Analysis and Synthesis of Nonlinear Reciprocal Networks Containing Two Element Types and Transformers

NGUYEN TAN HUNG AND BRIAN D. O. ANDERSON, FELLOW, IEEE

Abstract—This paper presents results for some analysis and synthesis problems of nonlinear reciprocal networks. In particular, for special systems such as lossless reciprocal networks and nonlinear *RC* reciprocal networks, synthesis techniques are introduced. The central feature of these techniques is the method of finding an appropriate stored energy function for the synthesizing network from prescribed network equations. Under certain assumptions, the stored energy function can be determined directly from the controllability and observability matrices of a system obtained by linearization around any state of the original system.

IEEE: CIRCUITS AND SYSTEMS
Vol. 27 Issue 4
Au: Hung
Galley No. 1 4/9

I. Introduction

This paper is concerned with the problem of synthesizing a nonlinear reciprocal network from given equations describing that network. The results obtained are restricted to lossless or *RC* networks which can contain constant turns-ratio transformers, and with the describing equations being of state-variable form, with entries of the state vector corresponding to inductor fluxes and/or capacitor voltages.

There are two ideas in the study which appear crucial, those of state function and the viewing of a network as an interconnection of a nondynamic or memoryless reciprocal system and a dynamic, but lossless reciprocal system.

In the study of nonlinear reciprocal networks, the concept of state functions has already proven very useful as an analysis tool [1]. The most familiar example (but not the only example) of a state function is the stored energy function which may be defined for any device exhibiting the attribute of conservation of energy, and a major part of the paper is devoted to proving that the synthesis of some special passive reciprocal networks may be reduced to the problem of determining an appropriate total stored energy function for a network synthesizing the prescribed network equations. As it turns out, for these networks—lossless reciprocal networks and nonlinear *RC* reciprocal networks—a unique stored energy function can be obtained.

As noted above, another important feature in the study of the synthesis of nonlinear reciprocal networks is to view the synthesizing network as a cascade interconnection of a memoryless reciprocal system and a dynamic lossless system [2]. A network synthesizing prescribed network equations may contain some or all of the following nonlinear reciprocal time-invariant network elements: inductive elements, capacitive elements, and nondynamic elements. Here nondynamic elements which are memoryless comprise ideal transformers and resistive elements in the usual sense.

The ideas of this paper also draw on two streams of prior work. There has of course been much work on passive, reciprocal linear network synthesis and in recent years, state-variable approaches have been developed, e.g., [3]–[5], presumably providing a better jump-off point for nonlinear theories. Such nonlinear theories have in part been developed for passive synthesis, for example, [6] introduces a nonlinear generalization of the state-variable characterization of operator passivity provided in the linear case by the Kalman–Yakubovic Lemma [7]–[8]. This idea is applied in [2] to nonlinear synthesis. Again, Willems has studied a number of questions relating to the stored energy of nonlinear passive systems [9].

The second stream is embodied more in such work as that of Chua and Lam [10], Brayton and Moser [11], and Varaiya and Verma [12]. This work in a sense is concerned with building up network properties from element properties, and in this may have more of an analysis than synthesis flavor. It has been highly successful in developing properties of single-element kind networks.

The structure of this paper is as follows. In Section II, we review the notions of reciprocity, passivity, and losslessness for nonlinear network elements. Section III is concerned with developing synthesis procedures for lossless nonlinear reciprocal networks. Section IV deals with the synthesis of nonlinear resistor, capacitor, transformer (RCT) reciprocal complete networks, defined in more detail in that section. A systematic synthesis procedure is presented and finally illustrated with a simple example. Section V studies general RCT networks and Section VI, in addition to containing concluding remarks, mentions some open questions for future research.

II. Nonlinear Reciprocity, Passivity, and Losslessness

Most of this section is concerned with reviewing definitions and standard properties, and explaining the rationale for some standing assumptions.

A. Nonlinear Reciprocity

We shall regard the n-port networks considered in this paper as interconnections of resistors, capacitors, inductors, transformers, and independent voltage or current sources. In order to define the reciprocity concept for such networks [10]–[15] we shall first define the concepts for three classes of algebraic n-ports, viz. the resistive (or nondynamic) n-port, the inductive n-port, and the capacitive n-port. The general resistive n-port is usually characterized by a set of n algebraic relations between the currents i and the voltages v across every port. In the inductive n-port, the voltages v are replaced by the magnetic flux linkages λ, with $d\lambda/dt = v$. In the capacitive n-port, the currents i are replaced by the electric charges q, with $dq/dt = i$.

The various n-ports are not restricted to being interconnections of 2-terminal elements. Coupling either by ideal transformers or coupling internal to a multiport element is permitted. We require that such n-ports can be represented by a constitutive relation of the form

$$R(y,x)=0 \tag{2.1}$$

between two n-vectors y,x with, in the case of a resistive n-port, $[x,y]=[v,i]$, in the case of an inductive n-port, $[x,y]=[\lambda,i]$, and in the case of a capacitive n-port, $[x,y]=[v,q]$.

Now suppose that it is possible to parametrize the relations in terms of an m-vector s, which is not necessarily a vector of all currents, or all voltages, etc., so that the relation becomes

$$y=y(s) \quad \text{and} \quad x=x(s) \tag{2.2}$$

IEEE: CIRCUITS AND SYSTEMS
Vol. 27 Issue 4
Au. Hung
Galley No. 2

where $x, y \in R^n$ are C^1 functions, $s \in R^m$ for $0 \leqslant m \leqslant n$, and the rank of the $2n \times m$ Jacobian matrix $\partial(y,x)/\partial s$ is equal to m. In this case the n-port is said to have dimension m and will be denoted by $N(m)$.

IEEE: CIRCUITS AND SYSTEMS
Vol. 27 Issue 4
Au: Hung
Galley No. 3

The following reciprocity definition was first introduced in [10] as a generalization of the definition in [15].

03700010 *Algebraic n-Port Reciprocity Definition:* An algebraic n-port $N(m)$ represented by the parametric form (2.2) is reciprocal if and only if its associated reciprocity matrix

$$R(s) \triangleq \left(\frac{\partial x}{\partial s}\right)'\left(\frac{\partial y}{\partial s}\right) \triangleq J_x'J_y \tag{2.3}$$

is symmetric where the Jacobian matrix $J_f(s)$ for a vector $f = f(s)$ is defined as

$$J_f(s) = \frac{\partial f}{\partial s} = \begin{bmatrix} \frac{\partial f_1}{\partial s_1} & \cdots & \frac{\partial f_1}{\partial s_m} \\ \frac{\partial f_n}{\partial s_1} & & \frac{\partial f_n}{\partial s_m} \end{bmatrix}.$$

Closely related to the concept of reciprocity are the state functions and the potential functions which may be defined as follows.

A C^1-function $g\colon R^n \to R^n$ is said to be a state function if the Jacobian matrix $J_g(s)$ is symmetric for all $s \in R^n$. As a consequence, the line integral of $g(\cdot)$ between any two points is independent of the path of integration. In order to obtain a state function for a reciprocal algebraic n-port, the following result is available.

Let $N(n)$ be a reciprocal n-port represented by the parametric form (2.2) with x a C^2-function on R^n. Then the function

$$g(s) = \left[\frac{\partial x(s)}{\partial s}\right] y(s) \tag{2.4}$$

is a state function on R^n [10].

The potential function $y(s)$ for this reciprocal n-port is then defined in terms of the state function $g(s)$ as

$$\phi(s) \triangleq \int_0^s g'(s)\,ds. \tag{2.5}$$

Hence the relationship between the potential function y and the state function g of a reciprocal n-port may be expressed as

$$g(s) \triangleq \left(\frac{\partial \phi}{\partial s}\right)' \triangleq \nabla\phi. \tag{2.6}$$

Remarks

(1) A resistive reciprocal n-port may be expressed explicitly as

$$y = v(s) \quad \text{and} \quad x = i(s) \tag{2.7}$$

where v, i are voltage and current vectors, and s is a parameter vector. It is clear that a state function and a potential function can be obtained from (2.4) and (2.5), and are by no means unique, depending on the choice of s:

(a) If $s = [i_1, i_2]'$, then $R(s) = J_x'J_y = J_y$, and reciprocity is equivalent to symmetry of the incremental resistance matrix.

(b) If $s = [i_1, v_2]'$ the associated reciprocity matrix is

$$R(s) = \left[\frac{\partial(i_1,i_2)}{\partial(i_1,v_2)}\right]'\left[\frac{\partial(v_1,v_2)}{\partial(i_1,v_2)}\right]$$

$$= \begin{bmatrix} \frac{\partial v_1}{\partial i_1} & \frac{\partial v_1}{\partial v_2} + \left(\frac{\partial i_2}{\partial i_1}\right)' \\ 0 & \left(\frac{\partial i_2}{\partial v_2}\right)' \end{bmatrix}. \tag{2.8}$$

IEEE: CIRCUITS AND SYSTEMS
Vol. 27 Issue 4
Au: Hung
Galley No. 4

So reciprocity is equivalent to the following matrix (an incremental hybrid matrix with sign change) being symmetric (a result well known in the linear case);

$$\begin{bmatrix} \dfrac{\partial v_1}{\partial i_1} & \dfrac{\partial v_1}{\partial v_2} \\ -\dfrac{\partial i_2}{\partial i_1} & -\dfrac{\partial i_2}{\partial v_2} \end{bmatrix}. \tag{2.9}$$

Resistive n-ports include ideal linear transformers. An ideal transformer can be defined by $v_1 = Tv_2$, $i_2 = -T'i_1$ for some matrix T, so that $s=[i_1 v_2]'$ is a suitable parameter. One finds $R(s)$ in (2.8) to be zero. Therefore, a linear transformer n-port is reciprocal. Note that in general, a *nonlinear transformer n-port* is not reciprocal [15]. State and potential functions can also be calculated.

(2) An inductive reciprocal n-port is expressed in the parametric form

$$y=i(s) \quad \text{and} \quad x=\lambda(s)$$

where i,λ are current and flux linkage vectors.

Usually, a convenient choice for s is

$$s=x=\lambda(s)$$

and in this case, the associated reciprocity matrix, $R(s) = J_x'J_y = J_y(s)$ is symmetric. As a consequence, $y=i(s)$ is a state function and the corresponding potential function $W_m(s)$ is defined as

$$i(s)=\nabla W_m(s), \quad W_m(0)=0. \tag{2.11}$$

(3) Similarly, a capacitive reciprocal n-port may be expressed as

$$y=v(s) \quad \text{and} \quad x=q(s) \tag{2.12}$$

where v,q are voltage and charge vectors. A convenient choice for s is

$$s=x=q(s)$$

and the associated reciprocity matrix $R(s)=J_x'J_y=J_y(s)$ is symmetric. Therefore, $y=v(s)$ is a state function and the corresponding potential function $W_e(s)$ is

$$v(s)=\nabla W_e(s), \quad W_e(0)=0. \tag{2.13}$$

(4) A constant current source I and a constant voltage source V, may be considered as a nonlinear inductive element and a nonlinear capacitive element respectively, with

$$W_m = I'\lambda \quad \text{and} \quad W_e = V'q.$$

Consider an arbitrary n-port, comprising an interconnection of resistive, inductive and capacitive multiports. *We shall term the n-port reciprocal when the constituent algebraic multiports are individually reciprocal.*

If the constituent multiports are all of the one kind or of the one kind other than for ideal transformers, the resulting interconnection is also an algebraic multiport. Some analysis will show that, as an interconnection of reciprocal multiports [15], [10] it possesses the reciprocity property of (2.3). So the definition of reciprocity of an interconnected network in terms of its constituent subnetworks is consistent with the definition of reciprocity for algebraic multiports. In Section V, a particular case will be considered.

B. Nonlinear Passivity and Losslessness

Passivity and reciprocity are of course different concepts. For example, a constant voltage source is reciprocal but not passive, and a gyrator is passive but not reciprocal. Here we briefly define nonlinear passivity and losslessness and show that despite the difference of the properties, reciprocity plays a very important role in determining several fundamental aspects of passivity and losslessness.

IEEE: CIRCUITS AND SYSTEMS
Vol. 27 Issue 4
Au: Duang
Galley No. 5

Passivity and losslessness may be defined in terms of energy and power as follows.

An n-port N is said to be passive if, with N unexcited at $t=0$,

$$w(t) = \int_0^t \langle v(\tau), i(\tau) \rangle \, d\tau, \qquad \forall t \geq 0$$

for all continuous voltage–current pairs (v, i).

An n-port N is said to be lossless if it is passive and if the average power

03700026

$$P_{av}[v(t), i(t)] \triangleq \lim_{T \to \infty} \frac{1}{T} \int_0^T \langle v(t), i(t) \rangle ft = 0$$

for all bounded continuous voltage-current pairs of N.

The lemma below relates reciprocity, passivity, and losslessness. For these lemmas, we require the notion of a diffeomorphism: we say that $y=f(x)$ is a C^1-diffeomorphism R^n onto itself if [16]:

1) $f(x)$ is continuously differentiable.
2) $|J_f(x)| \neq 0$ for all $x \in R^n$.
3) $\|f(x)\| \to \infty$ as $\|x\| \to \infty$.

Note that these properties imply that f^{-1} exists and is also continuously differentiable. In particular, when $n=1$, the function f must have nonzero slope everywhere.

Lemma 1: Let $L\{C\}$ be a reciprocal inductive {capacitive} n-port represented by $i=f(\lambda)$ $\{v=f(q)\}$ where $i, \lambda \{v, q\}$ are the current and flux linkage {voltage and charge} vectors, and $f(\cdot)$ is a C^1-diffeomorphism with $f(0)=0$. Then $L\{C\}$ is passive if and only if $J_f(\lambda) = \nabla^2 W_m(\lambda) > 0$, $\forall \lambda \in R^m$ $\{J_f(q) = \nabla^2 W_e(q) > 0, \ \forall q \in R^m\}$ where $W_m(\lambda)\{W_e(q)\}$ is the potential function associated with the state function $f(\lambda)$ $\{f(q)\}$.

\# *Proof:* It suffices to consider the inductor case. *Sufficiency*: As L is reciprocal, from (2.11) there exists a potential function $W_m(\lambda)$ so that

$$f(\lambda) = \nabla W_m(\lambda)$$

with $W_m(0)=0$. If

$$\nabla^2 W_m(\lambda) = J_f(\lambda) > o, \qquad \forall \lambda \in R^m$$

then $f(\lambda)$ is nondecreasing in the sense that $[f(\lambda_1) - f(\lambda_2)]'[\lambda_1 - \lambda_2] > 0$ for all λ_1, λ_2. [17]. Hence L is passive [10].

Necessity: If L is passive, then $W_m(\lambda) \geq 0$, $\forall \lambda \in R^n$ [2]. From (2.11) as $W_m(0)=0$, the potential function $W_m(\lambda)$ has its minimum point at $\lambda=0$, so $\nabla W_m(0)=0$. Furthermore, as $i=f(\lambda)=\nabla W_m(\lambda)$ is a C^1-diffeomorphism, $J_f(\lambda) = \nabla^2 W_m(\lambda)$ is continuous, symmetric, and invertible.

Now for arbitrary but fixed λ and $d \in R^n$ and variable $\alpha \in R^1$, we have

$$W_m(\lambda + \alpha d) = W_m(\lambda) + \alpha[\nabla W_m(\lambda)]'d + \tfrac{1}{2}\alpha^2 d'[\nabla^2 W_m(\lambda)]d + o(\alpha^2).$$

Taking $\lambda=0$, we obtain

$$W_m(\alpha d) = \tfrac{1}{2}\alpha^2 d'[\nabla^2 W_m(0)]d + o(\alpha^2).$$

Now $\nabla^2 W_m(0)$ is nonsingular and symmetric and so has all real nonzero eigenvalues. If $\nabla^2 W_m(0)$ is not positive definite, there exists d such that $d'[\nabla^2 W_m(0)]d < 0$, and then by taking α small enough, we contradict the passivity. So $\nabla^2 W_m(0)$ is positive definite. Since $\nabla^2 W_m(\cdot)$ is continuous, symmetric, and nonsingular the eigenvalues of $\nabla^2 W_m(\alpha\lambda)$ for fixed $\lambda \in R^n$ and $\alpha \in [0,1]$ vary continuously with α, are real for all α, are zero for no α and are positive for $\alpha=0$. Hence they are positive for $\alpha=1$, i.e., $\nabla^2 W_m(\lambda) > 0$.

IEEE: CIRCUITS AND SYSTEMS
Vol. 27 Issue 4
Au: Hung
Galley No. 6

Lemma 2: Let R be a reciprocal resistive n-port represented by $i=g(v)$ where i,v are the current and voltage vectors, and $g(\cdot)$ is a C^1-diffeomorphism with $g(0)=0$. Then R is passive if and only if $J_g(v)=\nabla^2 W_r(v)>0$, $\forall \in R^m$ where $W_r(v)$ is the potential function associated with the state vector $g(v)$.

Proof: Sufficiency: As R is reciprocal, $J_g(v)$ is symmetric. Therefore, $i=\nabla W_r(v)$ with $i(0)=0$ where $W_r(v)$ is the potential function. If $\nabla^2 W_r(v)>0$, $\forall v\in R^n$, then $i=g(v)$ is nondecreasing on R^n. Hence R is passive [10].

Necessity: If R is passive, then $\langle v,i\rangle=\langle v,g(v)\rangle \geq 0$, $\forall v\in R^n$ [10]. Define $v=\alpha d$ for fixed $d\in R^n$ and variable $\alpha\in R^1$. Then a Taylor series analysis yields

$$\langle \alpha d, g(\alpha d)\rangle = \alpha^2 d'\left[\nabla^2 W_r(0)\right]d + o(\alpha^2)$$

and the proof then follows as for the preceding lemma, *mutatis mutandis.*

Lemma 3: Let $L\{C\}$ be a reciprocal inductive {capacitive} n-port represented by $i=f(\lambda)$ $\{v=f(q)\}$ where $i,\lambda\{v,q\}$ are the current and flux linkage {voltage and charge} vectors, and $f(\cdot)$ is a C^1-diffeomorphism with $f(0)=0$. In addition $L\{C\}$ is passive. Then $L\{C\}$ is lossless.

Proof: The proof is obvious from [10].

Remarks

(1) We can summarize a number of the preceding ideas for algebraic n-ports as follows. If the n-port N can be described by a C^1-diffeomorphism

$$y=f(x)$$

then N is reciprocal if and only if the Jacobian matrix $J_f(x)$ is symmetric. In addition, the network N is also passive if and only if $J_f(x)$ is positive definite, and is lossless if it is an inductive or capacitive n-port.

(2) A linear transformer n-port is also passive and lossless, as is well known, and easy to check using the definition.

(3) Note that by our definition, a lossless element must be passive. By Lemma 3, with C^1-diffeomorphisms between appropriate port vectors, passive reciprocal inductive n-port and capacitive n-port are lossless. Henceforth in this paper, we shall regard *a general lossless reciprocal network as an interconnection of passive reciprocal inductors and capacitors, described by C^1-diffeomorphisms and linear transformers.*

(4) In a dynamic network which contains only reciprocal capacitors and inductors, the total stored energy function is defined as $\phi(\lambda,q)-W_m(\lambda)+W_e(q)$. As an immediate consequence of Lemma 1, with C^1-diffeomorphisms between the port vectors of the constituent algebraic multiports, the network is passive if and only if $\nabla^2\phi(x)>0$ $\forall\phi\in R^{m+n}$, where

$$\nabla^2\phi(x)=\begin{bmatrix}\dfrac{\partial^2 W_m}{\partial\lambda^2} & 0\\ 0 & \dfrac{\partial^2 W_e}{\partial q^2}\end{bmatrix}.$$

III. Lossless Nonlinear Reciprocal Networks

In this section, we study lossless nonlinear reciprocal networks which are interconnections of ideal linear transformers, and inductor and capacitor multiports representable using C_1-diffeomorphisms as in Lemma 1. We shall use the fact that the input–output mapping of ideal linear transformers has a very simple form in order to show how one may reduce the synthesis problem for lossless nonlinear reciprocal networks to the problem of determining an appropriate potential function.

IEEE: CIRCUITS AND SYSTEMS
Vol. 27 Issue 4
Au: Hung
Galley No. 7

A. Analysis Results

Consider a general lossless reciprocal network which is composed of a reciprocal inductive multiport L, a passive reciprocal capacitive multiport C, a linear transformer multiport T, and voltage sources u. The schemes depicted in Fig. 1, in which T_e denotes the linear transformer multiport T together with all the interconnections. Fig. 1

Suppose that L is represented by the C^1-diffeomorphism $i_L = f_L(\lambda)$, with i_L, $\lambda \in R^m$ the current and flux linkage vectors, and C by the C^1-diffeomorphism $v_c = f_c(q)$ where v_c, $q \in R^n$ are the voltage and charge vectors. Let the corresponding potential functions be $W_m(\lambda)$ and $W_e(q)$.

Suppose also that it is possible to specify independently i_L, v_c, and u, i.e., interconnections and T do not force a linear relation among the entries of i_L, v_c, and u. Further, suppose that specification of i_L, v_c, and u at any instant of time uniquely determines all other variables in the network at that time.[1] As a consequence, there exists a constant matrix T such that Footnote 1

0570044

$$v_1 = T'\begin{bmatrix} v_2 \\ u \end{bmatrix}\begin{bmatrix} -i_1 \\ -y \end{bmatrix} = Ti_1. \tag{3.1}$$

The detailed argument leading to (3.1) is developed, for example, [4, see sec. 4.3], for the case when T_e contains resistors, but the lossless case is easy to recover.

Partition the matrix T as

$$T = \begin{bmatrix} T_1 \\ T_2 \end{bmatrix}. \tag{3.2}$$

Equations (3.1) then can be rearranged as

$$\begin{bmatrix} v_L \\ i_c \end{bmatrix} = \begin{bmatrix} v_1 \\ -i_2 \end{bmatrix} = \begin{bmatrix} 0 & T_1' \\ -T_1 & 0 \end{bmatrix}\begin{bmatrix} i_L \\ v_c \end{bmatrix} + \begin{bmatrix} T_2' \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} T_2 & 0 \end{bmatrix}\begin{bmatrix} i_L \\ v_c \end{bmatrix}. \tag{3.3}$$

Choosing the state vector x of the network state[2] equations to contain inductor flux linkages λ and capacitor charges q, we obtain with total stored energy function $\phi(x) = W_m(\lambda) + W_e(q)$ the relations Footnote 2

$$\begin{bmatrix} i_L \\ v_c \end{bmatrix} = \begin{bmatrix} \left(\dfrac{\partial}{\partial\lambda} W_m(\lambda)\right)' \\ \left(\dfrac{\partial}{\partial q} W_e(q)\right)' \end{bmatrix} = \nabla\phi$$

and

$$\dot{x} = \begin{bmatrix} d\lambda/dt \\ dq/dt \end{bmatrix} = \begin{bmatrix} v_L \\ i_c \end{bmatrix}. \tag{3.4}$$

Therefore, the state-space network equations for the

$$\dot{x} = \begin{bmatrix} 0 & T_1' \\ -T_1 & 0 \end{bmatrix}\nabla\phi + \begin{bmatrix} T_2' \\ 0 \end{bmatrix}u \quad y = \begin{bmatrix} T_1 & 0 \end{bmatrix}\nabla\phi. \tag{3.5}$$

B. Synthesis Results

When the state vector of a lossless reciprocal p-port network is known to comprise flux linkage and capacitor charges, it is clear from analysis results that its state-space network equations should have the following form:

$$\dot{x} = f(x) + Gu \quad y = h(x) \tag{3.6}$$

where $f(\cdot)$: $R^{m+n} \to R^{m+n}$, $G \in R^{(m+n)\times p}$, $h(\cdot)$: $R^{m+n} \to R^p$, $f(0)=0$, $h(0)=0$.

The synthesis problem is to determine whether equations (3.6) can be obtained by constructing an appropriate lossless reciprocal network. In other words, one is asked to search for a network consisting of a passive reciprocal capacitive n-port, a passive reciprocal inductive m-port, a linear transformer multiport and voltage sources so that its state-space equations coincide with (3.6). It is clear that if we can deduce from (3.6) an appropriate total stored energy function $\phi(x)$ and a constant matrix T defining the network T_e, then the synthesis problem is almost solved as one can construct the corresponding reciprocal network. Since any linear transformer multiport is lossless, it remains to prove that the capacitive and inductive n-ports

IEEE: CIRCUITS AND SYSTEMS
Vol. 27 Issue 4
Au: Hang
Galley No. 8

Theorem 1: A necessary and sufficient condition for equations (3.6) to be realized by a lossless reciprocal network of the type described at the start of the section, is that there exist a constant matrix $T=\begin{bmatrix} T_1 \\ T_2 \end{bmatrix}$ and a C^2-function $\phi(\cdot)$, with $\nabla^2\phi(x)>0$ for all x, $\nabla\phi(0)=0$ and $\phi(0)=0$ such that

$$G=\begin{bmatrix} T_2' \\ 0 \end{bmatrix} \quad h(x)=\begin{bmatrix} T_2 & 0 \end{bmatrix}\nabla\phi \quad f(x)=\begin{bmatrix} 0 & T_1' \\ -T_1 & 0 \end{bmatrix}\nabla\phi \tag{3.7}$$

Proof: Sufficiency: Suppose that $T=\begin{bmatrix} T_1 \\ T_2 \end{bmatrix}$ and ϕ exist such that equations (3.7) are satisfied. Then T gives adequate information for the linear transformer n-port and the interconnection structure of the network. In addition, the reciprocal capacitive and inductive n-ports may be obtained directly from $\phi(x)$ where

$$\begin{bmatrix} f_L(\lambda) \\ f_c(q) \end{bmatrix} \triangleq \begin{bmatrix} i_L \\ v_c \end{bmatrix} = \nabla\phi.$$

As $\nabla^2\phi(x)>0$ for all x, $f_c(q)$ and $f_L(\lambda)$ are C^1-diffeomorphisms with $f_c(0)=0$, $f_L(\lambda)=0$ since

$$\begin{bmatrix} 0 & T_1' \\ -T_1 & 0 \end{bmatrix}\nabla\phi(0)=f(0)=0.$$

Hence the synthesizing network is lossless.

Necessity: Conversely, if (3.6) are realized by a lossless reciprocal network, its network equations have the form as described by (3.5). Consequently, there exist T and $\phi(x)$ so that (3.7) are satisfied. In addition, as the system in lossless, it is also passive and consequently, $\nabla^2\phi(x)>0$ for all x, while also $\phi(0)=0$, $\nabla\phi(0)=0$.

It is clear that from (3.7) that T_2 can be found directly from G, and T_1 as shown later is readily obtained if $\phi(x)$ is known. There are a number of ways that can be used to deduce the total stored energy function $\phi(x)$ from the given functions G, f, and h. However, the most interesting practical method which is very similar to one synthesis technique widely used for linear networks [4] relies on the assumption that the system derived from (3.6) by linearization around any nominal state is completely controllable [19]. The linearized system[3] has the form: Footnote 3

$$\begin{aligned} \Delta\dot{x} &= J_f\Delta x + G\Delta u \\ \Delta y &= J_h\Delta x \end{aligned} \tag{3.8}$$

and this system is completely controllable if the controllability matrix

$$V_c=\begin{bmatrix} G & J_fG & \cdots & J_f^{m+n-1}G \end{bmatrix}$$

has full rank. The original system is then said to possess incremental controllability if V_c has full rank for all linearizations.

Lemma 1: Assume that the system described by (3.6) is incrementally controllable. Then the total stored energy function $\phi(x)$ may be uniquely defined from

$$\nabla^2\phi=\left(\overline{V}_0'V_c'\right)\left(V_cC_c'\right)^{-1} \tag{3.9}$$

where

$$\begin{aligned} &\phi(0)=0, \quad \nabla\phi(0)=0 \\ &\overline{V}_0'=\begin{bmatrix} J_h' & -J_f'J_h' & \cdots & (-J_f')^{m+n-1}J_h' \end{bmatrix} \\ &V_c=\begin{bmatrix} G & J_fG & \cdots & J_f^{m+n-1}G \end{bmatrix}. \end{aligned}$$

Proof: Equations (3.7) imply

$$J_h=G'\nabla^2\phi J_f=\begin{bmatrix} 0 & T_1' \\ -T_1 & 0 \end{bmatrix}\nabla^2\phi \tag{3.10}$$

and the second equation implies

$$J_f'\nabla^2\phi=-\nabla^2\phi J_f. \tag{3.11}$$

IEEE: CIRCUITS AND SYSTEMS
Vol. 27 Issue 4
Au. Hung
Galley No. 9

Combining Equations (3.10) and (3.11), a solution for $\nabla^2\phi$ can be obtained easily:

$$\nabla^2\phi G = J_h'$$
$$\nabla^2\phi J_f G = -J_f'\nabla^2\phi G = -J_f'J_h'$$
$$\cdots \quad \cdots$$
$$\nabla^2\phi J_f^{m+n-1}G = (-J_f')^{m+n-1}\nabla^2\phi G = (-J_f')^{m+n-1}J_h'.$$

Consequently, we have

$$\nabla^2\phi V_c = \overline{V}_0'. \tag{3.12}$$

Since the system is incrementally controllable, V_c has full rank, and there exists an inverse for V_cV_c' Thus $\nabla^2\phi$ is uniquely defined by (3.9). Furthermore since $\phi(0)=0$ and $\nabla\phi(0)=0$, the total stored energy function $\phi(x)$ is uniquely defined by integration. $\nabla\nabla\nabla$

In brief, the synthesis problem may be solved as follows. Consider an incremental controllable system with the following state-space equations

$$\dot{x} = f(x) + Gu$$
$$v = h(x)$$

03700061 where $f(0)=0$, $h(0)=0$. The problem is to determine T and $\phi(x)$ so that (3.7) are satisfied.

By Lemma 1, the total stored energy function for the system can be uniquely determined from

$$\nabla^2\phi(x) = \left(\overline{V}_0'V_c'\right)\left(V_cV_c'\right)^{-1}$$

and $\phi(0)=0$, $\nabla\phi(0)=0$. If V_c is square, it is clear that

$$\nabla^2\phi(x) = \overline{V}_0'V_c^{-1}.$$

The constant matrix T can be easily recovered using

$$\begin{bmatrix} T_2 \\ 0 \end{bmatrix} = G$$

and

$$\begin{bmatrix} 0 & T_1' \\ -T_1 & 0 \end{bmatrix} = (J_f)(\nabla^2\phi)^{-1}. \tag{3.13}$$

Hence it only remains to check that $\nabla^2\phi(x) > 0 \forall x \in R^{m+n}$ in order to ensure that the synthesizing network is passive.

There are in fact two other ways be which $\phi(x)$ can theoretically be found. Recognizing that $\phi(x)$ is the energy stored by the network when the vector of inductor currents and capacitor voltages is x, it is clear that

$$\phi(x) = \int_0^T y'(t)u(t)\,dt \tag{3.14}$$

where $x(0)=0$ (i.e., the network stores no energy at the initial time), and $u(\cdot)$ is a voltage excitation causing the state at time T to be x. Of course, for a given x, it is not obvious what $u(\cdot)$ will achieve that x at time T, starting with $x(0)=0$. Put another way, the calculation of ϕ is reduced to a nonlinear controllability problem.

Second, one can recognize that if x is the initial state vector in the network, and if the network is resistively loaded under zero input conditions, then the resistive load will absorb the network energy, and as $t\to\infty$, the total power dissipated by the resistor should equal the initially stored energy. A resistive load of 1 Ω at each port corresponds to $u=-y$, and so

$$\phi(x_0) = \int_0^\infty h^2[x(t)]\,dt \tag{3.15}$$

with

$$\dot{x} = f(x) - Gh(x), \qquad x(0) = x_0. \tag{3.16}$$

To avoid the possibility of some of the inductors and capacitors not being connected to the ports, and thus not being discharged by the resistor, some form of nonlinear observability assumption is needed.

C. Example

The system equations to be synthesized is

$$\dot{x}=\begin{bmatrix} 3\left(x_2+\dfrac{x_2^3}{3}\right) \\ -3\left(2x_1+\dfrac{x_1^3}{3}\right)\end{bmatrix}+\begin{bmatrix}1\\0\end{bmatrix}u,\quad y=2x_1+\frac{x_1^3}{3}.$$

IEEE: CIRCUITS AND SYSTEMS
Vol. 27 Issue 4
Au: Hung
Galley No. 10

One easily obtains

$$J_f=\begin{bmatrix} 0 & 3(1+x_2^2) \\ -3(2+x_1^2) & 0\end{bmatrix},\quad J_h=\begin{bmatrix}2+x_1^2 & 0\end{bmatrix}$$

so that the incremental controllability matrix V_c is

$$V_c=\begin{bmatrix}G & J_fG\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & -3(2+x_2^2)\end{bmatrix}.$$

Since V_c has full rank for all x the system is incrementally controllable. Using Lemma 1, we obtain

$$\overline{V}_0'=\begin{bmatrix}J_h' & -J_f'J_h'\end{bmatrix}=\begin{bmatrix}2+x_1^2 & 0\\ 0 & -3(2+x_1^2)(1+x_2^2)\end{bmatrix}$$

$$\nabla^2\phi=\overline{V}_0'V_c^{-1}=\begin{bmatrix}2+x_1^2 & 0\\ 0 & 1+x_2^2\end{bmatrix}.$$

As $\phi(0)=0$ and $\nabla\phi(0)=0$, the total stored energy function is determined uniquely:

$$\phi(x)=x_1^2+\frac{x_1^4}{12}+\frac{x_2^2}{2}+\frac{x^4}{12}$$

with

$$\begin{bmatrix}i_L\\ v_c\end{bmatrix}=\nabla\phi(x)=\begin{bmatrix}2x_1+\dfrac{x_1^3}{3}\\ x_2+\dfrac{x_2^3}{3}\end{bmatrix}.$$

02100022 The constant matrix is easily recovered as follows:

$$\begin{bmatrix}T_2\\0\end{bmatrix}=G=\begin{bmatrix}1\\0\end{bmatrix}$$

and

$$\begin{bmatrix}0 & T_1'\\ -T_1 & 0\end{bmatrix}=J_f(\nabla^2\phi)^{-1}=\begin{bmatrix}0 & 3\\ -3 & 0\end{bmatrix}.$$

Hence

$$T=\begin{bmatrix}T_1\\T_2\end{bmatrix}=\begin{bmatrix}3\\1\end{bmatrix}.$$

In addition, since $\nabla^2\phi(x)>0$ for all $x\in R^2$, the given equations can be synthesized by a lossless reciprocal network which is described in Fig. 2. Fig. 2

IV. Nonlinear Passive Reciprocal *RC* Complete Networks

An *RC* network is complete if all capacitors form a tree for the network. Although it is a special form of network, the general complete *RC* network may be considered as quite typical in the sense that an arbitrary *RC* circuit is a limiting case of the *RC* complete network which is made from it by adding capacitive elements of negligible size. This fact can be justified on physical grounds because reactances are usually present as parasistic elements [11].

In this section, we shall study complete multiport networks comprising algebraic resistor, linear transformer, and capacitor multiports, together with access ports. The introduction of the transformers complicates the network topology picture, and so we take the following operational definition of completeness: a multiport network comprising an interconnection of resistor, linear transformer, and capacitor multiports is complete if arbitrary specification of the voltages at the ports of the capacitor multiports is possible, and when these voltages have been specified, all other voltages in the network are thereby specified with values independent of the resistors and capacitors. This definition is consistent in the transformerless case with the standard one.

IEEE: CIRCUITS AND SYSTEMS
Vol. 27 Issue 4
Au: Hung
Galley No. 11

A. Analysis Resuts

Consider a general passive reciprocal RC complete network which is composed of passive reciprocal capacitors, passive reciprocal resistors, linear transformers, and current sources, as shown in Fig. 3. The capacitors are represented by $v_c = f_c(q)$ where v_c, $q \in R^m$ are the voltages and charges of all capacitors, the resistors are represented by $iR = f_R(v_R)$ where i_R, $v_R \in R^n$ are the currents and voltages of all resistors, and $f_c(\cdot)$, $f_R(\cdot)$ are C^1-diffeomorphisms with $f_c(0) = 0$, $f_R(0) = 0$.

Fig. 3

The completeness implies that for some matrices F_1 and F_2 determined by T_e only,

$$v_R = F_1' v_c \quad y = F_2' v_c. \tag{4.1}$$

The losslessness of T_e implies that

$$i_c = -F_1 i_R + F_2 u. \tag{4.2}$$

Furthermore, as the capacitors are reciprocal

$$i_c = dq/dt = \dot{x}$$

and

$$v_c = \nabla\phi(x) \tag{4.4}$$

where ϕ is the total stored energy function.

The state-space equations for a general RC complete network then become

$$\begin{aligned} \dot{x} &= -F_1 f_R(v_R) + F_2 u \\ y &= F_2' \nabla\phi. \end{aligned} \tag{4.5}$$

03700076 Let $\hat{f}_R(x) \triangleq f_R[v_R(x)] = f_R[F_1'\nabla\phi]$; then we have

$$\dot{x} = -F_1 \hat{f}_R(x) + F_2 u, \quad y = F_2' \nabla\phi(x) \tag{4.6}$$

B. Synthesis Results

From the analysis results we see that if the state vector is known to comprise capacitor changes, the state-space network equations for a general passive reciprocal RC complete network with current sources should have the following form:

$$x = f(x) + Gu, \quad y = h(x) \tag{4.7}$$

where $f(\cdot)$: $R^m \to R^m$, $h(\cdot)$: $R^m \to R^p$, $u, y \in R^p$, $G \in R^{m\times p}$.

The synthesis problem is to determine a complete RC network with current sources so that its state-space equations are the same as (4.7); obviously, one needs to recover the corresponding total stored energy function $\phi(x)$ and the potential function $W_r(v_r)$.

Theorem 2: A necessary and sufficient condition for (4.7) to be realized by a passive reciprocal RC complete network (of the type described above) and driven by current sources is that there exist a constant matrix $F = [F_1 \quad F_2]$ and C^2-functions $\phi(x) R^m \to R$ and $W_r(c_R)$: $R^n \to R$ with $\phi(0) = 0$, $\nabla\phi(0) = 0$, $\nabla^2\phi(x) > 0$, $W_r(0) = 0$, $\partial W_r(0)/\partial v_R = 0$, $\partial^2 W_r(v_R)/\partial v_R^2 > 0$ for all x, v_R such that, with $f_R(v_R) = (\partial W_r/\partial v_R)'$, #

$$G = F_2, \quad h(x) = F_2'\nabla\phi(x), \quad f(x) = -F_1 f_R[F_1'\nabla\phi(x)] \tag{4.8}$$

Proof: Sufficiency: Suppose that $F = [F_1 \quad F_2]$, (x) and $W_r(v_R)$ exist with the stated properties, and are such that Equations (4.8) are satisfied. Then F gives adequate information for the interconnection structure of the network, and the capacitors and resistors are sufficiently represented by their potential functions $\phi(x)$ and $W_r(v_R)$. In addition, with C^1-diffeomorphisms between appropriate vectors, and $\nabla^2\phi(x) > 0$, $\partial^2 W_r(v_R)/\partial^2 v_R^2 > 0$, the network is passive.

Necessity: Conversely, if (4.8) are realized by a passive reciprocal RC complete network, its network equations have the form as described (4.7). Consequently, there exist F, $\phi(x)$, and $W_r(v_R)$ such that (4.8) are satisfied. The remaining conditions on $\phi(x)$ and $W_r(v_R)$ are fulfilled through the passivity, reciprocity and C^1-diffeomorphism assumption.

Similarity to the results obtained for lossless reciprocal networks, the total stored energy function $\phi(x)$ can be obtained directly from the incremental controllability matrix and observability matrix of the given system. It can also be shown that F and $W_R(v_R)$ may be recovered easily from $\phi(x)$. These results rely on the assumption that the system is incrementally controllable..

Lemma 2: With the notation as above, assume that the system described by (4.7) is incrementally controllable. Then the total stored energy function $\phi(x)$ for the synthesizing network may be uniquely defined from

$$\nabla^2\phi=(V_0'V_c')(V_cV_c')^{-1} \tag{4.9}$$

where

$$\phi(0)=0 \quad \nabla\phi(0)=0$$
$$V_0'=\left[J_f' \quad J_f'J_h' \quad \cdots \quad (J_f')^{m-1}J_h'\right]$$
$$V_c=\left[G \quad J_fG \quad \cdots \quad (J_f)^{m-1}G\right].$$

Proof: Equations (4.8) imply

$$h(x)=G'\nabla\phi(x)$$

or

$$\nabla^2\phi(x)G=\left(\frac{\partial h}{\partial x}\right)'=J_h'. \tag{4.10}$$

In addition, with $v_R=F_1'\nabla\phi(x)$,

$$\begin{aligned}\frac{\partial f}{\partial x}&=-F_1\frac{\partial f_R(v_R)}{\partial v_R}\frac{\partial v_R}{\partial x}\\&=-F_1\frac{\partial^2W_r(v_R)}{\partial v_R^2}F_1'\nabla^2\phi.\end{aligned} \tag{4.11}$$

Hence

$$\nabla^2\phi(J_f)=J_f'\nabla^2\phi. \tag{4.12}$$

Combining (4.10) and (4.12), the unique solution for $\phi(x)$ may be obtained from

$$(\nabla^2\phi)V_c=V_0' \tag{4.13}$$

or from

$$\nabla^2\phi=(V_0'V_c')(V_c'V_c')^{-1} \tag{4.14}$$

with $\phi(0)=0$ and $\nabla\phi(0)=0$.

Therefore, the procedure to solve the synthesis problem for a passive reciprocal RC complete network may be briefly summarized as follows.

Consider an incrementally controllable system with the following state-space equations

$$\dot{x}=f(x)+Gu, \quad y=h(x)$$

where $f(0)=0$, $h(0)=0$. The problem is to calculate $F=[F_{1\beta} \quad F_2]$ and the potential functions (x) and $W_r(v_R)$. By Lemma 2, the total stored energy function can be uniquely determined from

$$\nabla^2\phi(x)=(V_0'V_c')(V_cV_c')^{-1}$$

and

$$\phi(0)=0, \quad \nabla\phi(0)=0.$$

The matrix F, the capacitor voltage v_c, and the potential function $W_r(v_R)$ can be easily recovered from (4.8) and (4.11). First we take

$$F_2=G \quad F_1=I. \tag{4.15}$$

Now (4.11) requires

$$\frac{\partial^2W_r(v_R)}{\partial v_R^2}=-J_f(\nabla^2\phi)^{-1} \tag{4.16}$$

while as we also know, $v_R=F_1'\nabla\phi(x)=\nabla\phi(x)$ for the choice $F_1=I$. Because of the C^1-diffeomorphism property, $x=\psi(v_R)$ for some v_R, allowing the right-hand side of (4.16), which is a symmetric matrix function of x, to be rewritten as a function of v_R:

$$\frac{\partial^2W_r(v_R)}{\partial v_R^2}=\psi(v_R). \tag{4.17}$$

IEEE: CIRCUITS AND SYSTEMS
Vol. 27 Issue 4
Au: Hung
Galley No. 12

IEEE: CIRCUITS AND SYSTEMS
Vol. 27 Issue 4
Au: Hung
Galley No. 13

Then $W_r(v_R)$ can be obtained by integration.

Note that other choice of F_1 are possible and lead to different $W_r(v_R)$. These choices correspond to the possibility of varying the transformers with consequential variation of the resistive network to leave the input/state/output behaviour invariant.

C. Example

We synthesize the following equations:

$$\dot{x}=\begin{bmatrix} -2\left(x_1+\frac{x_1^3}{3}+2x_2+\frac{x_2^3}{3}\right) \\ -2\left(x_1+\frac{x_1^3}{3}\right)-3\left(2x_2+\frac{x_2^3}{3}\right)-\left(2x_2+\frac{x_2^3}{3}\right)^3 \end{bmatrix}+\begin{bmatrix}1\\0\end{bmatrix}u$$

The incremental controllability matrix V_c and the incremental observability matrix V_0' are

$$J_f=\begin{bmatrix} -2(1+x_1^2) & -2(2+x_2^2) \\ -2(1+x_1^2) & -3(2+x_2^2)\left[1+\left(2x_2+\frac{x_2^3}{3}\right)^2\right] \end{bmatrix}$$

$$J_h'=\begin{bmatrix}1+x_1^2\\0\end{bmatrix}$$

$$V_c=\begin{bmatrix}G & J_fG\end{bmatrix}=\begin{bmatrix}1 & -2(1+x_1^2)\\ 0 & -2(1+x_1^2)\end{bmatrix}$$

$$V_0'=\begin{bmatrix}J_h' & J_f'J_h'\end{bmatrix}=\begin{bmatrix}1+x_1^2 & -2(1+x_1^2)^2\\ 0 & -2(1+x_1^2)(2+x_2^2)\end{bmatrix}.$$

Since V_c has full rank for all x, the system is incrementally controllable. Using Lemma 2, we obtain

$$\nabla^2\phi=V_0'V_c^{-1}=\begin{bmatrix}1+x_1^2 & 0\\ 0 & 2+x_2^2\end{bmatrix}.$$

As $\phi(0)=0$ and $\nabla\phi(0)=0$, the total stored energy function is

$$\phi(x)=\frac{x_1^2}{2}+\frac{x_1^4}{12}+x_2^2+\frac{x_2^4}{12}$$

with

$$\begin{bmatrix}v_{c_1}\\ v_{c_2}\end{bmatrix}=\nabla\phi(x)=\begin{bmatrix}x_1+\frac{x_1^3}{3}\\ 2x_2+\frac{x_2^3}{3}\end{bmatrix}.$$

The matrix F and the potential function $W_r(v_R)$ are received as follows:

$$F_2=G=\begin{bmatrix}1\\0\end{bmatrix}$$

$$F_1\frac{\partial^2 W_r}{\partial v_R^2}F_1'=-J_f(\nabla^2\phi)^{-1}$$

$$=\begin{bmatrix}2 & 2\\ 2 & 3\left[1+\left(2x_2+\frac{x_2^3}{3}\right)^2\right]\end{bmatrix}.$$

Although we could take $F_1=I$, some simplification is achieved by making $\partial^2 W_r/\partial v_R^2$ diagonal. As a consequence, we obtain

$$F_1=\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}$$

and then

$$v_R=F_1'\nabla\phi(x)=\begin{bmatrix}x_1+\frac{x_1^3}{3}+2x_2+\frac{x_2^3}{3}\\ 2x_2+\frac{x_2^3}{2}\end{bmatrix}$$

and

$$\frac{\partial^2 W_r}{\partial v_R^2} = F_1^{-1}\begin{bmatrix} 2 & 2 \\ 2 & 3\left(1+v_{R_2}^2\right) \end{bmatrix}(F_1')^{-1}$$

$$= \begin{bmatrix} 2 & 0 \\ 0 & 1+3v_{R_2}^2 \end{bmatrix}.$$

Hence

$$\begin{bmatrix} i_{R_1} \\ i_{R_2} \end{bmatrix} = \left(\frac{\partial W_r}{\partial v_R}\right)' = \begin{bmatrix} 2v_{R_1} \\ v_{R_2}+v_{R_2}^3 \end{bmatrix}.$$

In addition, since $\nabla^2\phi$ and $\partial^2 W_r/\partial^2_{v_R}$ are positive definite, the (4.18) can be synthesized by the passive reciprocal RC complete network with

$$F = [F_1 \quad F_2] = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

depicted in Fig. 4 Fig. 4

$$v_{c_1} = q_1 + \frac{q_1^3}{3}$$

$$v_{c_2} = 2q_2 + \frac{q_2^3}{3}$$

$$i_{R_1} = 2v_{R_1}$$

$$i_{R_2} = v_{R_2} + \frac{v_{R_2}^3}{3}.$$

V. Nonlinear Passive Reciprocal RC Networks

A slightly more complicated network is the general nonlinear passive reciprocal RC (RL) network which may be viewed as an interconnection of transformers, resistors, and capacitors. Again, it is possible to reduce the synthesis problem to the problem of determining the appropriate total stored energy function ϕ.

A. Resistor Transformer Networks

Before introducing capacitors into the picture, we shall study the network depicted in Fig. 5. FIg. 5

We shall assume that R is a reciprocal, passive, resistive multiport defined by a C^1-diffeomorphism $i_R = g(v_R)$, and T is a network of constant ideal transformers. Our goal is to show that the interconnection has a certain hybrid description, reflecting the reciprocity and passivity properties. Exhibiting existence of the hybrid description is a task in itself.

Being lossless, T has a hybrid matrix description; suppose the excitations are as depicted in Fig. 6. We shall show that the interconnections has a hybrid description with excitation variables v_1, i_2. Fig. 6

For some constant T_{ij}, we have

$$\begin{bmatrix} v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} T_{21} & T_{24} \\ T_{31} & T_{34} \end{bmatrix}\begin{bmatrix} v_1 \\ v_4 \end{bmatrix}$$

$$\begin{bmatrix} i_1 \\ i_4 \end{bmatrix} = -\begin{bmatrix} T_{21}' & T_{31}' \\ T_{24}' & T_{34}' \end{bmatrix}\begin{bmatrix} i_2 \\ i_3 \end{bmatrix}. \qquad (5.1)$$

The properties assumed for R guarantee, see [20], that there exists a hybrid description for R of the form:

$$\begin{bmatrix} v_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} h_3(-i_3, v_4) \\ h_4(-i_3, v_4) \end{bmatrix}. \qquad (5.2)$$

IEEE: CIRCUITS AND SYSTEMS
Vol. 27 Issue 4
Au: Hung
Galley No. 14

IEEE: CIRCUITS AND SYSTEMS
Vol. 27 Issue 4
Au: Hung
Galley No. 5

Let us define

$$H=\begin{bmatrix} \dfrac{\partial h_3(x,y)}{\partial x} & \dfrac{\partial h_3(x,y)}{\partial y} \\ \dfrac{\partial h_4(x,y)}{\partial x} & \dfrac{\partial h_4(x,y)}{\partial y} \end{bmatrix}_{x=-i_3,\, y=v_4.} \tag{5.3}$$

Reciprocity implies symmetry of $\partial h_3/\partial x$ and $\partial h_4/\partial y$ and also that

$$\frac{\partial h_4}{\partial x}=-\left(\frac{\partial h_3}{\partial y}\right)'. \tag{5.4}$$

Passivity implies that

$$\frac{\partial h_3}{\partial x}>0 \quad \frac{\partial h_4}{\partial y}>0 \tag{5.5}$$

for all x and y. Also, see [20],

$$\lim_{\|x\|\to\infty}\|h_3(x,y)\|=\infty \quad \lim_{\|v\|\to\infty}\|h_4(x,y)\|=\infty. \tag{5.6}$$

Now define

$$\begin{aligned} k(x,y)&=\begin{bmatrix} k_3(x,y) \\ k_4(x,y) \end{bmatrix} \\ &=\begin{bmatrix} h_3(x,y) \\ h_4(x,y) \end{bmatrix}+\begin{bmatrix} 0 & -T_{34} \\ T'_{34} & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}. \end{aligned} \tag{5.7}$$

Then (5.4), (5.5), and (5.6) hold with h_3,h_4 replaced by k_3,k_4. Now for each x,y, the Jacobian of $[k'_3 \;\; k'_4]'$ has the form

$$\begin{bmatrix} K_{33} & K_{34} \\ -K'_{34} & K_{44} \end{bmatrix}$$

with $K_{33}=K'_{33}>0$, $K_{44}=K'_{44}>0$. Its determinant, viz.,

$$|K_{33}|\,|K_{44}+K'_{34}K_{33}^{-1}K_{34}|$$

is accordingly nonsingular for all x,y and thus the mapping $(x,y)\to(k_3,k_4)$ is globally invertible.

03700114

Now observe that (5.2) and the transformer equations force

$$\begin{bmatrix} h_3(-i_3,v_4) \\ h_4(-i_3,v_4) \end{bmatrix}+\begin{bmatrix} 0 & -T_{34} \\ T'_{34} & 0 \end{bmatrix}\begin{bmatrix} -i_3 \\ v_4 \end{bmatrix} =\begin{bmatrix} T_{31} & 0 \\ 0 & T'_{24} \end{bmatrix}\begin{bmatrix} v_1 \\ i_2 \end{bmatrix}$$

so we have

$$\begin{bmatrix} -i_3 \\ v_4 \end{bmatrix}=k^{U-1}\left(\begin{bmatrix} T_{31} & 0 \\ 0 & T'_{24} \end{bmatrix}\begin{bmatrix} v_1 \\ i_2 \end{bmatrix}\right)$$

and on using the transformer equations again, we have

$$\begin{aligned} \begin{bmatrix} i_1 \\ v_2 \end{bmatrix}=\begin{bmatrix} C & -T'_{21} \\ T_{21} & 0 \end{bmatrix}\begin{bmatrix} v_1 \\ i_2 \end{bmatrix}+\begin{bmatrix} T'_{31} & 0 \\ 0 & T_{24} \end{bmatrix}k^{-1} \\ \cdot\left(\begin{bmatrix} T_{31} & 0 \\ 0 & T'_{24} \end{bmatrix}\begin{bmatrix} v_1 \\ i_2 \end{bmatrix}\right). \end{aligned} \tag{5.8}$$

Accordingly, we have exhibited a hybrid description for the interconnection of T and R, the existence of which stems from the fact that any T must have a hybrid description, and from an assumption that R is passive and reciprocal and defined by a C^1-diffeomorphism.

It is not hard to check explicitly that this new description satisfies reciprocity and passivity constraints. With h_1,h_2 defined by

$$\begin{bmatrix} i_1 \\ v_2 \end{bmatrix}=\begin{bmatrix} h_1(v_1,i_2) \\ h_2(v_1,i_2) \end{bmatrix} \tag{5.9}$$

we have

$$\begin{aligned} \begin{bmatrix} \dfrac{\partial h_1}{\partial v_1} & \dfrac{\partial h_1}{\partial i_2} \\ \dfrac{\partial h_2}{\partial v_1} & \dfrac{\partial h_2}{\partial i_2} \end{bmatrix}=\begin{bmatrix} 0 & -T'_{21} \\ T_{21} & 0 \end{bmatrix}+\begin{bmatrix} T'_{31} & 0 \\ 0 & T_{24} \end{bmatrix} \\ \cdot\left(H+\begin{bmatrix} 0 & -T_{34} \\ T'_{34} & 0 \end{bmatrix}\right)^{-1}\begin{bmatrix} T_{31} & 0 \\ 0 & T'_{24} \end{bmatrix} \end{aligned} \tag{5.10}$$

and this formula can be used to check the reciprocity and passivity.

Given the left side of (5.10) and the fact that the left side is expressible in the form shown on the right, the fact that

$$H+\begin{bmatrix} 0 & -T_{34} \\ T'_{34} & 0 \end{bmatrix}$$

is nonsingular allows one to perform a decomposition of the type shown on the right of (5.10). Many such decompositions will exist, but will differ only trivially.

B. Resistor–Capacitor–Transformer Networks–Analysis

Consider a passive reciprocal network comprising a passive reciprocal capacitive n-port C, a passive reciprocal resistive p-port R, and a transformer/interconnection $(m+n+p)$-port T, as shown in Fig. 7. Fig. 7

Throughout this section, we shall assume that the networks T and R have the properties described in the last section, while the assumptions on C guarantee, for example, the existence of a strictly convex stored energy function, zero together with its gradient at the origin.

Suppose that those ports of T which are connected to C can be taken as voltage excited, and the ports connected to the generators u can be taken as current excited in providing a hybrid matrix description of T. Let $\tilde{R}$ denote the interconnection of T and R, see Fig. 8. Then as the last subsection showed, there is a hybrid description for $\tilde{R}$ of the form: Fig. 8

$$\begin{aligned} -i_c &= h_1(v_c,u) \\ y &= h_2(v_c,u). \end{aligned} \tag{5.11}$$

The reciprocity and passivity of $\tilde{R}$ impose certain restrictions of h_1, h_2. These restrictions, given in effect by (5.10) and (5.3) through (5.6), imply the existence of a potential function

$$W_r(v_c,u)=\int_0^{v_c,u}\left[h_1'dv_c-h_2'du\right]. \tag{5.12}$$

With ϕ the stored energy function of the capacitor and $x=q$, we have

$$v_c=\nabla\phi(x) \tag{5.13}$$

and then (5.11) can be written

$$\begin{aligned} \dot{x} &= -\left(\frac{\partial W_r(v_c,u)}{\partial v_c}\right)'\bigg|_{v_c=\nabla\phi(x)} \\ y &= -\left(\frac{\partial W_r(v_c,u)}{\partial u}\right)'\bigg|_{v_c=\nabla\phi(x)}. \end{aligned} \tag{5.14}$$

C. Synthesis Results

The analysis results show that if the state vector is known to comprise capacitor charges, the state-space network equations for a general passive reciprocal RC network have the following form:

$$x=f(x,u) \quad y=h(x,u) \tag{5.15}$$

where f: $R^{m+n}\to R^n$ and h: $R^{m+n}\to R^n$ satisfy certain side conditions.

The synthesis problem can be posed in two ways. First, one can search for a network consisting of a lossless reciprocal multiport and a passive reciprocal nondynamic multiport, driven by current sources. Thus one seeks separate descriptions of of the networks C and R of Fig. 8. Second, one can seek descriptions of C, T, and R. The first problem is solved by recovering $\phi(x)$ and $W_r(v_c,u)$, the second by recovering $\phi(x)$ and obtaining a description of $\tilde{R}$ in the decomposed form of (5.8), or equivalently (5.10).

IEEE: CIRCUITS AND SYSTEMS
Vol. 27 Issue 4
Au. Hung
Galley No. 16

IEEE: CIRCUITS AND SYSTEMS
Vol. 27 Issue 4
Au: Hung
Galley No. 17

Theorem 3: A necessary and sufficient condition for (5.15) to be realized by a passive reciprocal resistor–capacitor–transformer network is that there exist C^2-functions $\phi(x)$ and $W_r(v_c,u)$ such that

$$f(x,u) = -\left(\frac{\partial W_r(v_c,u)}{\partial v_c}\right)\bigg|_{v_c=\nabla\phi(x)}$$

$$h(x,u) = -\left(\frac{\partial W_r(v_c,u)}{\partial u}\right)'\bigg|_{v_c=\nabla\phi(x)} \qquad (5.16)$$

with $\phi(0)=0$, $\nabla\phi(0)=0$, $\nabla^2\phi(x)>0$, $W_r(0,0)=0$, $\partial W_r/\partial v_c(0,0)=0$, $\partial W_r/\partial u(0,0)=0$.and

$$\begin{bmatrix} \dfrac{\partial^2 W_r}{\partial v_c^2} & \dfrac{\partial^2 W_r}{\partial u \partial v_c} \\ \dfrac{\partial^2 W_r}{\partial v_c \partial u} & \dfrac{\partial^2 W_r}{\partial u^2} \end{bmatrix}$$

expressible in the form of the right side of (5.10), where H, defined in (5.3), satisfies (5.4) through (5.6).

Proof: Sufficiency: Suppose that $\phi(x)$ and $W_r(v_c,u)$ exist with the listed properties such that (5.16) are satisfied. It is obvious that a passive reciprocal RC network can be constructed .

Necessity: Conversely, if equations (5.15) are realized by a passive reciprocal RC network, its network equations have the form as described by (5.14). It is clear that the potential functions $\phi(x)$ and $W_r(v_c,u)$ satisfy (5.16).

For the synthesis problem, one must recover the appropriate functions $\phi(x)$ and $W_r(v_c,u)$ from $f(x,u)$ and $h(x,u)$. We shall now show how, first, the stored energy function $\phi(x)$ can be obtained and then how $W_r(v_c,u)$ can be obtained.

Similarly to the study of lossless reciprocal networks, the system described by (5.6) is assumed to be incrementally controllable, i.e., the linearized system derived from the system described by (5.6), which is

$$\Delta\dot{x} = \frac{\partial f}{\partial x}\Delta x + \frac{\partial f}{\partial u}\Delta u \quad \Delta y = \frac{\partial h}{\partial x}\Delta x + \frac{\partial h}{\partial u}\Delta u \quad (5.17)$$

is required to be completely controllable, i.e.,

$$V_c = \left[\frac{\partial f}{\partial u} \quad \frac{\partial f}{\partial x}\frac{\partial f}{\partial u} \quad \cdots \quad \left(\frac{\partial f}{\partial x}\right)^{n-1}\frac{\partial f}{\partial u}\right]$$

has full rank.

Lemma 3: Assume that the system described by (5.15) is incrementally controllable. Then the total stored energy function $\phi(x)$ may be uniquely determined from

$$\nabla^2\phi = (V_0' V_c')(V_c' V_c')^{-1} \qquad (5.18)$$

with $\phi(0)=0$, $\nabla\phi(0)=0$, where

$$V_0'\left[\left(\frac{\partial h}{\partial x}\right) \quad \left(\frac{\partial f}{\partial x}\right)'\left(\frac{\partial h}{\partial x}\right)' \quad \cdots \quad \left(\frac{\partial f}{\partial x}\right)'^{m+n-1}\left(\frac{\partial h}{\partial x}\right)'\right]$$

Proof: Equations (5.16) imply

$$\frac{\partial h}{\partial x} = -\frac{\partial^2 W_r}{\partial x \partial u} = -\frac{\partial^2 W_r}{\partial v_c \partial u}\frac{\partial v_c}{\partial x} = -\frac{\partial^2 W_r}{\partial v_c \partial u}\nabla^2\phi.$$

Since

$$\frac{\partial^2 W_r}{\partial v_c \partial u} = \left(\frac{\partial^2 W_r}{\partial u \partial v_c}\right)' = -\left(\frac{\partial f}{\partial u}\right)',$$

we obtain

$$\nabla^2\phi\left(\frac{\partial f}{\partial u}\right) = \left(\frac{\partial h}{\partial x}\right)'. \qquad (5.19)$$

In addition, we have

$$\frac{\partial f}{\partial x} = -\frac{\partial^2 W_r}{\partial_x \partial v_c} = -\frac{\partial^2 W_r}{\partial v_c^2}\frac{\partial v_c}{\partial x} = -\frac{\partial^2 W_r}{\partial v_c^2}\nabla^2\phi$$

$$\nabla^2\phi\left(\frac{\partial f}{\partial x}\right) = \left(\frac{\partial f}{\partial x}\right)'\nabla^2\phi. \qquad (5.20)$$

IEEE: CIRCUITS AND SYSTEMS
Vol. 27 Issue 4
Au: Huang
Galley No. 18

The conclusion of the lemma follows from (5.19) and (5.20) as in the case of lossless reciprocal networks.

Once the function $\phi(x)$ has been obtained, the potential function $W_r(v_c,u)$ has to be recovered. For this purpose, we need to assume incremental observability, which is equivalent to V_0 having full rank and in turn equivalent to $\nabla^2\phi(x)$ having full rank. But, as we know, this is consistent with an assumption that $v_c=\nabla\phi(x)$ is invertible, i.e., we have $x=q(v_c)$ for some function q. Now in the course or proving (5.19) and (5.20), we showed that

$$\begin{bmatrix} \dfrac{\partial^2 W_r}{\partial v_c^2} & \dfrac{\partial^2 W_r}{\partial u \partial v_c} \\ \dfrac{\partial^2 W_r}{\partial v_c \partial u} & \dfrac{\partial^2 W_r}{\partial u^2} \end{bmatrix} = \begin{bmatrix} -\left(\dfrac{\partial f}{\partial x}\right)(\nabla^2\phi)^{-1} & -\left(\dfrac{\partial f}{\partial u}\right) \\ -\left(\dfrac{\partial f}{\partial u}\right)' & -\left(\dfrac{\partial h}{\partial u}\right) \end{bmatrix}. \tag{5.21}$$

The right-hand side of the equality can be expressed as a function of v_c and u since $x=q(v_c)$. Then integration using the conditions $\partial W/\partial v_c(0,0)=0$, $\partial W_r/\partial u(0,0)$, $W_r(0,0)=0$ yields W_r.

C. Example

We shall synthesize the following network equations:

$$\dot{x}=f(x,u)=\begin{bmatrix} -\dfrac{4}{11}\left(x_1+\dfrac{x_1^3}{3}\right)-\dfrac{1}{11}\left(2x_2+\dfrac{x_2^3}{3}\right)+\dfrac{5}{11}u \\ -\dfrac{1}{11}\left(x_1+\dfrac{x_1^3}{3}\right)-\dfrac{3}{11}\left(2x_2+\dfrac{x_2^3}{3}\right)+\dfrac{4}{11}u \end{bmatrix} \tag{5.15}$$

$$y=\frac{5}{11}\left(x_1+\frac{x_1^3}{3}\right)+\frac{4}{11}\left(2x_2+\frac{x_2^3}{3}\right)+\frac{57}{11}u+u^3.$$

The incremental controllability and observability matrices V_c and V_0' can be determined as follows:

$$\frac{\partial f}{\partial u}=\begin{bmatrix} 5/11 \\ 4/11 \end{bmatrix}$$

$$\frac{\partial f}{\partial x}=\begin{bmatrix} -\dfrac{4}{11}(1+x_1^2) & -\dfrac{1}{11}(2+x_2^2) \\ -\dfrac{1}{11}(1+x_1^2) & -\dfrac{3}{11}(2+x_2^2) \end{bmatrix}$$

$$\left(\frac{\partial h}{\partial x}\right)'=\begin{bmatrix} \dfrac{5}{11}(1+x_1^2) \\ \dfrac{4}{11}(2+x_2^2) \end{bmatrix}$$

$$V_c=\left[\frac{\partial f}{\partial u}\;\frac{\partial f}{\partial x}\;\frac{\partial f}{\partial u}\right]$$

$$=\begin{bmatrix} 5/11 & -\dfrac{20}{121}(1+x_1^2)-\dfrac{4}{121}(2+x_2^2) \\ 4/11 & -\dfrac{5}{121}(1+x_1^2)-\dfrac{12}{121}(2+x_2^2) \end{bmatrix}$$

$$V_0'=\left[\left(\frac{\partial h}{\partial x}\right)'\left(\frac{\partial f}{\partial x}\right)'\left(\frac{\partial h}{\partial x}\right)'\right]$$

$$=\begin{bmatrix} \dfrac{5}{11}(1+x_1^2) & -\dfrac{20}{121}(1+x_1^2)^2-\dfrac{4}{121}(1+x_1^2)(2+x_2^2) \\ \dfrac{4}{11}(2+x_2^2) & -\dfrac{5}{121}(1+x_1^2)(2+x_2^2)-\dfrac{12}{121}(2+x_2^2)^2 \end{bmatrix}.$$

It is easily checked that V_c has full rank for all x_1,x_2; using Lemma 3, we obtain

$$\nabla^2\phi=V_0'V_c^{-1}=\begin{bmatrix} 1+x_1^2 & 0 \\ 0 & 2+x_2^2 \end{bmatrix}.$$

IEEE: CIRCUITS AND SYSTEMS
Vol. 27 Issue 4
Au: Hung
Galley No. 19

With $\phi(0)=0$ and $\nabla\phi(0)=0$, the corresponding energy function $\phi(x)$ is

$$\phi(x)=\frac{x_1^2}{2}+\frac{x_1^4}{12}+x_2^2+\frac{x_2^4}{12}$$

and

$$v_c=\nabla\phi(x)=\begin{bmatrix} x_1+x_1^3/3 \\ 2x_2+x_2^3/3 \end{bmatrix}. \tag{5.16}$$

The potential function $W_r(v_c,u)$ is also easily recovered. We have, using (5.12),

$$\begin{bmatrix} \dfrac{\partial^2 W_r}{\partial v_c^2} & \dfrac{\partial^2 W_r}{\partial u \partial v_c} \\ \dfrac{\partial^2 W_r}{\partial v_c \partial u} & \dfrac{\partial^2 W_r}{\partial u^2} \end{bmatrix}=\left[\begin{array}{cc:c} 4/11 & 1/11 & -5/11 \\ 1/11 & 3/11 & -4/11 \\ \hdashline -5/11 & -4/11 & -57/11-3u^2 \end{array}\right]$$

from which

$$\frac{\partial W_r(v_c,u)}{\partial v_c}=\begin{bmatrix} 4/11v_{c_1}+1/11v_{c_2} & -5/11u \\ 1/11v_{c_1}+3/11v_{c_2} & -4/11u \end{bmatrix},$$

and

$$\frac{\partial W_r(v_c,u)}{\partial u}=-5/11v_{c_1}-4/11v_{c_2}-57/11u-6u^3.$$

Observing (5.1) and (5.3), we see that the equations of the hybrid coupling network $\tilde{R}$ are

$$-i_c=\begin{bmatrix} \dfrac{4}{11}v_{c_1}+\dfrac{1}{11}v_{c_2} & -\dfrac{5}{11}u \\ \dfrac{1}{11}v_{c_1}+\dfrac{3}{11}v_{c_2} & -\dfrac{4}{11}u \end{bmatrix}$$

$$y=\frac{5}{11}v_{c_1}+\frac{4}{11}v_{c_2}+\frac{57}{11}u+6u^3. \tag{5.17}$$

It is not difficult to synthesize (5.17). One way is to observe that (5.17) are equivalent to

$$v_{c_1}=-3i_{c_1}+i_{c_2}+u$$
$$v_{c_2}=i_{c_1}-4i_{c_2}+u$$
$$y=-i_{c_1}-i_{c_2}+6u+u^3. \tag{5.18}$$

03700156 Together, (5.16) and (5.18) are synthesized by the network of Fig. 9. Fig. 9

If we proceed according to the second synthesis problem, viz., identifying R and T, we have for the hybrid equations of $\tilde{R}$, see (5.10),

$$\begin{bmatrix} \dfrac{\partial h_1}{\partial v_1} & \dfrac{\partial h_1}{\partial i_2} \\ \dfrac{\partial h_2}{\partial v_1} & \dfrac{\partial h_2}{\partial i_2} \end{bmatrix}=\left[\begin{array}{cc:c} 4/11 & 1/11 & -5/11 \\ 1/11 & 3/11 & -4/11 \\ \hdashline 5/11 & 4/11 & 57/11+3u^2 \end{array}\right].$$

From (5.10), we see that $T_{21}=[5/11 \quad 4/11]$, $T_{31}=i$, $T_{24}=I$, $T_{34}=0$ (this identifies T) and

$$H^{-1}=\left[\begin{array}{cc:cc} 4/11 & 1/11 & 0 & 0 \\ 1/11 & 3/11 & 0 & 0 \\ \hdashline 0 & 0 & \multicolumn{2}{c}{\dfrac{57}{11}+3u^2} \end{array}\right]$$

whence the equations of R become

$$v_3=\begin{bmatrix} 3 & -1 \\ -1 & 4 \end{bmatrix}i_3$$

$$v_4=\frac{57}{11}i_4+i_4^3.$$

VI. Conclusions

It has been shown how the synthesis problem for special types of nonlinear reciprocal networks such as lossless reciprocal networks or nonlinear reciprocal *RCT* networks can be reduced to the problem of determining an appropriate stored energy function of the corresponding network.

The problems solved are far less numerous and less important than those remaining. Among the latter, we can note the following: reciprocal synthesis of a *LRC* network, reciprocal synthesis of a lossless network with state-vector entries not equal to inductor fluxes or capacitor charges, and the developing of a property on the operator mapping the port inputs $u(\cdot)$ into port outputs $y(\cdot)$ of a reciprocal network which reflects that reciprocity.

IEEE: CIRCUITS AND SYSTEMS
Vol. 27 Issue 4
Au: Hung
Galley No. 20

References

[1] T. E. Stern. *Theory of Nonlinear Networks and Systems*. Reading, MA: Addison-Wesley, 1965.

[2] B. D. O. Anderson and P. J. Moylan, "Structure result for nonlinear passive systems," in *Int. Symp. on Operator Theory of Networks and Systems*, Montreal, Canada, Aug. 1975.

[3] S. Vongpanitlerd, "Reciprocal lossless synthesis via state-variable techniques," *IEEE Trans. Circuit Theory*, vol. CT-17, pp. 680–632, Nov. 1970.

[4] B. D. O. Anderson and S. Vongpanitlerd, *Network Analysis and Synthesis: A Modern Systems Theory Approach*. NJ: Prentice-Hall, 1973.

[5] B. D. O. Anderson and R. W. Newcomb, "Impedance Synthesis via state-space techniques," *Proc IEEE* vol. 115, pp. 928–936, July 1968.

[6] P. J. Moylan, "Implications of Passivity in a Class of Nonlinear Systems," *IEEE Trans. on Automat. Contr.*, vol. AC-19, pp. 373–381, Aug. 1974.

[7] V. A. Yakubovich, "Absolute Stability of nonlinear control in critical cases—Part I," *Automat. Remote Contr.*, vol. 24, pp. 273–282, 1963.—, "Absolute stability of nonlinear control in critical cases—Part II," *Automat. Remote Contr.*, pp. 655–688, 1963.

[8] R. E. Kalman, "On a new characterization of linear passive systems," in *Proc. 1st Allerton Conf. Circuit and System Theory*, Univ. of Illinois, Urbana, pp. 456–470, Nov. 1963.

[9] J. C. Willems, "Dissipative dynamical Systems, Part I: General Theory; Part II: Linear Systems with quadratic Supply Rates:," *Arch. Rational Mechanics and Anal.*, vol. 45, no. 5, pp. 321–393, 1972.

[10] L. O. Chua and Y. F. Lam, "A Theory of algebraic *n*-ports," *IEEE Trans. Circuit Theory*, vol. 20, pp. 370–381, July 1973.

[11] R. K. Brayton and J. K. Moser, "A Theory of nonlinear networks I and II," *Quart of Appl. Math.*, vol. 2, no. 2, pp. 1–33; pp. 81–104, July 1964.

[12] P. P. Varaija and J. P. Verma, "Equivalent nonlinear reciprocal networks, *IEEE Trans Circuit Theory*, vol. CT-18, pp. 214–217, March 1971.

[13] W. Millar, "Some general theorems for nonlinear systems possessing resistance," *Phil Mag.*, Ser. 7, vol. 42, no. 333, pp. 1150–1161, October 1951.

[14] C. Cherry, "Some general theorems for nonlinear systems possessing reactance," *Phil Mag.*, Ser. 7, vol. 42, no. 333, pp. 1161–1177, Oct. 1951.

[15] R. K. Brayton, "Nonlinear reciprocal networks," IBM Rep. RC2606 (12427), pp. 1–15, 1969.

[16] R. S. Palais, "Natural operations on differential forms," *Trans Amer. Math. Soc.*, vol. 92, pp. 125–141, July 1959.

[17] L. O. Chua and Y. F. Lam, "Global homeomorphism of vector-valued functions," *J. Math. Anal. Appl.*, vol. 39, no. 3, pp. 600–624, Sept. 1972.

[18] R. W. Newcomb, *Linear Multiport Synthesis*. New York: McGraw-Hill, 1966.

[19] R. W. Brockett, *Finite Dimensional Linear Systems*. New York: Wiley, 1970.

[20] C. A. Desoer and G. F. Oster, "Globally reciprocal stationary systems," *Int. J. Eng. Sci.*, vol. 11, pp. 141–155, 1973.

Nguyen Tan Hung was born in Binh Thuan, Vietnam, in 1952. He received the B.E. and M.E. degrees in electrical engineering from the University of Newcastle, Newcastle, Australia, in 1976 and 1977, respectively. He has also completed requirements for the Ph.D. degree in electrical engineering at the University of Newcastle.

He is currently a Design Engineer in the Power Electronics division of Warman International Limited, Sydney, Australia. His research interests are in multivariate systems control, nonlinear networks, and power electronics.

IEEE: CIRCUITS AND SYSTEMS
Vol. 27 Issue 4
Au: Hung
Galley No. 21

Brian D. O. Anderson (S'62–M'66–SM'74–F'75) was born in Sydney, Australia, in 1941. He received the B.S. degrees in pure mathematics and electrical engineering from the University of Sydney, Sydney, Australia, and the Ph.D. degree in electrical engineering from Stanford University, Stanford, CA.

Before joining the University of Newcastle in 1967, where he is a Professor in the Department of Electrical Engineering, he was a Faculty Member of the Department of Electrical Engineering at Stanford University and Staff Consultant at Vidar Corporation, Mountain View, CA. He had also held appointments as a Visiting Professor at Southern Methodist University, University of Massachusetts, University of California at Berkeley, and Stanford University. He was Head of the Department of Electrical Engineering at the University of Newcastle from 1967 to 1975. He is coauthor of five books, *Linear Optimal Control*, *Network Analysis and Synthesis*, *Foundation of Systems Theory*, and most recently *Singular Optimal Control*, with D. J. Clements, and *Optimal Filtering*, with J. B. Moore. His research interests are in control, communication systems, and networks, and he has written many papers in these areas. From 1971 to 1977, he was a member of the Australian Research Grants Committee and since 1977 has been a member of the Australian Science and Technology Council.

He is a Fellow of the Australian Academy of Science and Institution of Engineers, Australia, and a member of Sigma Xi and Siam.

Manuscript received July 18, 1977; revised May 1, 1979. This work was supported by the Australian Research Grants Committee.
The authors are with the Department of Electrical Engineering, University of Newcastle, New South Wales 2308, Australia.

---

[1]An equivalent requirement is that T_e has a hybrid matrix description with excitation variables i_1, v_2, and u. Of course, T_e always has a hybrid matrix description with some selection of variables [18], but the choice here is important. The effect of the assumption is to disallow the possibility that y can depend instantaneously on, inter alia, $\dot{u}$—a situation that could arise if there was, in effect, a capacitor in parallel with the input. Such a topology could be handled by a new choice of input and output variables, i.e., some or all of the entries of u become entries of y, and vice versa.

---

[2]The state vector of the network state equations should be distinguished from the state function of reciprocal algebraic multiports of the previous section.

---

[3]We are not linearizing around a trajectory, so that J_p and J_n are not time-varying.

03700170