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Adaptive Systems, Lack of Persistency of Excitation and Bursting Phenomena*

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When persistency of excitation is lost in adaptive systems, the system variables from time to time exhibit bursts of oscillatory behaviour.

Key Words—Adaptive control; adaptive systems; system identification.

Abstract—The paper considers four algorithms arising in adaptive systems problems, equation error identification, output error identification, reference trajectory following and adaptive pole positioning. Theoretical explanation, backed up by simulations, is offered for the phenomenon of bursts of oscillatory behaviour in the presence of excitations which are not persistently exciting.

1. INTRODUCTION

THE PURPOSE of this paper is to describe what happens in several adaptive algorithms when persistency of excitation conditions are lost. To put the problem in perspective, suppose a fourth order single-input, single-output system is to be identified: there are eight real parameters, the numerator and denominator coefficients, to be found. If the system is excited with a single sinusoid, there is no algorithm, adaptive or nonadaptive, which could possibly determine all eight parameters. Only two functions, maybe linear, of the parameters, can be obtained. Were an adaptive identifier being used we would say that the input was not persistently exciting. (A formal definition will be given subsequently. For the moment, regard a signal as persistently exciting if it is frequency rich.)

A result of Anderson and Johnson (1982) is that persistently exciting inputs ensure robust identification exponentially fast of the system parameters for a stable system with no pole-zero cancellations, if an equation error or output error identification procedure is used. The point of this paper in relation to these identification methods is that, in the *absence* of persistency of excitation, the output error algorithm will not only fail to identify the parameters correctly, but can exhibit a bursting phenomenon. Its output can, as a result of noise, nonlinearities, round-off errors and so on, exhibit periods (bursts) of unstable, possibly oscillatory, behaviour between quiescent periods. (Since trajectories are not actually unbounded, a purist would not describe such behaviour as unstable.) During these quiescent periods, the identifier output will track the output of the plant being identified.

A similar sort of phenomenon can be observed for an adaptive control algorithm aimed at reference trajectory following, see Goodwin, Ramadge and Caines (1980). This algorithm is claimed in Goodwin, Ramadge and Caines (1980) to be stable, to give bounded signals and in the limit, to ensure that the plant output approaches the reference trajectory, given a minimum phase, single-input, single-output discrete-time plant with known degree and known delay. That is of course correct for the ideal algorithm. Also if the reference trajectory is persistently exciting, and there are no pole-zero cancellations in the plant, the ideal algorithm of Goodwin, Ramadge and Caines (1980) is robust, see Anderson and Johnson (1982) so that small departures from the ideal will not significantly affect the conclusion. However, this paper argues that if the reference trajectory is not persistently exciting, the bursting phenomenon will be encountered in practice. Signals will still be bounded, but because of the bursting, correct tracking of the reference trajectory by the plant output will not occur, except during the quiescent periods between the bursts of instability.

It will often be the case that the desired reference trajectory is a setpoint. This paper is then making the case that with such reference trajectories, adaptive controllers can be expected to misbehave, since such trajectories are obviously not persistently exciting. Such a conclusion appears to correspond with anecdotal reports of field performance of adaptive controllers.

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A third illustration of bursting phenomena is provided by an adaptive pole positioning problem. In Anderson and Johnstone (1982) an algorithm for adaptive pole positioning of a first order system is presented; the algorithm involves a minor adjustment of the intuitively attractive strategy of running an identifier and implementing a feedback based on the current parameter estimates of the identifier. This latter procedure, when unadjusted, provides a locally convergent algorithm, see Goodwin and Sin (1981) while the adjusted procedure, when used with a persistently exciting external control, provides a globally convergent procedure, see Anderson and Johnstone (1982). In this paper, we argue that bursting phenomena will be likely when the external control is not persistently exciting.

In all three problems considered, we shall present both theoretical analysis and simulation data.

There has certainly been for some time general recognition of the possibility of bursting phenomena in the absence of persistency of excitation. The fairly simple analysis here helps to explain the phenomena. It is important to note that we are not studying the effects of "covariance wind-up" when recursive least squares algorithms are used, and the weighting matrix becomes more and more ill-conditioned as time goes on in those situations where there is not persistency of excitation. In fact, in both the theoretical discussions and simulations, we have chosen to use gradient-type parameter estimation algorithms in order to disentangle the effect we are describing from covariance wind-up. The effect of the latter will be to heighten the inaccuracies in the parameter estimation algorithms. It is the existence of such inaccuracies in practice, stemming from noise and so on, which is fundamentally responsible for the phenomena we describe. Least squares parameter estimation will simply make avoidance of the phenomena that much more difficult.

The plan of the paper is as follows. Sections 2, 3 and 4 discuss respectively output error identification, reference trajectory following and adaptive pole positioning. We use first order systems throughout, to keep the analysis simple, our aim being to illustrate. Section 5 contains concluding remarks.

2. OUTPUT ERROR IDENTIFICATION

We shall consider identification, using the output error method, of the plant

$$y_k = ay_{k-1} + bu_{k-1} \tag{2.1}$$

where $|a| < 1$, for stability. As is well known, the output error identifier is governed by the equations, see Landau (1979)

$$\hat{y}_k = \hat{a}_k z_{k-1} + \hat{b}_k u_{k-1} \tag{2.2a}$$

$$\begin{bmatrix} \hat{a}_{k+1} \\ \hat{b}_{k+1} \end{bmatrix} = \begin{bmatrix} \hat{a}_k \\ \hat{b}_k \end{bmatrix} + \mu \begin{bmatrix} z_{k-1} \\ u_{k-1} \end{bmatrix} v_k \tag{2.2b}$$

$$v_k = y_k - z_k + c_1(y_{k-1} - z_{k-1}) \tag{2.2c}$$

$$z_k = \hat{a}_{k+1} z_{k-1} + \hat{b}_{k+1} u_{k-1}. \tag{2.2d}$$

The quantities $\hat{y}_k$, z_k are the *a priori* and *a posteriori* output, and the constant c_1 is such that $(1 + c_1 z^{-1})(1 + az^{-1})^{-1}$ is strictly positive real. Implementation of (2.2c) is usually replaced by implementation of the following equivalent equation

$$v_k = \frac{y_k - \hat{y}_k + c_1(y_{k-1} - z_{k-1})}{1 + \mu z_{k-1}^2 + \mu u_{k-1}^2}. \tag{2.2e}$$

With $|a| < 1$, $c_1 = 0$ ensures $(1 + c_1 z^{-1})(1 + az^{-1})^{-1}$ is strictly positive real. So we shall assume $c_1 = 0$.

Our strategy will be to consider the behaviour of the ideal algorithm, by appealing to known results and simple algebra. Then we shall argue that what happens in practice can be markedly different, at least for a particular sort of input.

It is known, see Landau (1979), that the ideal algorithm causes $y_k - \hat{y}_k \to 0$, $y_k - z_k \to 0$. Suppose that $u_k \equiv 1$. Then (2.1) and (2.2) show that $\hat{b}_k(1 - \hat{a}_k)^{-1} \to b(1 - a)^{-1}$. (Thus, as one could reasonably expect, the DC gain of the adjustable model approaches that of the plant.)

Now suppose that convergence has actually occurred, and that we adjust the time origin so that $\hat{b}_1(1 - \hat{a}_1)^{-1} = b(1 - a)^{-1}$, and $y_0 = z_0 = b(1 - a)^{-1}$. Then (2.2a) yields $y_1 = b(1 - a)^{-1}$, (2.2c) yields $v_1 = 0$, (2.2b) yields $\hat{a}_2 = \hat{a}_1$, $\hat{b}_2 = \hat{b}_1$. Continuing the argument yields

$$\hat{b}_k \equiv \hat{b}_0 \quad \hat{a}_k \equiv \hat{a}_0 \quad z_k \equiv \hat{y}_k \equiv y_k \equiv b(1 - a)^{-1}. \tag{2.3}$$

The matching of the unknown system output with the adjustable model output suggests that identification has occurred, and no adjustment of parameter estimates takes place.

Now consider what can happen with real implementation, using floating point arithmetic. While $\hat{b}_k(1 - \hat{a}_k)^{-1}$ will stay virtually constant, $\hat{a}_k$ and $\hat{b}_k$ can separately drift. Once $\hat{a}_k$ drifts outside of $[-1, 1]$, (2.2d) shows that the adjustable model becomes unstable, and tracking of y_k by z_k will no longer persist, until the constant parameters b and a have been learnt again, or at least until $\hat{a}_k$ has come within the $(-1, 1)$ interval. At that point, z_k will again be well-behaved, but the drifting of $\hat{a}_k$, $\hat{b}_k$ can occur again. So here, there is a possibility of bursting phenomena. [Of course, had the input been persistently exciting, the drifting is not possible, see Anderson and Johnson (1982).]

Notice that such bursting phenomena will not be encoutered in equation error identification although drifting is still to be expected. Equations (2.2c) and (2.2d) are replaced by

$$\hat{y}_k = \hat{a}_k y_{k-1} + \hat{b}_k u_{k-1} \tag{2.4}$$

and whether or not $|\hat{a}_k| > 1$, $\hat{y}_k$ cannot go temporarily unstable (in the same manner as z_k) since there is no feedback mechanism to produce the instability.

Simulation data is illustrated in Figs 1–6.

Figure 1 shows the envelope of the $\hat{y}_k$ sequence for 10,000 points. The simulation is for the plant (3.1) with $a = 0.9$ and $b = 1$, $y_0 = z_0 = 10$, $\hat{a}_1 = -0.98$, $\hat{b}_1 = 19.8$, and $u_k = 1$ for all k. A small level of measurement noise is added to y_k and a small level additive noise is also inserted in the update equations for $\hat{a}_k$, $\hat{b}_k$. The standard deviation of the noise was 0.002 in each case. Notice that $b(1-a)^{-1} = \hat{b}_1(1-\hat{a}_1)^{-1}$, i.e. the d.c. gains of the plant and model are initially the same. Also, $\hat{a}_1$ is very close to -1, which is the boundary of the instability point.

Figure 2 shows the variation in $\hat{a}_k$. Notice that each of the two bursting phenomena causes an improvement in the error $|a - \hat{a}_k|$ (though the error remains very substantial).

Figures 3 and 4 show the fine structure of the first burst.

Figures 5 and 6 show the effect on the envelope of the $\hat{y}_k$ sequence and the estimate $\hat{a}_k$ of a step change in input at $k = 1000$. Notice the improvement in the estimate of $\hat{a}_k$ caused by the step change (around which u_k can be regarded as locally rich). Notice also that although $\hat{y}_k$ behaves well before the step change, suggesting that convergence has occurred, this is far from the case in that a bursting-like mode is exhibited when the step change hits.

3. BURSTING PHENOMENA IN ADAPTIVE CONTROL—REFERENCE TRAJECTORY FOLLOWING

Consider the plant

$$y_k = a y_{k-1} + b u_{k-1} \tag{3.1}$$

with a, b unknown, $b \neq 0$, and suppose the adaptive control task is to follow a reference trajectory y_k^*. As illustrated in Goodwin, Ramadge and Caines (1981) the general approach is to estimate a, b with estimates $\hat{a}_k$, $\hat{b}_k$ assumed available just after measurement of y_k; then u_k is chosen to secure the right y_{k+1}^* under the assumption that $\hat{a}_k$, $\hat{b}_k$ are the correct a, b. Thus

$$y_{k+1}^* = \hat{a}_k y_k + \hat{b}_k u_k. \tag{3.2}$$

(If $\hat{b}_k$ is very small, the algorithm is adjusted.)

One can use either a gradient algorithm or least-squares type algorithm to estimate a, b. Let us suppose a gradient algorithm is used, so that questions of "covariance wind-up" do not confuse the picture. Our interest in this section will be the performance of the algorithm for a particular

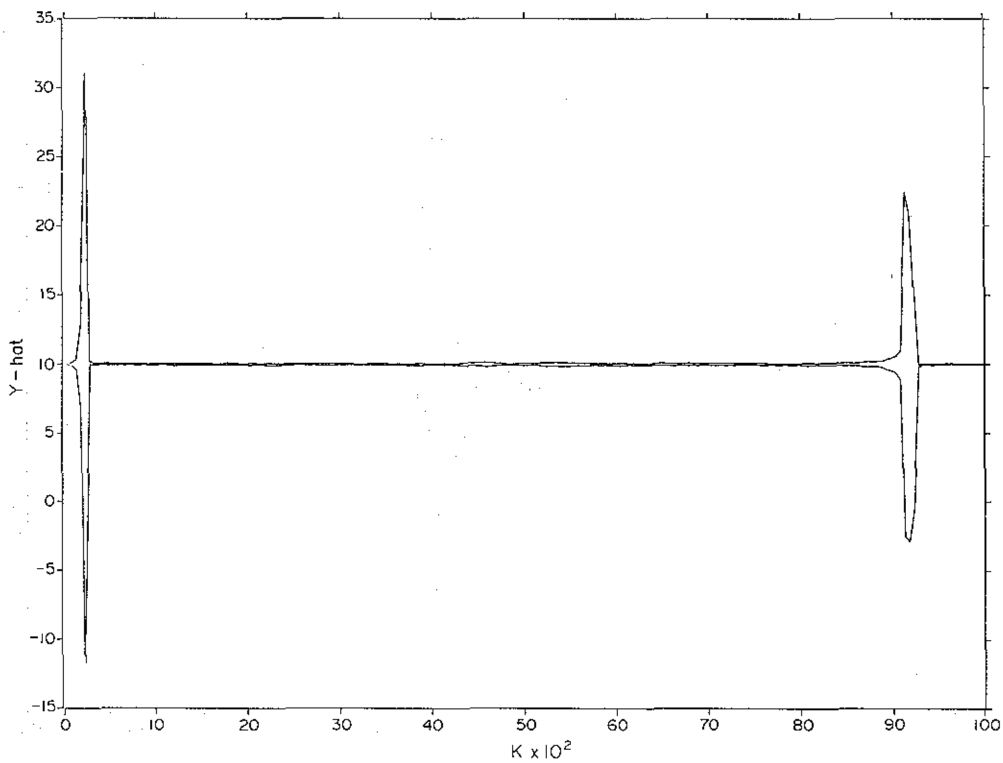

FIG. 1. Envelope of adaptive identifier output showing bursting.

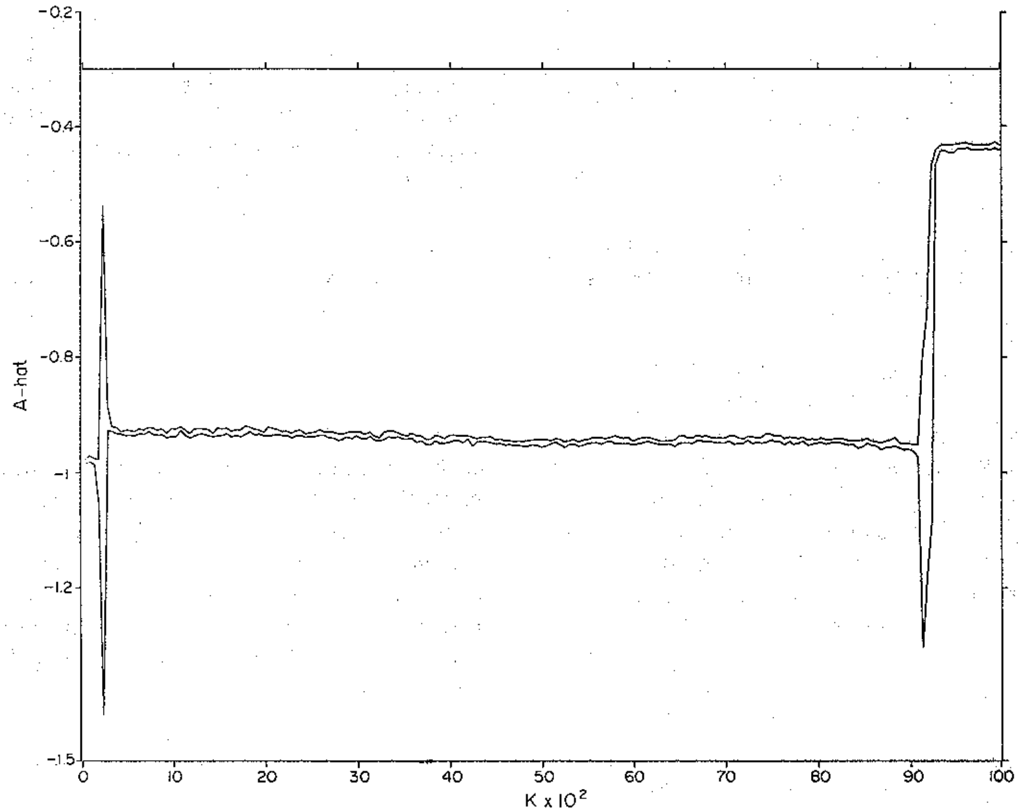

FIG. 2. Envelope of estimate of denominator coefficient.

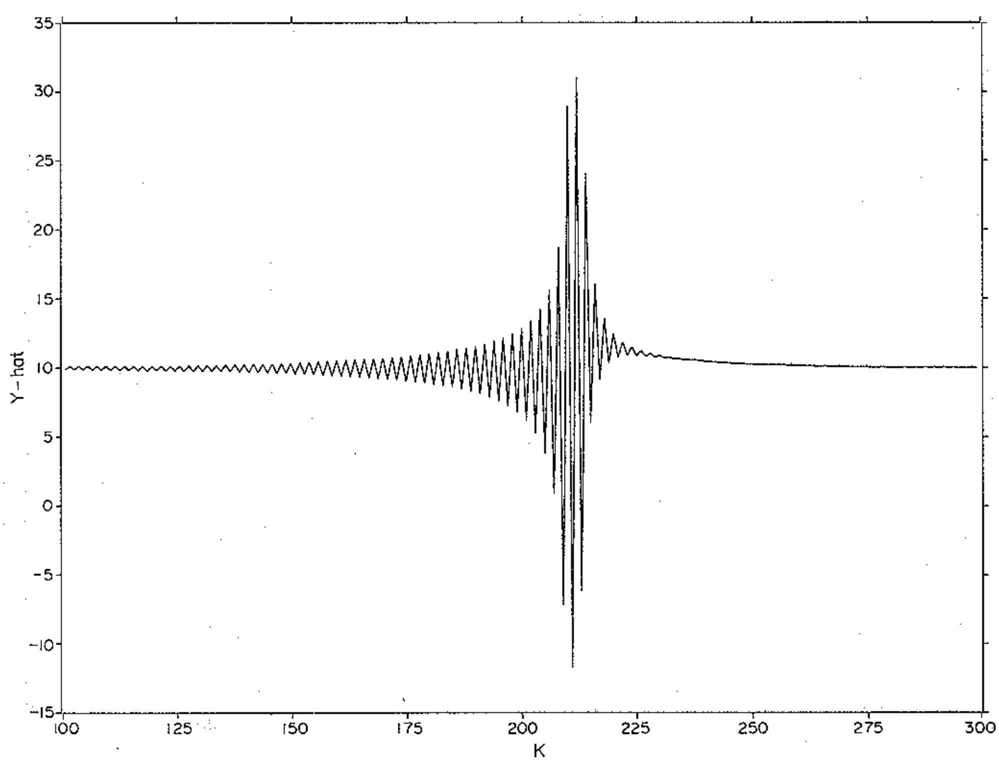

FIG. 3. Detail of bursting in adaptive identifier output.

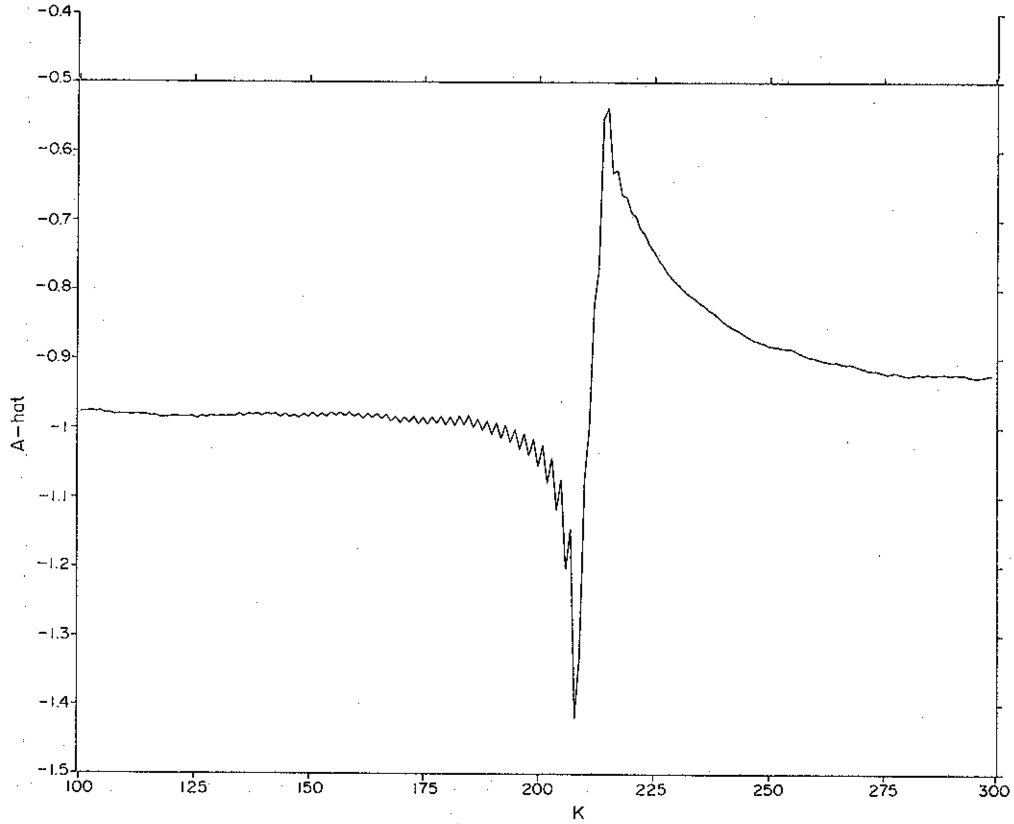

FIG. 4. Detail of bursting in estimate of denominator coefficient.

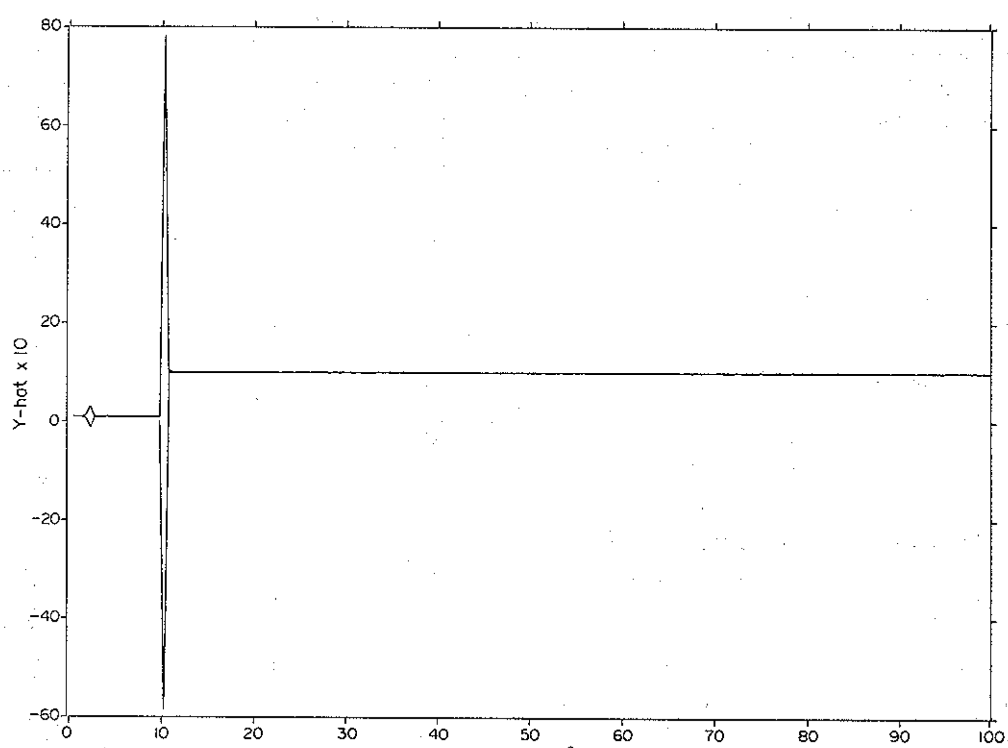

FIG. 5. Envelope of adaptive identifier output including bursting triggered by level change.

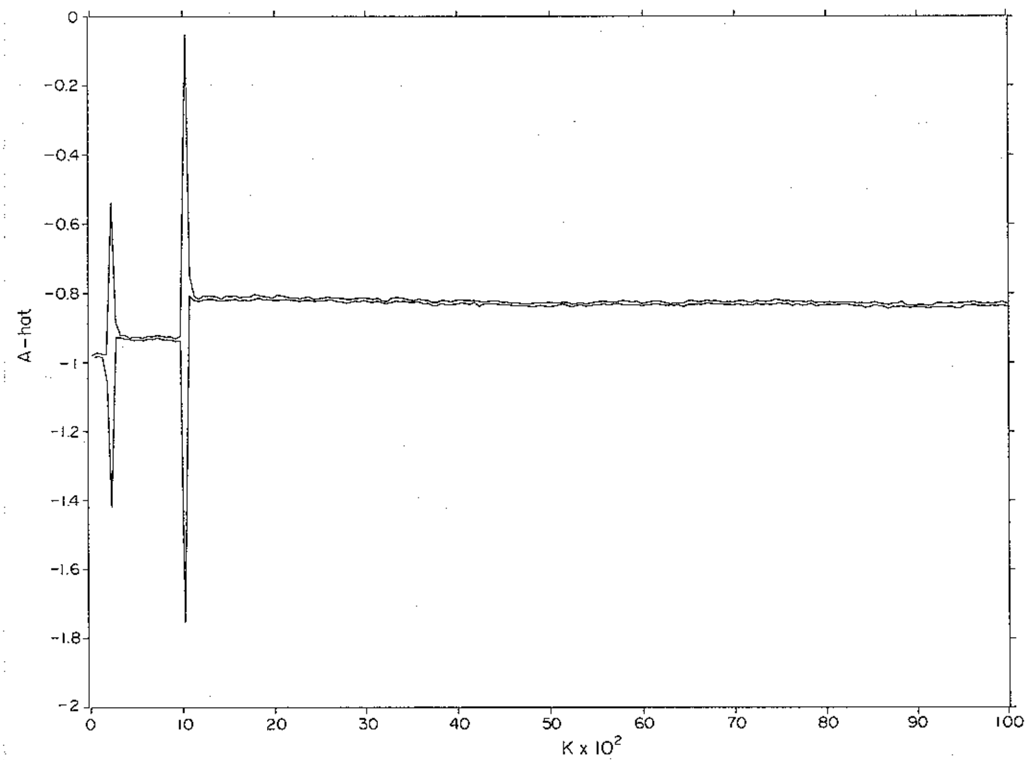

FIG. 6. Envelope of estimate of denominator coefficient, including bursting triggered by level change.

reference trajectory: we shall assume

$$y_k^* \equiv 1. \tag{3.3}$$

To begin with, we shall explain what is supposed to happen, using a combination of published results and a little algebra. Then we shall describe what will happen in practice; this is quite different to what should happen. We also illustrate what happens in practice with simulation data.

According to the algorithm of Goodwin, Ramadge and Caines (1981) we have $y_k - y_k^* \to 0$ as $k \to \infty$. Then (3.2) implies

$$u_k \to \frac{1 - a_k}{\hat{b}_k} \tag{3.4}$$

and (3.1) then implies

$$1 - a - b\frac{1 - \hat{a}_k}{\hat{b}_k} \to 0 \tag{3.5}$$

as $k \to \infty$. This has a simple interpretation. The d.c. gain of the plant is $b/(1-a)$ and (3.5) says that the estimated d.c. gain of the plant approaches the true d.c. gain, something which is entirely reasonable given that the excitation y_k^* is a d.c. signal.

Suppose $a = 2$, $b = 1$. Then $\hat{a}_k - \hat{b}_k \to 1$. However, it appears that we can get no further information about $\hat{a}_k$, $\hat{b}_k$ from this algorithm which would allow their separate identification. Indeed, this is easy to check. Suppose in fact that the convergence of $\hat{a}_k - \hat{b}_k$ to 1 has occurred and we reinitialize the time origin so that $\hat{a}_0 - \hat{b}_0 = 1$, and $y_0 = 1$. Then (3.2) yields $u_0 = (1 - \hat{a}_0)/\hat{b}_0 = -1$, (3.1) yields $y_1 = 1$, and so $y_1^* - y_1 = 0$. This zero error ensures that $\hat{a}_1 = \hat{a}_0$, $\hat{b}_1 = \hat{b}_0$. In fact, the process repeats, and we get $\hat{a}_j = \hat{a}_0$, $\hat{b}_j = \hat{b}_0$ for all j. One also has $y_j^* - y_j = 0$ for all j, i.e. perfect tracking, despite the nonidentification of a, b. Given this perfect tracking, one might well ask what the problem is.

Let us consider what we are actually implementing. Combining (3.2) and (3.1) (and with $a = 2$, $b = 1$), we obtain

$$y_{k+1} = \left(2 - \frac{\hat{a}_k}{\hat{b}_k}\right) y_k + \frac{1}{\hat{b}_k} y_{k+1}^*. \tag{3.6}$$

With $\hat{b}_k = \hat{a}_k - 1$, then

$$y_{k+1} = \frac{\hat{a}_k - 2}{\hat{a}_k - 1} y_k + \frac{1}{\hat{a}_k - 1} y_{k+1}^*. \tag{3.7}$$

Should we have $|(\hat{a}_k - 2)(\hat{a}_k - 1)^{-1}| > 1$, or equivalently $\hat{a}_k < 3/2$, (3.6) or (3.7) will be unstable: while in principle, one can have $y_k = y_k^* = 1$ for all k, this will place (3.7) at an unstable steady-state operating point.

The algebraic calculations done using the adaptive control algorithm which assert that the system can remain at an unstable equilibrium point cannot be reflected in any practical implementation using floating point arithmetic, and the actual trajectories to be observed will be quite unlike those predicted from the algebraic calculation; they will depend on the outgoing direction from the unstable operating point. In the sense that the theory in unamended form predicts trajectories which are unlike those actually encountered, we can say that the adaptive algorithm is not robust.

More generally for arbitrary a and b, if $(1-\hat{a}_0)/\hat{b}_0 = (1-a)/b$, $y_0 = 1$ and $y_k^* = 1$ for all k, algebra suggests that $y_k = 1$ for all k, and what is actually being implemented is

$$y_{k+1} = \left(a - \frac{\hat{a}_k}{\hat{b}_k} b\right) y_k + \frac{1}{\hat{b}_k} u \qquad (3.8)$$

and with $(1-\hat{a}_k)/\hat{b}_k = (1-a)/b$ this is

$$y_{k+1} = \frac{a-\hat{a}_k}{1-\hat{a}_k} y_k + \frac{1}{\hat{b}_k} u_k. \qquad (3.9)$$

So whether or not $|a| < 1$, i.e. the original plant is stable, there is obviously the potential for instability.

Above, we indicated that the adaptive algorithm will not be robust if a situation of instability is encountered, and this is confirmed below by simulation results. Let us however now make further remarks about what a real algorithm will do. As has already been indicated, with constant y_k^*, we can expect that $\hat{b}_k(1-a) - b(1-\hat{a}_k) \to 0$. Thus one linear functional of $[\hat{a}_k \hat{b}_k]'$ is learnt by the identification part of the adaptive control algorithm. However, a second linear functional is not learnt. We must then expect that, as a result of measurement and or algorithm noise, the orthogonal linear functional will drift in a random manner. While the stability condition

$$\left|\frac{a-\hat{a}_k}{1-\hat{a}_k}\right| < 1 \qquad (3.10)$$

persists, nothing untoward will be noticed. However, when the drifting causes violation of this condition, instabilities will arise. The effective excitation of the plant is no longer a steady (d.c.) signal but there is also a growing exponential. This increase in the complexity or the frequency content of the exciting signals will allow improved, perhaps correct, identification of the *two* plant parameters, i.e. after a limited period of instability, we can expect that a, b will be better identified by $\hat{a}_k$, $\hat{b}_k$. Once this happens, the closed loop becomes stable again, and we are back in the situation where the plant excitation is not sufficiently rich as to maintain identification of all linear functionals of $[\hat{a}_k, \hat{b}_k]'$. Accordingly, the drifting process starts again, to be followed by a limited interval of unstable behaviour in which correct parameters are learnt, and so the cycle repeats.

When $a = 2$, $b = 1$, (3.7) represents a stable system for all $\hat{a}_k > 3/2$, and an unstable system for all $\hat{a}_k < 3/2$. With $\hat{a}_k = 3/2$ (3.7) becomes

$$y_{k+1} = -y_k + \frac{1}{\hat{a}_k - 1} y_{k+1}^*.$$

The significance of this is that the boundary between stability and instability is defined by a mode at -1, rather than $+1$. Therefore, the periods of unstable behaviour referred to in the above discussion will be oscillatory. The same conclusion holds for any $a > 1$. For any $a < 1$ however, the boundary is defined by a mode at $+1$, and so is less likely to be so immediately apparent. (The case $a = 1$ is special, and has been tacitly ruled out in the preceding discussion, since the d.c. gain of the plant is then infinity.)

Simulation data is shown in Figs 7–11. In the simulation, small levels of noise were added to y_k and to the parameter update equations; initial estimates for $\hat{a}$ and $\hat{b}$ were chosen as 1.5 and 0.5 (so that the closed loop system lay right on the stability boundary, while the d.c. gain of the plant and model were matched). Also, the reference trajectory for the first four figures was $y_k^* \equiv 1$, with $y_0 = 1$.

Figure 7 shows the envelope of values of y_k over 500,000 points and Fig. 8 the corresponding envelope of values of $\hat{a}_k$. Notice that $\hat{a}_k$ certainly reaches the correct value of $a = 2$ then leaves it. The variation in $\hat{b}_k$ is not shown, but very closely obeys $\hat{b}_k = \hat{a}_k - 1$, as expected. There is a very significant interval between the early bursting in y_k and the later bursting, and the bursting heights are clearly different. Figures 9 and 10 show more detail again, with Fig. 9 showing the envelope of y_k over the first 4000 points and Figs 10 and 11 showing the bursting detail over the time interval (3300, 3499), as well as the variation of $\hat{a}_k$ over this range. Note that the onset of bursting is not precisely synchronous with $\hat{a}_k$ equalling 1.5. Simulation can be used also to see the effect of a change in level of y^* for 1–10, occurring at time 500,000, by plotting the envelope of y_k. The bursting around the time of the level change is considerable.

Bursting should not be expected with a persistently exciting reference trajectory y_k^*. As established in Anderson and Johnson (1982), stability of the adaptive algorithm is robust. [For a precise definition of persistency of excitation, consult Anderson and Johnson (1982).] At the intuitive level, persistently exciting y_k^* are those which contain sufficient frequencies as to allow continuously updatable estimation of all parameters (i.e. a and b) above, rather than a restricted

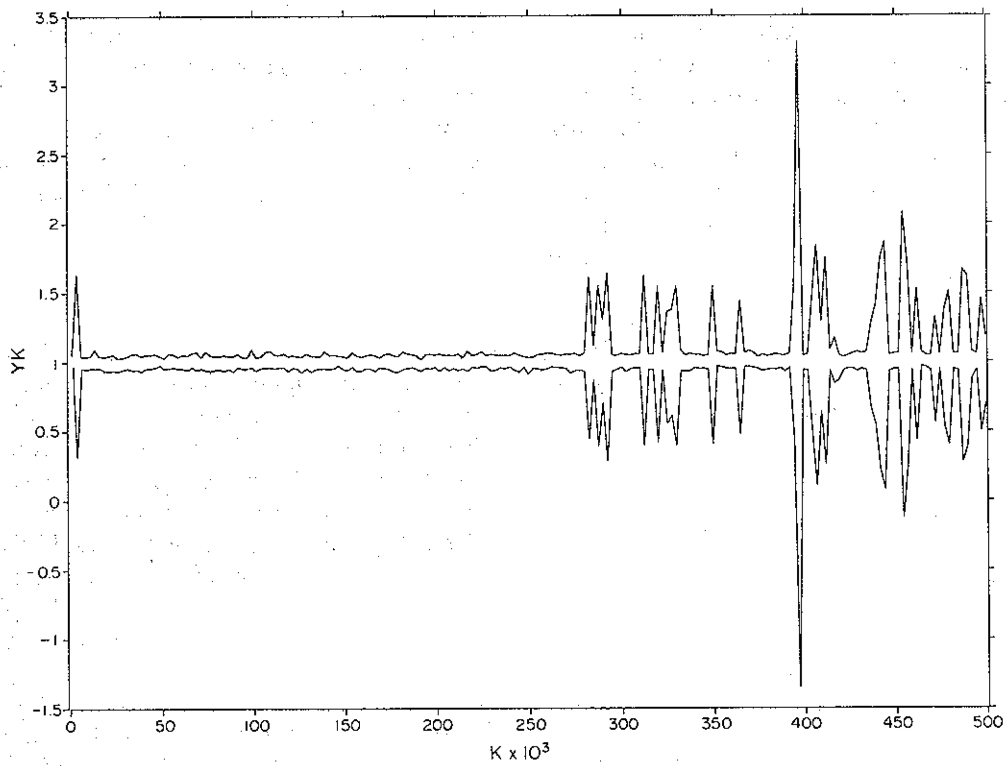

FIG. 7. Envelope of plant output using reference trajectory following adaptive controller.

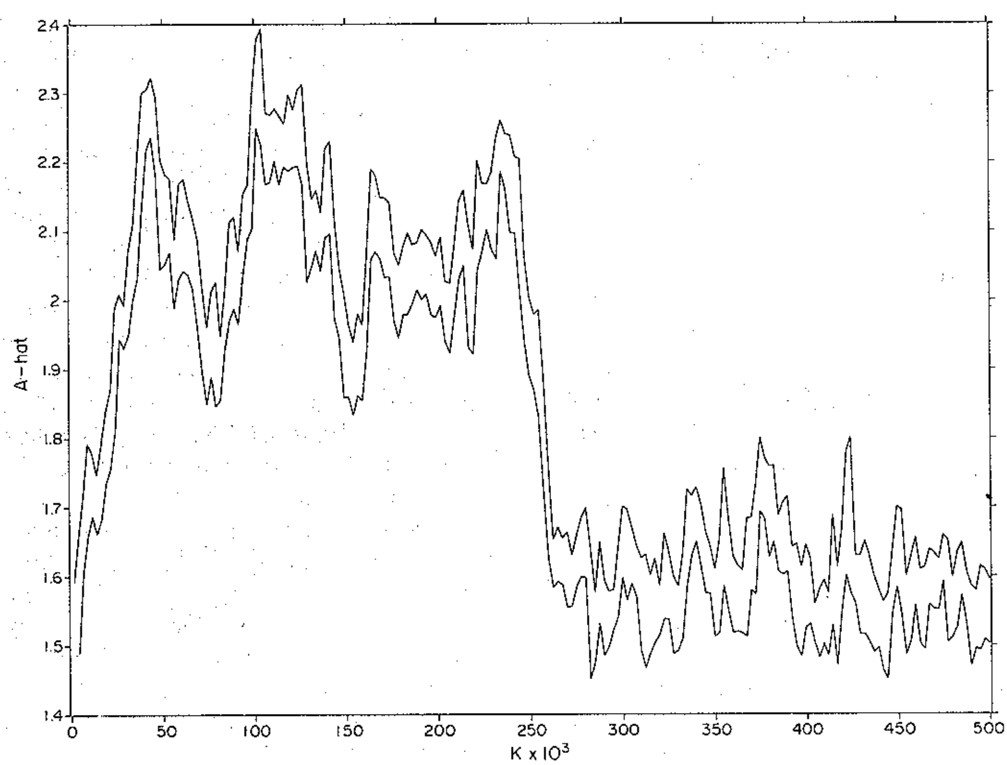

FIG. 8. Envelope of estimate of denominator coefficient using reference trajectory following adaptive controller.

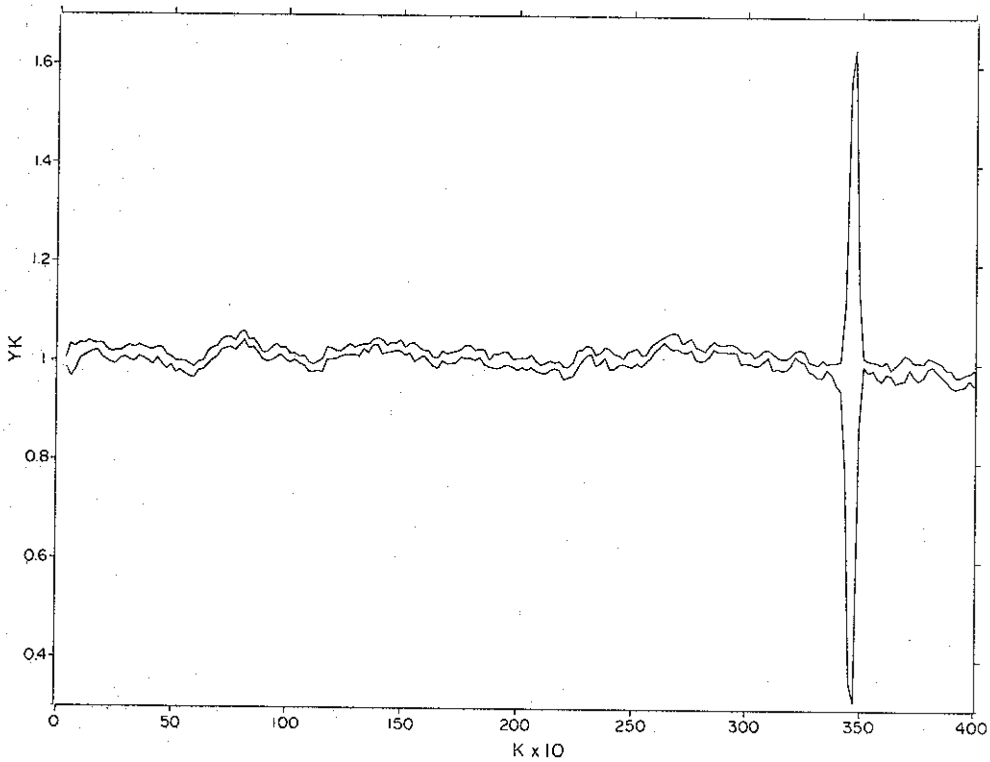

FIG. 9. Envelope of plant output using reference trajectory following adaptive controller.

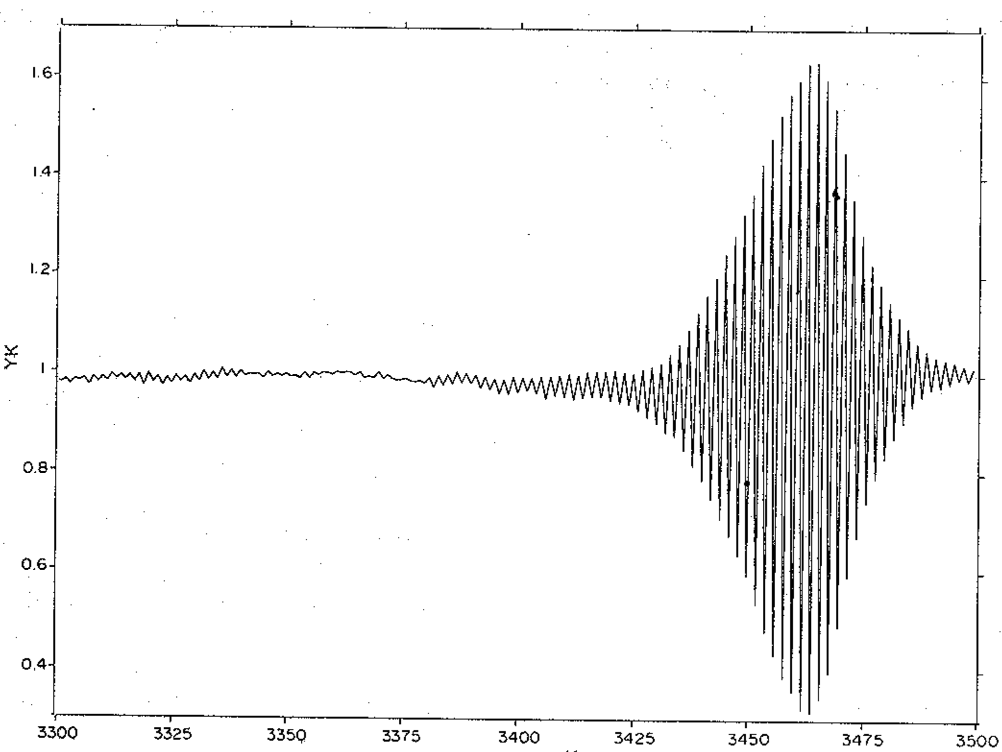

FIG. 10. Plant output showing bursting during use of reference trajectory following adaptive controller.

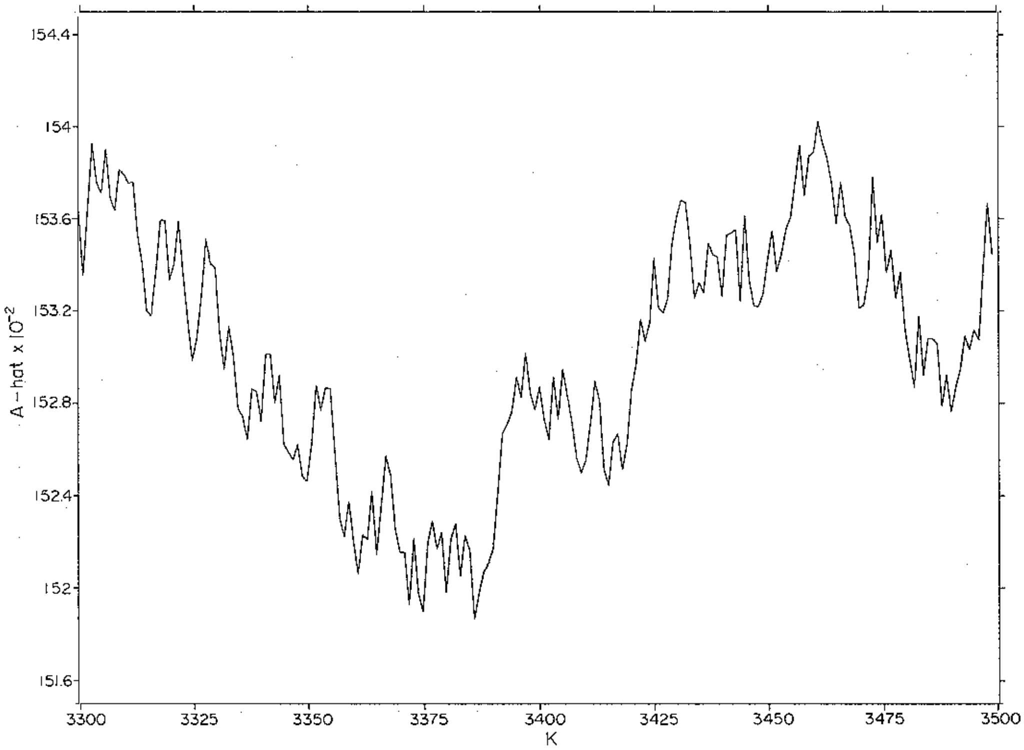

FIG. 11. Estimate of denominator coefficient using reference trajectory following adaptive controller.

number of linear functionals of the parameters (1 or 0 above).

4. ADAPTIVE POLE POSITIONING

Suppose the plant is, as before,

$$y_{k+1} = ay_k + bu_k \tag{4.1}$$

and our aim is to find a feedback law $u_k = fy_k + u_k^*$ so that $a + bf = d$ where $0 < d < 1$ and with d prescribed. The values of a, b are unknown, but one knows that $|b| \geqslant \rho > 0$ for some ρ. The general approach is to estimate a, b via $\hat{a}_k$, $\hat{b}_k$ and implement

$$u_k = \hat{f}_k y_k + u_k^* \tag{4.2}$$

where $\hat{f}_k$ is chosen so that

$$(\hat{a}_k + \hat{b}_k \hat{f}_k) = d. \tag{4.3}$$

If $|\hat{b}_k| < \rho$, an alternative procedure is used to define $\hat{f}_k$, and with persistency of excitation of u_k^*, the algorithm converges in a robust manner, see Anderson and Johnstone (1982).

Let us suppose that the initial estimates $\hat{b}_0$, $\hat{a}_0$ of b, a satisfy

$$\frac{\hat{b}_0}{1 - \hat{a}_0} = \frac{b}{1 - a}. \tag{4.4}$$

(For convenience, suppose also that $a \neq 1$.) The closed loop plant at time k is

$$y_k = \left(a + b\frac{d - \hat{a}_{k-1}}{\hat{b}_{k-1}}\right) y_{k-1} + bu_{k-1}^* \tag{4.5}$$

and the prediction $\hat{y}_k$ of y_k made by the identifier is

$$\hat{y}_k = dy_{k-1} + \hat{b}_{k-1} u_{k-1}^*. \tag{4.6}$$

Suppose $u_k^* \equiv 1$, and that $y_0 = \hat{b}_0/1 - d$. Then from (4.5) and (4.6) we obtain $y_1 = \hat{y}_1 = \hat{b}_0(1 - d)^{-1}$. Since $y_1 - \hat{y}_1 = 0$ the identifier makes no change to the estimate of a and b, and so $\hat{a}_1 = \hat{a}_0$, $\hat{b}_1 = \hat{b}_0$. Thus generally, for all k,

$$y_k \equiv \hat{y}_k \equiv \frac{\hat{b}_0}{1 - d}$$

$$\hat{a}_k \equiv \hat{a}_0 \quad \hat{b}_k \equiv \hat{b}_0.$$

Now suppose that in particular, $a = 0$, $b = 1$, $d = 0.9$ and $1 = \hat{a}_0 + \hat{b}_0$. From (4.5) the actual closed loop pole will be at

$$0 + 1\frac{0.9 - (1 - \hat{b}_0)}{\hat{b}_0} = 1 - \frac{0.1}{\hat{b}_0}.$$

Obviously if $\hat{b}_0$ is small, the closed loop pole can be unstable, far from 0.9. Again, there is a possibility of bursts of instability, as $\hat{b}_0$ drifts so as to introduce instability. If $\hat{b}_0$ starts off with a positive value greater than 0.05, instability will arise when $\hat{b}_0$ drifts to 0.05, and in this case, the unstable mode will be at -1, i.e. an oscillation form of instability will result.

This instability is revealed in the simulations. Figure 12 shows the envelope of y_k over many iterations—with all bursts shown. Note the great

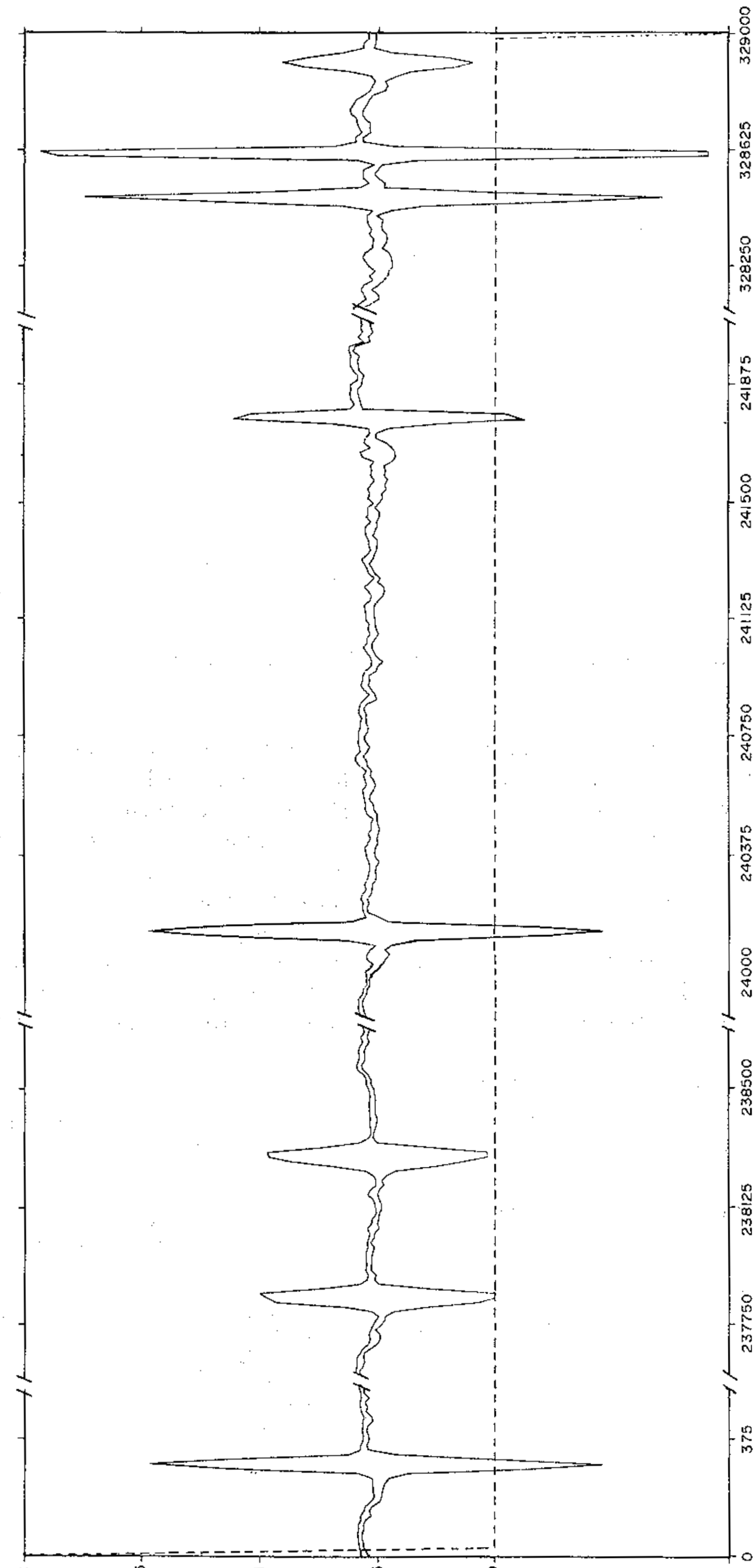

FIG. 12. Envelope of plant output for adaptive pole positioning controller.

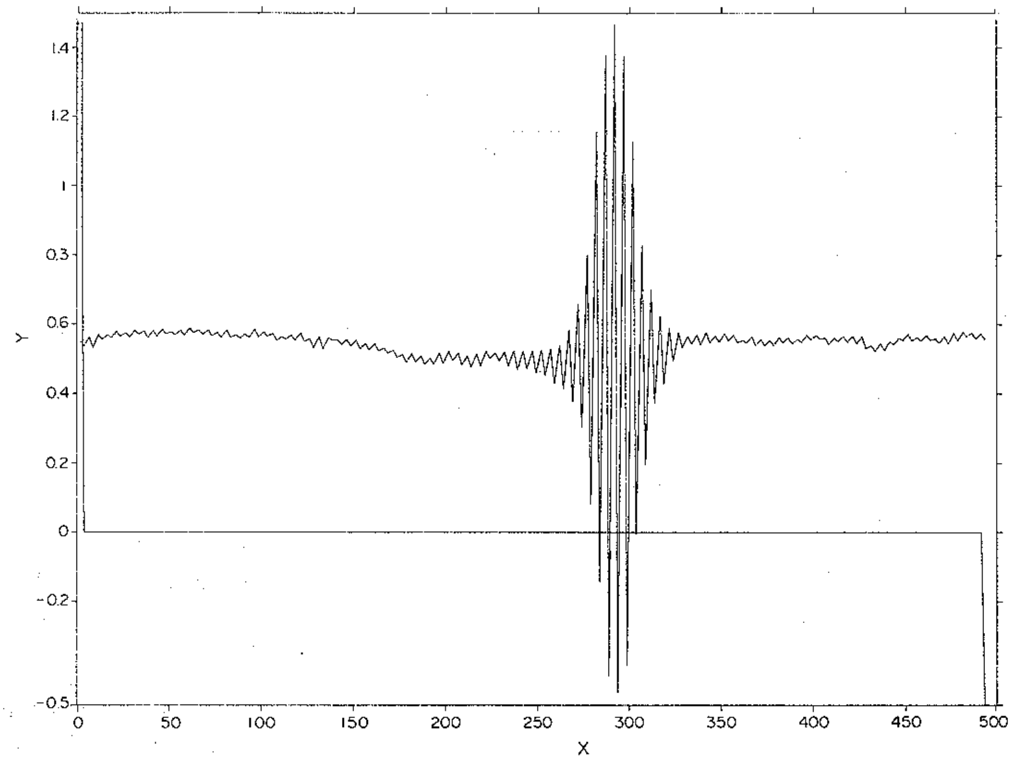

FIG. 13. Detail of bursting in plant output with adaptive pole positioning controller.

variation in gaps between some of the bursts (over 237,000 iterations between the first two, and then less 1000 than iterations till the next).

The detail of the first burst is shown in Fig. 13.

5. CONCLUSIONS

The previous three sections have studied three different adaptive systems problems, and argued theoretically and with simulation data that with nonpersistently exciting inputs, bursting can occur. More specifically, when inputs (using the word in an extended sense, to include a reference trajectory) are not frequency rich, there is the possibility that estimates of individual system parameters can drift, because of noise or other unmodelled or unmodellable effects; if the adaptive system is such as to have within it a closed loop the stability of which is in part determined by parameter estimates, bursting phenomena must then be expected. There can however be very long intervals in which no bursting occurs, times of occurrences of bursts being, to all practical purposes, random. The duration and maximum amplitude of the burst are not completely predictable.

These results do not invalidate the mathematical theories associated with the three adaptive systems problems considered. They do invalidate generalized assertions about nice behaviour in practice, to the extent that they demonstrate the need to qualify such assertions. As commented in the introduction, persistency of excitation [defined in detail in Anderson and Johnson (1982), but roughly frequency richness] is sufficient to guarantee some measure of robustness; this paper has argued why it is necessary.

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