begin{aligned}
A(s) &= \frac{s + a}{x_1}, & B(s) &= \frac{cs + b}{x_1} \\
C(s) &= \frac{-cs + b}{x_1}, & D(s) &= \frac{-s + a}{x_1}
\end{aligned}
\]

(13a)

#### Type II

**Chain Parameters:**

\[
\begin{aligned}
A(s) &= \frac{s + a}{x_0s}, & B(s) &= \frac{cs + b}{x_0s} \\
C(s) &= \frac{cs - b}{x_0s}, & D(s) &= \frac{s - a}{x_0s}
\end{aligned}
\]

(14a)

**Transfer Parameters:**

\[
\begin{aligned}
T_{11}(s) &= \frac{cs - b}{s + a} \\
T_{12}(s) &= \frac{x_0s}{s + a} \\
T_{22}(s) &= -\frac{cs + b}{s + a}
\end{aligned}
\]

(14b)

Note that the Type I two-pair has a transmission zero (zeros of \(T_{12}\) and \(T_{21}\)) at \(s = \infty\) whereas the Type II two-pair has a transmission zero at \(s = 0\). For the LBR-based synthesis, with suitable choices of \(b, c,\) and \(x_1\), we arrive at four different forms of first-order LBR two-pairs, as shown in Table I. Note that Type IA has \(T_{11}(\infty) = -1\) whereas Type IB has \(T_{11}(\infty) = 1.\) Similarly Type IIA has \(T_{11}(0) = -1\) whereas Type IIB has \(T_{11}(0) = 1.\)

Property 4 of LBR functions as given in the previous section can be used to develop second- (and higher) order LBR two-pairs from the two-pairs listed in Table I. Thus the following reactance transformation:

\[
s \rightarrow \frac{s^2 + \beta}{s}
\]

(15)

where \(\beta = \omega_0^2\), maps the point \(s = 0\) onto \(s = \pm j\omega_0\) and can be used to derive the LBR two-pairs, as shown in Table II. Specifically, Type IIIA of Table II is obtained by applying this transformation on Type IIB and Type IIIB is obtained by transforming Type IIA. Tables I and II give us a complete set of LBR two-pairs that are needed in the synthesis of BR function.
### TABLE I
FIRST-ORDER RECIPROCAL LBR TWO-PAIRS

<table>
<thead>
<tr>
<th>Type</th>
<th>Chain Parameters</th>
<th>Transfer Parameters</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>IA</td>
<td>( A = \frac{s}{a} + a )</td>
<td>( T_{11} = \frac{s}{s+a} )</td>
<td>To be used when ( G_m(\infty) = -1 )</td>
</tr>
<tr>
<td></td>
<td>( B = \frac{s}{a} )</td>
<td>( T_{12} = \frac{s}{s+a} )</td>
<td>( a = \frac{D}{ds} ) ( s \to \infty )</td>
</tr>
<tr>
<td></td>
<td>( C = -\frac{a}{s} )</td>
<td>( T_{22} = \frac{s}{s+a} )</td>
<td>( a = \frac{D}{ds} ) ( s \to \infty )</td>
</tr>
<tr>
<td></td>
<td>( D = \frac{-s+a}{a} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>IB</td>
<td>( A = \frac{s+a}{s} )</td>
<td>( T_{11} = \frac{s}{s+a} )</td>
<td>To be used when ( G_m(\infty) = 1 )</td>
</tr>
<tr>
<td></td>
<td>( B = \frac{s}{a} )</td>
<td>( T_{12} = \frac{s}{s+a} )</td>
<td>( a = \frac{D}{ds} ) ( s \to \infty )</td>
</tr>
<tr>
<td></td>
<td>( C = \frac{a}{s} )</td>
<td>( T_{22} = \frac{s}{s+a} )</td>
<td>( a = \frac{D}{ds} ) ( s \to \infty )</td>
</tr>
<tr>
<td></td>
<td>( D = \frac{-s+a}{a} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>II A</td>
<td>( A = \frac{s+a}{s} )</td>
<td>( T_{11} = \frac{s}{s+a} )</td>
<td>To be used when ( G_m(0) = 1 )</td>
</tr>
<tr>
<td></td>
<td>( B = \frac{a}{s} )</td>
<td>( T_{12} = \frac{s}{s+a} )</td>
<td>( a = \frac{D}{ds} ) ( s = 0 )</td>
</tr>
<tr>
<td></td>
<td>( C = -\frac{a}{s} )</td>
<td>( T_{22} = \frac{s}{s+a} )</td>
<td>( a = \frac{D}{ds} ) ( s = 0 )</td>
</tr>
<tr>
<td></td>
<td>( D = \frac{s-a}{s} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>II B</td>
<td>( A = \frac{s}{s-a} )</td>
<td>( T_{11} = \frac{s}{s+a} )</td>
<td>To be used when ( G_m(0) = -1 )</td>
</tr>
<tr>
<td></td>
<td>( B = \frac{a}{s} )</td>
<td>( T_{12} = \frac{s}{s+a} )</td>
<td>( a = \frac{D}{ds} ) ( s = 0 )</td>
</tr>
<tr>
<td></td>
<td>( C = -\frac{a}{s} )</td>
<td>( T_{22} = \frac{s}{s+a} )</td>
<td>( a = \frac{D}{ds} ) ( s = 0 )</td>
</tr>
<tr>
<td></td>
<td>( D = \frac{s-a}{s} )</td>
<td></td>
<td></td>
</tr>
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</table>

### TABLE II
SECOND-ORDER RECIPROCAL LBR TWO-PAIRS

<table>
<thead>
<tr>
<th>Type</th>
<th>Chain Parameters</th>
<th>Transfer Parameters</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>IIA</td>
<td>( A = \frac{s}{s+a} ) ( s+\beta )</td>
<td>( T_{11} = \frac{s}{s+a} ) ( s+\beta )</td>
<td>To be used when ( G_m(j\omega) = 1 )</td>
</tr>
<tr>
<td></td>
<td>( B = \frac{-s-a}{s+\beta} )</td>
<td>( T_{12} = \frac{s}{s+a} ) ( s+\beta )</td>
<td>( a = \frac{D}{ds} ) ( s=j\omega )</td>
</tr>
<tr>
<td></td>
<td>( C = \frac{a}{s+\beta} )</td>
<td>( T_{22} = \frac{s}{s+a} ) ( s+\beta )</td>
<td>( a = \frac{D}{ds} ) ( s = j\omega )</td>
</tr>
<tr>
<td></td>
<td>( D = \frac{s^{2}-as+\beta}{s+\beta} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>II B</td>
<td>( A = \frac{s^{2}}{s+\beta} ) ( s+\beta )</td>
<td>( T_{11} = \frac{s^{2}}{s+\beta} ) ( s+\beta )</td>
<td>To be used when ( G_m(j\omega) = -1 )</td>
</tr>
<tr>
<td></td>
<td>( B = \frac{a}{s+\beta} )</td>
<td>( T_{12} = \frac{s}{s+\beta} ) ( s+\beta )</td>
<td>( a = \frac{D}{ds} ) ( s = j\omega )</td>
</tr>
<tr>
<td></td>
<td>( C = -\frac{a}{s+\beta} )</td>
<td>( T_{22} = \frac{s}{s+\beta} ) ( s+\beta )</td>
<td>( a = \frac{D}{ds} ) ( s = j\omega )</td>
</tr>
<tr>
<td></td>
<td>( D = \frac{s^{2}-as+\beta}{s+\beta} )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
3.2. Active RC Realization of Basic LBR Two-Pairs

Consider first the realization of the Type IB two-pair for which the input and output relations are given by

\[ Y_1 = \frac{s}{s+a} X_1 + \frac{a}{s+a} X_2 \]
\[ Y_2 = \frac{a}{s+a} X_1 + \frac{s}{s+a} X_2. \]  
(16)

Rewriting above in the form shown below

\[ Y_1 = -\left(-X_1 - \frac{a}{s+a} (X_2 - X_1)\right) \]
\[ Y_2 = -\frac{a}{s+a} (X_2 - X_1) + X_2 \]  
(17)

we obtain the integrator-based realization sketched in Fig. 5(a). Note that, an active RC implementation of this structure in general will require three operational amplifiers. Another realization of the Type IB two-pair is obtained by observing that \( Y_1 \) can be generated using a resistor–capacitor voltage divider, and \( Y_2 \) then obtained by subtracting \( Y_1 \) from \( X_1 + X_2 \). The corresponding circuit realization is sketched in Fig. 5(b) which can be implemented using two operational amplifiers. Note that the operational amplifier implementation of Fig. 5(b) is precisely the active RC wave-equivalent of a series inductor two-port advanced by Wuper and Meerkotter [14], and by Constantinides et al. [15], [16].

Circuit realization of Type IA two-pair can be readily obtained from that of Type IB two-pair by connecting an inverter at the input \( X_1 \) and at the \( Y_2 \) output as shown in Fig. 6(a). In fact a two-pair containing a single inverter in one-path (the two-pairs shown by dotted lines in Fig. 6(a)) is precisely the active RC wave-equivalent of a gyrator. Hence, by the equivalence outlined in [14], connections of two such two-pairs to the Type IB two-pair implies that the Type IA two-pair represents the wave equivalent of a shunt capacitor two-port. Another circuit realization of the Type IA two-pair implementable using only two operational amplifiers is obtained by rewriting its input–output relations as

\[ Y_1 = 2 \left[ \frac{a}{2(a + s)} (X_1 + X_2) \right] - X_1 \]
\[ Y_2 = 2 \left[ \frac{a}{2(a + s)} (X_1 + X_2) \right] - X_2 \]  
(18)

which leads to the circuit of Fig. 6(b).

Circuit realization of Type IIA and Type IIB two-pairs are readily obtained from that of Type IA and Type IB, respectively. For example, Type IIB is obtained from Type IB by interchanging \( Y_1 \) and \( Y_2 \) terminals (or equivalently, \( X_1 \) and \( X_2 \) terminals). Type IIA is obtained from Type IA in an identical manner, except that the signs of the outputs should also be changed.

Circuit realization of the Type IIIA two-pair can be obtained by cascading the circuit realizations of the Type IB and IIB two-pairs, and interchanging the roles of \( Y_1 \) and \( Y_2 \) output terminals. To show this we observe that the II-matrix of the cascade is given by

\[ \frac{1}{as} \begin{bmatrix} s + a & -s \\ s & -s + a \end{bmatrix} \begin{bmatrix} s + b & -b \\ b & s - b \end{bmatrix} \]
\[ = \frac{1}{as} \begin{bmatrix} (s^2 + as + \beta) & -(s^2 + \beta) \\ s^2 + \beta & -(s - as + \beta) \end{bmatrix} \]  
(19)

where we have used \( \beta = ab \). Interchanging the roles of the output terminals \( Y_1 \) and \( Y_2 \) we get a new two-pair characterized by the transfer matrix

\[ \begin{bmatrix} \frac{1}{A} & -B \\ C & AD - BC \end{bmatrix} \frac{1}{s^2 + \frac{1}{A}} \begin{bmatrix} as & s^2 + \beta \\ s^2 + \beta & as \end{bmatrix} \]  
(20)

which is exactly the transfer matrix of the Type IIIA two-pair.

Another interesting realization of the Type IIIA two-pair is obtained by rewriting its input and output relations as given below:

\[ Y_1 = X_1 + X_2 - Y_2 \]
\[ Y_2 = \frac{s^2 + \beta}{s^2 + as + \beta} X_1 + \frac{as}{s^2 + as + \beta} X_2. \]  
(21)

Note that the transfer function \( T_{\text{III}} \) is that of a symmetric biquadratic notch filter. A convenient realization of such a transfer function is shown in Fig. 7(a) [18]. By interchang-
The Basic Two-Pair Extraction Procedure

Basic to our synthesis procedure is the extraction of an LBR two-pair from a BR function, such that the remainder is a lower order BR function. Given a BR function \( G_m(s) \) let us assume that it is scaled such that \( |G_m(j\omega)| = 1 \) for some \( s \) on the imaginary axis. Then there are four possible cases:

**Case I:** \( G_m(0) = \pm 1 \).

**Case II:** \( G_m(\infty) = \pm 1 \).

**Case III:** \( G_m(\pm j\omega_0) = \pm 1 \) for \( 0 < \omega_0 < \infty \).

**Case IV:** \( |G_m(\pm j\omega_0)| = 1 \) but \( G_m(\pm j\omega_0) \) is complex.

For each of these cases, we outline the "LBR extraction rules" leading to order-reduction.

**Case I:** Let \( G_m(0) = 1 \). The LBR two-pair Type IIB of Table I has the property \( T_{11}(0) = 1 \) and \( T_{21}(0) = T_{22}(0) = 0 \), and is clearly the most appropriate two-pair to be extracted. However, a suitable value of \( a \) should be found that ensures a reduced-order BR remainder \( G_{m-1}(s) \). From (11) we have

\[
G_{m-1}(s) = \frac{a - (s + a)G_m}{-aG_m - (s - a)}.
\]

Setting \( s = 0 \) we find

\[
G_{m-1}(0) = \frac{a - aG_m(0)}{-aG_m(0) + a} = \frac{a - a}{a - a} = 0
\]

which shows that, there is a cancellation of the factor \( s \) between the numerator and denominator of (22). The order of \( G_{m-1}(s) \) is, therefore, at most \( m \). The order can be reduced still further by cancellation of a second common factor. Applying L'Hospital's rule to (22) we get

\[
\lim_{s \to 0} G_{m-1}(s) = \frac{-G_m'(0)}{-aG_m'(0)} = \frac{-1}{-a} = \frac{1}{a}.
\]

In order to obtain the desired cancellation we have to choose \( a = -1/G_m(0) \). In the Appendix, we state and prove a theorem which justifies that all the order-reduction schemes employed in this paper always give rise to bounded real remainders.

If \( G_m(0) \) were equal to \(-1\) rather than \(1\), we can insert a negative sign ahead and proceed as above. Equivalently, we can extract a modified two-pair (Type IIA, Table I) with \( a = 1/G_m(0) \).

Case II can be handled in a similar manner, leading to the extraction of Type IA or Type IB LBR two-pairs, with values of \( a \) as given in Table I.

**Case III:** Let \( G_m(\pm j\omega_0) = 1 \), \( 0 < \omega_0 < \infty \). We now wish to extract a second-order two-pair such that order of \( G_{m-1}(s) \) is 2 less than that of \( G_m(s) \). The two-pair Type IIBA, Table II has \( T_{11}(\pm j\omega_0) = 1 \) and is the most appropriate LBR two-pair to be extracted. Once again, it remains to find the right value of \( a \). For this, we note

\[
G_{m-1}(s) = \frac{as - (s^2 + as + \beta)G_m}{-asG_m - (s^2 + as + \beta)}.
\]

Setting \( s = \pm j\omega_0 \) and remembering \( \beta = \omega_0^2 \), we get

\[
G_{m-1}(\pm j\omega_0) = \frac{a\omega_0\pm a\omega_0}{a\omega_0 - a\omega_0} = \frac{a\omega_0\pm a\omega_0}{a\omega_0 - a\omega_0},
\]

which shows that there is a cancellation of the factors \( s \pm j\omega_0 \), and this implies that the order of \( G_{m-1}(s) \) is at most that of \( G_m(s) \). To force a further order-reduction, we apply L'Hospital's rule to look at the limit of \( G_{m-1} \) as \( s \to \pm j\omega_0 \):

\[
\lim_{s \to \pm j\omega_0} G_{m-1}(s) = \frac{\pm 2j\omega_0 \pm a(j\omega_0)G_m'(\pm j\omega_0)}{\pm 2j\omega_0 \pm a(j\omega_0)G_m'(\pm j\omega_0)}.
\]

There is thus a further cancellation of factors \( s^2 + \omega_0^2 \) in \( G_{m-1} \) provided \( a \) is chosen such that the right-hand side of (27) is of the form \( 0/0 \). To this end we let

\[
a = \frac{-2}{G_m'(s)} \Bigg|_{s = \pm j\omega_0}.
\]

Note that by Property 3, \( a \) is real and positive, regardless of whether \( s = j\omega_0 \) or \( s = -j\omega_0 \) is used in (28).

The case where \( G_m(\pm j\omega_0) = -1 \) can be handled in a similar manner by extracting the two-pair Type IIB.
Note that in view of Properties 1, 2, and 3, the quantity \( a \) appearing in Tables I and II is always positive. Thus as long as parameter fluctuations in an implementation do not change the sign of \( a \), the LBR property is retained and this is the key to the low sensitivity properties. Two points are worth noting before proceeding to Case IV. First, referring back to (11), if \( G_m \) has order \( m \), and if \( A, B, C, D \) are of first-order, then \( G_{m-1} \) cannot have an order smaller than \( m - 1 \). Thus after a first-order LBR two-pair is extracted according to Table I, the order of the remainder is precisely one less than that of \( G_m \). An equivalent statement holds for the second-order extraction also. Second, it can be easily verified that if an LBR two-pair of a given type (say Type IA) is cascaded to another LBR two-pair of the same type, the resulting LBR two-pair is of the same type and, therefore, the same order. As a consequence, if \( G_m(s_0) = +1 \) (where \( s_0 = 0 \) or \( \infty \) or \( \pm j\omega_0 \)) and an appropriate two-pair if extracted, the remainder \( G_{m-1} \) of lower order cannot have a value "1" at \( s = s_0 \). (For if it did, another two-pair of the same type could be extracted to give a remainder \( G_{m-2} \) of still lower order, and at the same time the two two-pairs can be combined into one, of same order.) Thus the two-pair extraction has essentially "removed" the "one" from \( G_m \) to produce \( G_{m-1} \). The two-pair extractions of Types IB, IIB, and IIIA can, therefore, be considered as "1" removal operations from \( s = \infty \), \( s = 0 \) and \( s = \pm j\omega_0 \) whereas those of Type IA, IIA, and IIB can be considered as "+1" removal operations. If the BR function \( G_m(s) \) is such that \( G_m(j\omega) \) is neither "1" nor "-1" for any \( \omega \), then there is no more "1" or "-1" to be removed, and this leads to Case IV.

**Case IV:** If \( |G_m(j\omega_0)| = 1 \) but \( G_m(j\omega_0) \) itself is not real, then we can extract a first-order two-pair of the form given in Table I to obtain a remainder \( G_{m-1} \) such that \( G_{m-1}(\pm j\omega_0) \) is real, with magnitude equal to one. For example, in order to force \( G_{m-1}(\pm j\omega_0) = 1 \), Type IA or Type IIA can be used, whereas to force \( G_{m-1}(\pm j\omega_0) = -1 \), Type IB or Type IIB can be extracted. This extraction typically leads to a remainder \( G_{m-1} \) whose order is one higher than \( G_m \). To consider a specific case, let us extract Type IA. Then, in order to force \( G_{m-1}(\pm j\omega_0) = -1 \), \( a \) should be chosen such that

\[
\frac{-s - (s + a)G_m(s)}{sG_m(s) - (-s + a)} = 1
\]

which gives

\[
a = 2j\omega_0\frac{1}{1 - G_m(j\omega_0)}.
\]

In Table III we tabulate various cases, and call the corre-
sponding first-order two-pairs, Type IV. For a BR function $G_m(s)$ with $|G_m(j\omega_0)|=1$, it can be shown that "a" appearing in Table III is always real.

Now, we can extract a Type IIIA LBR two-pair from $G_{m-1}$ in order to get an order reduction by 2. Let the remainder function be denoted $G_{m-2}$. Then $G_{m-2}$ has order one lower than $G_m$. Thus further extraction of a first-order two-pair is necessary to obtain a final remainder $G_{m-3}$ of order two less than $G_{m}$. In order to see how to accomplish this, note that a Type IV two-pair gives rise to a transmission zero at $s=0$ (Type IVB and D) or at $s=\infty$ (Type IVA and C) which tends to force $G_m$ to be equal to ±1 for this value of $s$. In order to avoid this unintended behavior, the extraction of the first-order two pair from $G_{m-2}$ should be such as to cancel the transmission zero created by the Type IV two-pair. The following two rules of extraction cover all possible synthesis requirements falling under Case IV:

1. If extraction of Type IVA with $a=a_1$ is followed by that of Type IIIA with $a=a_2$, then a Type IVA two-pair with $a=a_3$ given by $a_3=-(a_1+4a_2)$ should follow. This gives an overall remainder whose order is two lower than that of $G_m$.

2. Similarly, if extraction of the Type IVC with $a=a_1$ is followed by that of IIIB with $a=a_2$, then a two-pair of the Type IVC with $a=a_3=-(a_1+4a_2)$ should follow.

The proofs are omitted here for brevity. This completes the set of all rules required for BR synthesis. Two numerical examples are next given to illustrate the synthesis procedure. Note that the above method of dealing with Case IV can in general lead to unstable intermediate remainder functions, and unstable intermediate two-pairs. This is analogous to the appearance of negative storage elements in a “Brune Section” [21]. However, the three two-pairs extracted in the above process can be combined into a single second-order LBR two-pair, as demonstrated in the example to follow. This LBR two-pair is terminated with a BR (hence stable) overall remainder $G_{m-3}$. The input function of the two-pair is $G_m$, which is the BR function we started with.

IV. ILLUSTRATIVE EXAMPLES

Example 1: Consider

$$G_4(s) = \frac{-27s^3+11s^2-17s+3}{27s^3+16s^2+19s+6}.$$  

It can be verified that $G_4(s)$ is BR with $G_4(\infty) = -1$. Extracting the LBR two-pair Type IA with suitable value of $a$ will therefore result in a second-order BR remainder. The appropriate value of $a$ is found from Table I to be

$$a_4 = \frac{dG_4}{ds}|_{s=\infty} = 1.$$  

Let the remainder function be denoted $G_{m-2}$. Then $G_{m-2}$ has order one lower than $G_m$. Thus further extraction of a first-order two-pair is necessary to obtain a final remainder $G_{m-3}$ of order two less than $G_m$. In order to see how to accomplish this, note that a Type IV two-pair gives rise to a transmission zero at $s=0$ (Type IVB and D) or at $s=\infty$ (Type IVA and C) which tends to force $G_m$ to be equal to ±1 for this value of $s$. In order to avoid this unintended behavior, the extraction of the first-order two pair from $G_{m-2}$ should be such as to cancel the transmission zero created by the Type IV two-pair. The following two rules of extraction cover all possible synthesis requirements falling under Case IV:

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2. Similarly, if extraction of the Type IVC with $a=a_1$ is followed by that of IIIB with $a=a_2$, then a two-pair of the Type IVC with $a=a_3=-(a_1+4a_2)$ should follow.

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$$a_4 = \frac{dG_4}{ds}|_{s=\infty} = 1.$$  

Leads to a remainder

$$G_3(s) = C - \frac{AG_4}{BG_4-D} = \frac{13s^2-8s+3}{14s^2+10s+6}.$$  

Clearly $G_3(0) = \frac{1}{2}$ and $G_3(\infty) = \frac{13}{14}$, which means, we cannot extract any more first-order LBR two-pairs for order reduction. However, $G_3(\pm j1) = (-10-8j)/(-8+10j)$ which means $G_3(\pm j1) \neq \pm 1$, but does have a magnitude of unity. Let us, therefore, extract a Type IVC two-pair with

$$a_2 = 2(j1) \frac{1-G_4(j1)}{1+G_3(j1)} = 2.$$  

The two-pair to be extracted is therefore,

$$A = \frac{s+2}{2}, \quad B = -\frac{s}{2}, \quad C = \frac{s}{2}, \quad D = -\frac{s+2}{2}.$$  

This leads to the unstable remainder function

$$G_2(s) = \frac{s^3-8s^2+19s-6}{s^3-10s^2-17s-12}.$$  

Now $G_2(j1) = -1$ as expected. Extracting therefore a Type IIIB two-pair with

$$\beta = \omega_0^2 = 1 \quad \text{and} \quad \sigma = \frac{dG_2}{ds}|_{s=-j1} = \frac{2}{7} = 1.$$  

we get the unstable remainder function

$$G_1(s) = \frac{s-6}{s-12}.$$  

To compensate for the unwanted transmission zero introduced by the Type IVC two-pair, we now extract another Type IVC two-pair with $a_3 = -(a_1+4a_2) = -(2+4) = -6$. This gives the remainder $G_0 = 1/2$ which is a BR constant. The complete realization is shown in Fig. 8(a). The unstable intermediate two-pairs and unstable intermediate remainders which appear in the synthesis procedure do not actually explicitly appear in the implementation. Thus the three two-pairs can be combined into one second-order LBR (hence stable) two-pair where $G_3$ and $G_0$ are BR (hence stable). The combined second-order LBR two-pair has the transfer matrix:

$$\mathcal{F}(s) = \frac{1}{(5s^2+4s+3)} \begin{bmatrix} 4s^2-2s & 3(s^2+1) \\ 4(s^2+1) & -(4s^2+2s) \end{bmatrix}$$

which can be implemented with two integrators.
TABLE IV

<table>
<thead>
<tr>
<th>Stage</th>
<th>Type of Two-Pair</th>
<th>Value of 'a'</th>
<th>Value of β</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>IA</td>
<td>1.5533</td>
<td>4.031</td>
</tr>
<tr>
<td>2</td>
<td>IIIA</td>
<td>2.2576</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>IA</td>
<td>1.0666</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>IIIA</td>
<td>1.1683</td>
<td>2.301</td>
</tr>
<tr>
<td>5</td>
<td>IA</td>
<td>1.0608</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>IB</td>
<td>2.2382</td>
<td></td>
</tr>
</tbody>
</table>

Example 2: Consider a sixth-order "modified" low-pass elliptic filter, with normalized cutoff frequency equal to 1 rad/s, and with the following transfer function [21]

\[ H(s) = \frac{K(s^2 + 2.301)(s^2 + 4.031)}{(s^2 + 0.132s + 1.045)(s^2 + 0.448s + 0.710)(s^2 + 0.763s + 0.252)}. \]

Assume \( K \) to be such that \( |H(j\omega)|_{\text{max}} = 1 \). This transfer function can be realized by synthesizing its "complement" \( G(s) \) as the input-function of a terminated cascade of LBR two-pairs. The function \( G(s) \) satisfies

\[ |G(j\omega)|^2 + |H(j\omega)|^2 = 1 \]

and is given by

\[ G(s) = \frac{-(s^2 + 0.086)(s^2 + 0.571)(s^2 + 0.949)}{D(s)} \]

where \( D(s) \) is the denominator of \( H(s) \). Note that \( G(s) \) is a BR function, and has magnitude equal to unity at \( s = \pm j/2.301, \pm j/4.031 \) and at infinity. \( G(s) \) can be synthesized by LBR two-pair extraction in the following manner.

1) Remove a Type IA LBR two-pair from \( G_m(s) \) so that the remainder \( G_{m-1}(s) \) satisfies, \( G_{m-1}(e^{j\omega}) = 1 \), where \( \omega = \sqrt{4.031} \).

2) Remove a Type IIIA LBR two-pair from \( G_{m-1}(s) \) so that the remainder \( G_{m-2}(s) \) is of order 4.

3) Remove a Type IA LBR two-pair from \( G_{m-3}(s) \) so that the remainder \( G_{m-4}(s) \) is of order 2.

4) Remove a Type IIIA LBR two-pair from \( G_{m-3}(s) \) so that the remainder \( G_{m-5}(s) \) is of order 2.

5) Finally remove a Type IA two-pair so that the remainder \( G_{m-6}(s) \) is of order 1, and then remove a Type IB two-pair to obtain a constant remainder, completing the synthesis.

Table IV shows the parameter values for the normalized design. Fig. 9 shows the ideal frequency response, with an actual denormalized passband cutoff of 10 kHz. In order to study the sensitivity properties, let us assume the components to be random variables, with mean equal to the ideal value, and a uniform statistical distribution in the tolerance range. The magnitude response \( |H(j\omega)| \) is thus a random variable, with a mean equal to \( m(j\omega) \) and a standard deviation \( \sigma(j\omega) \). If \( |H(j\omega)| \) were a Gaussian variable, then about 95 percent of the manufactured filters would have a magnitude response in the range \( m(j\omega) \pm 2\sigma(j\omega) \). Even though the exact distribution of the random variable \( |H(j\omega)| \) is not clear, a plot of \( m(j\omega) \pm 2\sigma(j\omega) \) gives a measure of the filter sensitivity. Fig. 10 shows some of
Fig. 10. Frequency responses of the circuit of Example 2 implemented with nonideal components. Dashed lines denote the response \( m(j\omega) \).

(a) Overall response with 1 percent tolerance components. (b) Passband details with 1-percent tolerance components. (c) Overall response with 2-percent tolerance components. (d) Passband details with 2-percent tolerance components. (See page 697.)

these plots for typical tolerances of 1 and 2 percent. The curves demonstrate the good sensitivity behavior of the circuit. (Note that 2-percent tolerance corresponds to less than 6 bits of accuracy in the equivalent digital world.)

V. LOW SENSITIVITY VERSUS TERMINATION REQUIREMENTS

It is well known that, in order to exhibit low passband sensitivity with respect to components, a continuous-time LC two-port must be suitably doubly terminated so that at the maxima of the magnitude in the passband the source transfers the maximum available power. As a consequence, a singly terminated, or an unterminated LC two-port network does not give rise to low-sensitivity realizations, because, the maximum available power is not bounded above in these cases.

In the case of active filters based on the LBR approach, the role of terminations is different. Thus the structures advanced in this paper are lossless analog two-pairs, which are typically terminated at one end, or even unterminated. In spite of this, they still exhibit low passband sensitivity. The reason for this can be seen by considering a general, doubly terminated LBR analog two-pair, as shown in Fig. 11, where \( H(s) = Y(s)/X(s) \).

Based on the losslessness of the two-pair, it can be shown that the maximum possible value of the quantity \(|H(j\omega)|\) is given by

\[
\frac{1}{\sqrt{(1-n^2)(1-m^2)}}.
\]  

(31)

According to Section II, if the transfer function magnitude attains this bound at certain frequencies in the passband this is sufficient to ensure low sensitivity. Wave-active filters naturally fall under this class, as they are obtained from continuous-time LC networks, which are "properly" terminated in order to attain the corresponding
“maximum available power” bound at the reflections zeros in the passband. The structures advanced in earlier sections of this paper also fall under this class, with “n” being typically zero. Note that, even if “n” and/or “m” is zero, the bound in (31) is finite and hence attainable, and this explains the reason why double terminations are not necessary for low passband sensitivity. Note however, that if m and/or n has a magnitude equal to unity, we have an unattainable bound, and therefore the resulting digital filter structure does not have low sensitivity. This is the situation, analogous to a singly terminated or unterminated continuous-time LC two-port.

VI. CONCLUDING REMARKS

A general theory for low passband sensitivity of analog filters has been presented along with one synthesis approach resulting in an active RC realization. Two examples illustrating the synthesis procedure have been outlined. The low passband sensitivity feature has been verified by computer simulation of the examples. Effect of gain-bandwidth product of the operational amplifiers is currently under investigation.

Several additional comments are here in order. The analog (LBR) two-pairs developed in this paper are indeed equivalent to the wave active two-pairs developed for various series and shunt lossless resonant two-ports advanced by Constantinides–Haritanis [15], [16] and Wupper–Meerkötter [14], [20]. As a result, the approaches proposed by these authors are special cases of our general framework as our theory does not hinge upon simulation of doubly terminated lossless two-ports. The other point to be mentioned here is that it is easy to show that the (BR) synthesis advanced in Section III is the active “wave” equivalent of the classical Brune synthesis [22] of positive real (PR) functions. The primary emphasis in this paper is, therefore, the development of a general theory of low passband sensitivity filters and one synthesis procedure that encompasses certain well-known methods. There is no intention, however, to compare the resulting structures with other
unrelated but well-known circuits, such as the leapfrog structures, GIC based structures etc.

**APPENDIX**

**Theorem:** Let \( G(s) = \frac{P(s)}{Q(s)} \) be an mth-order BR function and let an LBR two-pair with chain parameters \( A(s), B(s), C(s), D(s) \) be extracted from \( G(s) \). Assume that, when the chain parameters are written over a least common denominator, the numerator of \( A(s) \) is of nth degree. Let the remainder \( H(s) = R(s)/S(s) \) be of order \((m - n)\). Then \( H(s) \) is BR.

**Proof:** The proof has two parts:
1) Show that \( |H(j\omega)| \leq 1 \) for all \( \omega \).
2) Show that \( S(s) \) has \( m - n \) zeros in the open left-half plane.

Now recall that \( G(s) \) and \( H(s) \) are related through

\[
G(s) = \frac{P(s)}{Q(s)} = \frac{C(s)+D(s)H(s)}{A(s)+B(s)H(s)}.
\]

(A1)

In (A1), \( A(s), B(s), C(s), D(s) \) can be considered as polynomials, since the common denominator cancels. (In particular \( A(s) \) is an nth degree polynomial.) We know that on the \( j\omega \)-axis:

\[
AB^* = CD^*, \quad AA^* = DD^*, \quad BB^* = CC^*.
\]

(A2)

Moreover, \( |G(j\omega)| \leq 1 \) for all \( \omega \), hence we have

\[
A^2 + |B|^2|H|^2 \geq |B|^2 + |A|^2|H|^2, \quad \text{for all } \omega.
\]

(A3)

Recalling that \( T_{22} = -B/A \), we get the inequality:

\[
\frac{1+|T_{22}|^2|H|^2}{|T_{22}|^2 + |H|^2} \geq 1, \quad \text{for all } \omega.
\]

(A4)

In view of the following inequality:

\[
|T_{22}(j\omega)|^2 \leq 1, \quad \text{for all } \omega.
\]

(A5)

(which follows because the two-pair is LBR), we, therefore, conclude that (A4) implies

\[
|H(j\omega)| \leq 1, \quad \text{for all } \omega.
\]

(A6)

This completes one part of the proof. Next, \( Q(s) \) is given by

\[
Q(s) = A(s)S(s) + B(s)R(s).
\]

(A7)

For \( s = j\omega \), we note that

\[
|A(j\omega)S(j\omega)| \geq |B(j\omega)R(j\omega)|
\]

in view of (A5) and (A6). Moreover, the following statement is true:

\[
A(j\omega)S(j\omega) + B(j\omega)R(j\omega) \neq 0, \quad \text{for any } \omega.
\]

(A8)

In order to see this, note that if \( Q(j\omega_0) = 0 \), then \( P(j\omega_0) \) should also be zero, so that \( G(s) \) is stable. But this would imply that the degree of \( G(s) \) as given by (A1) is less than \( m \), because of the cancellation of \((s^2 + \omega_0^2)\).

Now refer to Fig. 12, where we have formed a closed contour with \( R \) "sufficiently large" so that all the zeros of the polynomials \( A(s)S(s) \) and \( B(s)R(s) \) in the left-half plane, are inside the contour. The following conditions are true:

a) \( A(s)S(s) \) and \( B(s)R(s) \) are analytic interior to the closed contour, and continuous on the closed contour.

b) \(|A(s)S(s)| > |B(s)R(s)| \) on the contour, and

c) \( A(s)S(s) + B(s)R(s) \neq 0 \) on the contour.

Consequently, Rouche's Theorem ([22, p. 5]) can be applied, leading to the following conclusion:

Number of zeros of \( Q(s) \) in \( \text{Re } s < 0 \)

\[
= \text{Number of zeros of } A(s)S(s) \text{ in } \text{Re } s < 0.
\]

(A10)

In conclusion, the number of zeros of \( S(s) \) in the open left-half plane is precisely \((m - n)\) and this completes the proof.

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