The effects of back contacts to both $n$type and $p$type silicon wafers were investigated. Measurements were made on wafers which had been lapped and subsequently chemically polished on the front face only. The probe head had a linear probe array with an interprobe spacing of 1·00mm.

(i) 0·5Ωcm ntype wafer (0·66mm thick and 25·1mm in diameter)

Resistivity measurements were taken across a diameter of the wafer, at 1mm intervals, with the probe head placed perpendicular to, and symmetrically across, this diameter. The readings are shown in Fig. 2*a*, where the apparent increase in resistivity towards the periphery of the wafer can

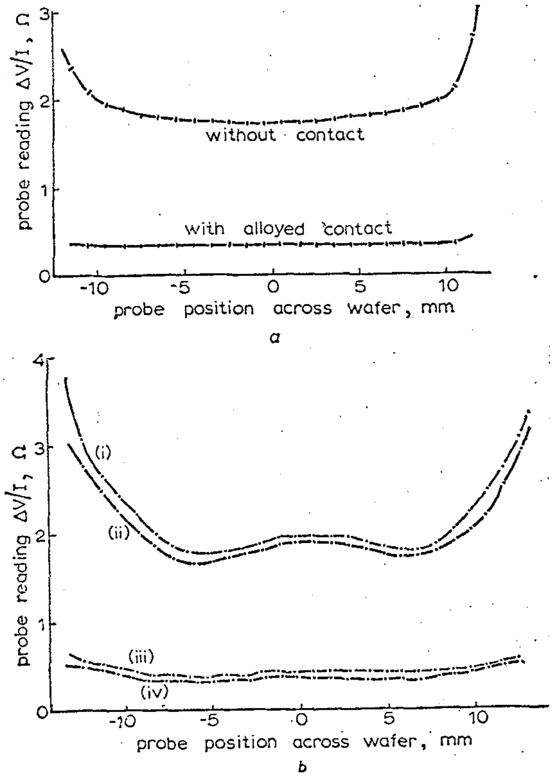

Fig. 2 *4-point-probe readings on silicon wafers*

a Aluminium-alloyed back contact on 0·5Ωcm $p$type silicon wafer
b Nickel-plated back contact on 0·7Ωcm $n$type silicon wafer
(i) Without plating
(ii) Nickel plated
(iii) Sintered
(iv) Second nickel plating

mainly be accounted for[3] by the correction factor C_1. Aluminium was then evaporated at less than 10^{-4}torr on to the back face of the wafer; the thickness of the evaporation was estimated to be 0·4μm. The resistivity of the top face of the wafer was measured again, but no change was found. The wafer was heated for 1min at 700°C in a horizontal furnace in a nitrogen atmosphere, again without any apparent change in resistivity. However, when the wafer was heated for 2min at 700°C, a marked decrease in the apparent resistivity was evident, and this is shown in Fig. 2*a*. For an s/w ratio of 1·52, the expected decrease in apparent resistivity of C_{20}/C_{2s} would be 0·19 times, which is in good agreement with the measured value.

(ii) 0·7Ωcm ptype wafer (0·66mm thick and 28·5mm in diameter)

A similar procedure was followed, except that the bottom face of the wafer was plated with nickel by immersing in a commercial electroplating solution.* The procedure involves masking the top face of the wafer with resistant tape and immersing the wafer for 5min in the plating solution, which is maintained between 80 and 85°C. A nickel layer of about 0·2μm is deposited. Fig. 2*b*(i) shows the 4-point-probe measurement before plating; it is noted that the $n$type variation in apparent resistivity can be attributed mainly to variation in doping during crystal growth. After the nickel plating, there is a small decrease in probe readings, as shown in Fig. 2*b*(ii). A dramatic reduction in apparent resistivity of 0·22 times was achieved by sintering the wafer at 700°C for 5min; this is slightly higher than the expected value of 0·19. It was thought that this discrepancy was probably due to the thinness of the nickel plating, and so the bottom face of the wafer was replated for a further 5min. Fig 2*b*(iv) shows that the apparent resistivity did, in fact, decrease to the final value of 0·19, and no further change was observed when the wafer was again sintered at 700°C for 5 min.

The outlined procedure has been shown to be a sensitive method for checking the ohmic metal–semiconductor contact. It allows a valuable check to be made in the process of the fabrication of alloyed-junction diodes and may also prove useful for the surface-barrier diode.[4]

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2nd September 1966

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SOLUTION OF QUADRATIC MATRIX EQUATIONS

This note considers the solution of equations of the form

$$A + TB + CT + TDT = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where A, B, C and D are prescribed $n \times n$ matrixes, and T is an unknown $n \times n$ matrix. Interest in this equation stems from its appearance in a number of electrical-engineering problems.[1–4] Potter[5] has solved a more restricted form of this equation, where C is restricted to being the Hermitian conjugate of B and the matrix M defined below is diagonalisable.

The solution of eqn. 1 proceeds by defining the $2n \times 2n$ matrix:

$$M = \begin{bmatrix} C & A \\ -D & -B \end{bmatrix} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

and selecting a matrix R bringing M into the Jordan form. Thus

$$R^{-1}MR = J \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

or, with an obvious partitioning and a rearrangement of eqn. 3,

$$\begin{bmatrix} C & A \\ -D & -B \end{bmatrix} \begin{bmatrix} R_1 & R_2 \\ R_3 & R_4 \end{bmatrix} = \begin{bmatrix} R_1 & R_2 \\ R_3 & R_4 \end{bmatrix} \begin{bmatrix} J_1 & J_2 \\ 0 & J_4 \end{bmatrix} \quad (4)$$

Note that J_1 and J_4 are both upper triangular matrixes, the diagonal elements being the eigenvalues of M. Note also that, if the eigenvalues were arranged in a different order, a different matrix R would appear in eqn. 3. Finally, note that the columns of R are (generalised) eigenvectors of M.

If R_3 is nonsingular,

$$T = R_1 R_3^{-1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

is a solution to eqn. 1. To see this, from eqn. 4, we have

$$CR_1 + AR_3 = R_1 J_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6a)$$

and $$-DR_1 - BR_3 = R_3 J_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6b)$$

from which

$$CR_1 R_3^{-1} + A = R_1 J_1 R_3^{-1} \quad . \quad . \quad . \quad . \quad . \quad . \quad (7a)$$

* Produced by PMD Chemicals Ltd.

and $\quad -R_1R_3^{-1}DR_1R_3^{-1} - R_1R_3^{-1}B = R_1J_1R_3^{-1} \quad . \quad . \quad (7b)$

The result immediately follows from eqns. 7*a* and *b*.

The matrix T is, of course, not unique. Since there are $\frac{(2n)!}{n!n!}$ ways of choosing n eigenvectors from $2n$, we would expect a maximum of this number of solutions (always assuming that the matrix R_3 was nonsingular).

This work was performed with the support of US Joint Services Contract NONR 225(83).

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WIDEBAND DISCRIMINATOR WITH OPERATIONAL AMPLIFIER

High-performance wideband low-frequency discriminators can be realised by the use of feedback. The principle of a discriminator with voltage negative feedback is presented.

The use of feedback permits the realisation of wideband low-frequency discriminators with high performance.[1,2] If the transfer function or the feedback circuit, or both, have suitable frequency characteristics, a linear dependence between the output voltage and the frequency of the input signal can be obtained.

For distortionless demodulation, the discriminator characteristic must have a linear dependence on the frequency:[3]

$$T_d\left(j\frac{\omega}{\omega_0}\right) = j\frac{\omega}{\omega_0} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Using an auxiliary polynomial $Q\left(j\frac{\omega}{\omega_0}\right)$, as in active-filter synthesis,[4,5] eqn. 1 can be written as follows:

$$T_d\left(j\frac{\omega}{\omega_0}\right) = \frac{T_d\left(j\frac{\omega}{\omega_0}\right)}{Q\left(j\frac{\omega}{\omega_0}\right)} Q\left(j\frac{\omega}{\omega_0}\right) = P\left(j\frac{\omega}{\omega_0}\right) Q\left(j\frac{\omega}{\omega_0}\right) \quad . \quad . \quad . \quad . \quad (2)$$

From this relation, it may be seen that the required transfer function can be obtained by connecting in cascade two quadrupoles, each with a suitable frequency dependence.

If one of the quadrupoles is a simple RC differentiating circuit, with the transfer function

$$P\left(j\frac{\omega}{\omega_0}\right) = \frac{j\frac{\omega}{\omega_0}}{1 + j\frac{\omega}{\omega_0}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

the second quadrupole must have a transfer function of the following form:

$$Q\left(j\frac{\omega}{\omega_0}\right) = 1 + j\frac{\omega}{\omega_0} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Such a frequency dependence may be obtained with a voltage-negative-feedback amplifier having a high feedback factor and using a simple integrating RC circuit as the feedback circuit. From the condition

$$T_f\left(j\frac{\omega}{\omega_0}\right) = \frac{T\left(j\frac{\omega}{\omega_0}\right)}{1 - \beta\left(j\frac{\omega}{\omega_0}\right) T\left(j\frac{\omega}{\omega_0}\right)} = Q\left(j\frac{\omega}{\omega_0}\right) \quad . \quad (5)$$

if $\quad \left|\beta\left(j\frac{\omega}{\omega_0}\right) . T\left(j\frac{\omega}{\omega_0}\right)\right| \gg 1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$

one obtains

$$\beta\left(j\frac{\omega}{\omega_0}\right) = -\frac{1}{1 + j\frac{\omega}{\omega_0}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Such a discriminator has the disadvantage that it requires two independent RC circuits. With the use of an operational amplifier, it is possible to use only one circuit.

Following this principle, a wideband discriminator was made. Fig. 1 shows the block diagram and the circuit of this

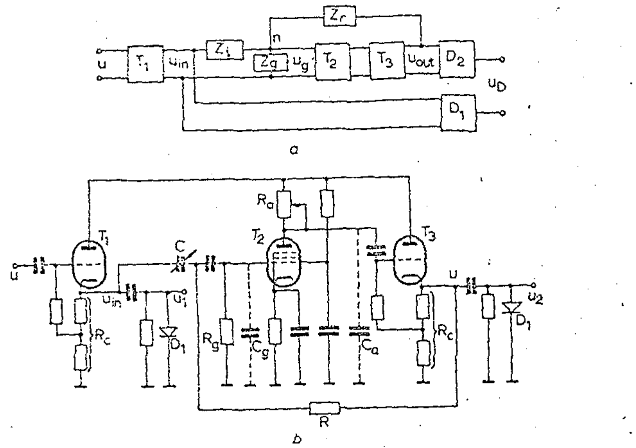

Fig. 1 *Discriminator*
a Block diagram
b Circuit diagram

discriminator. T_1 and T_3 are the buffer stages, with high input impedances and small output impedances; the amplifier T_2, with T_3 and the impedances Z_r, Z_i and Z_g, compose the operational amplifier, and D_1 and D_2 are the peak detectors. For simplicity, we shall consider the detection efficiency $\eta = 1$. The introduction of detector D_1 assures balance of the output.

The discrimination characteristic $T_D\left(\frac{\omega}{\omega_0}\right)$ is given by the expression

$$T_D\left(\frac{\omega}{\omega_0}\right) = \left|\frac{u_1}{u}\right| - \left|\frac{u_2}{u}\right| = \left\{T_d\left(\frac{\omega}{\omega_0}\right) - 1\right\} T_1\left(\frac{\omega}{\omega_0}\right) \quad (8)$$

The modulus of the transfer function of T_1 is not frequency dependent. Consequently, in the analysis of the discriminator, one can consider only the frequency dependence of the function $T_d\left(\frac{\omega}{\omega_0}\right)$.

Writing the current at the point n:

$$\frac{u_i - u_g}{Z_i(j\omega)} = \frac{u_g}{Z_g(j\omega)} + \frac{u_g - u_e}{Z_f(j\omega)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

putting

$$T_{23}\left(j\frac{\omega}{\omega_0}\right) = -\frac{u_e}{u_g} = T_2\left(j\frac{\omega}{\omega_0}\right) T_3\left(j\frac{\omega}{\omega_0}\right) \quad . \quad . \quad (10)$$

one obtains

$$T_d\left(j\frac{\omega}{\omega_0}\right) = -\frac{u_e}{u_i}$$

$$= \frac{\dfrac{Z_f(j\omega)}{Z_i(j\omega)}}{1 + \dfrac{1}{T_{23}\left(j\frac{\omega}{\omega_0}\right)}\left[1 + \dfrac{Z_f(j\omega)}{Z_i(j\omega)}\left\{1 + \dfrac{Z_i(j\omega)}{Z_g(j\omega)}\right\}\right]} \quad . \quad . \quad . \quad . \quad (11)$$

If the impedances are chosen so that